### **Xavier Pennec**

Asclepios team, INRIA Sophia-Antipolis – Mediterranee, France

With indebting contributions from: Pierre Fillard, Vincent Arsigny, Jean-Marc Peyrat, Thomas Yeo, Tom Vercauteren and others...

### Diffusion analysis using a Riemannian Framework



MICCAI Diffusion MRI Tutorial New York, Sept. 6th 2008



## **Diffusion Tensor Imaging**

Covariance of the Brownian motion of water -> Architecture of axonal fibers

#### Very noisy data

- Tensor image processing
  - Robust estimation
  - Filtering, regularization
  - Interpolation / extrapolation
- □ Information extraction (fibers)

#### Symmetric positive definite matrices

- Convex operations are stable
  - mean, interpolation
- □ More complex operations are not
  - PDEs, gradient descent...



Diffusion Tensor Filed (slice of a 3D volume)

### The Tensor Space is not a Vector Space

**Tensors = Space of positive definite matrices:** 



#### Matrices with null eigenvalues are reachable in a finite time

Null and negatives eigenvalues are not physical

Intrinsic computing on Manifold-valued images?

### Riemannian geometric approaches

#### **Shape spaces & directional statistics**

□ [Kendall StatSci 89, Small 96, Dryden & Mardia 98]

#### Numerical integration of dynamical systems

□ [Helmke & Moore 1994, Hairer et al 2002]

□ Matrix Lie groups [Owren BIT 2000, Mahony JGO 2002]

#### Information Geometry (statistical manifolds)

- □ [Amari 1990 & 2000, Kass & Vos 1997]
- □ [Oller Annals Stat. 1995, Battacharya Annals Stat. 2003 & 2005]

### **Statistics for Computer vision**

- Rigid body transformations [Pennec PhD96]
- □ General Riemannian manifolds [Pennec JMIV98, NSIP99, JMIV06]
- □ PGA for M-Reps [Fletcher IPMI03, TMI04]
- Planar curves [Klassen & Srivastava PAMI 2003]

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### The geometric framework: Riemannian Manifolds

#### **Riemannian metric :**

- Dot product on tangent space
- □ Speed, length of a curve
- Distance and geodesics
  - Closed form for simple metrics/manifolds
  - Optimization for more complex

#### Exponential map (Normal coord. syst.) :

- □ Geodesic shooting:  $Exp_x(v) = \gamma_{(x,v)}(1)$
- □ Log: find vector to shoot right

#### Unfolding (Log<sub>x</sub>), folding (Exp<sub>x</sub>)

Operator	Euclidean space	Riemannian manifold
Subtraction	$\overrightarrow{xy} = y - x$	$\overrightarrow{xy} = Log_x(y)$
Addition	$y = x + \overrightarrow{xy}$	$y = Exp_x(\overrightarrow{xy})$
Distance	$\operatorname{dist}\left(x,y\right) = \left\ y-x\right\ $	$\operatorname{dist}(x, y) = \left\  \overrightarrow{xy} \right\ _{x}$
Gradient descent	$x_{t+\varepsilon} = x_t - \varepsilon \nabla C(x_t)$	$x_{t+\varepsilon} = Exp_{x_t}(-\varepsilon \nabla C(x_t))$



Affine Invariant Metrics on TensorsAction of the Linear Group GL
$$A * \Sigma = A.\Sigma.A^T$$
Invariant distance $dist(A * \Sigma_1, A * \Sigma_2) = dist(\Sigma_1, \Sigma_2)$ Invariant metric $\langle W_1 | W_2 \rangle_{\Sigma} \stackrel{def}{=} \langle \Sigma^{-1/2} * W_1, \Sigma^{-1/2} * W_2 \rangle_{Id}$  $\Box$  All rotationally invariant scalar products at identity: $\langle W_1 | W_2 \rangle_{Id} \stackrel{def}{=} \operatorname{Tr}(W_1^T W_2) + \beta \operatorname{Tr}(W_1).\operatorname{Tr}(W_2) \quad (\beta > -1/n)$  $\Box$  Geodesics $\exp_{\Sigma}(\widetilde{\Sigma \Psi}) = \Sigma^{1/2} \exp(\Sigma^{-1/2}.\widetilde{\Sigma \Psi}.\Sigma^{-1/2})\Sigma^{1/2}$  $\Box$  Distance $dist(\Sigma, \Psi)^2 = \langle \widetilde{\Sigma \Psi} | \widetilde{\Sigma \Psi} \rangle_{\Sigma} = \left\| \log(\Sigma^{-1/2}.\Psi.\Sigma^{-1/2}) \right\|_{L_2}^2$ 

### Linear vs. Riemannian Interpolation: walking along geodesics



X Pennec, P.Fillard, N.Ayache: Riemannian Tensor Computing, IJCV 66(1), Jan 2006 <sup>8</sup>

### Affine Invariant Metrics on Tensors

 $\left\|W\right\|_{\Sigma}^{2} = \operatorname{Tr}\left(W.\Sigma^{-1}W\Sigma^{-1}\right) + \beta \operatorname{Tr}\left(W\Sigma^{-1}\right)^{2} \quad (\beta > -1/n)$ 

#### **Statistics on Riemannian spaces**

- □ [Pennec, Fillard, Ayache, IJCV 66(1), Jan 2006 / INRIA RR-5255, 2004]
- D PGA on tensors [Fletcher & Joshi CVMIA04, SigPro 87(2) 2007]

#### **Space of Gaussian distributions**

- Fisher information metric [Burbea & Rao J. Multivar Anal 12 1982, Skovgaard Scand J. Stat 11 1984, Calvo & Oller Stat & Dec. 9 1991]
- DTI segmentation [Lenglet RR04 & JMIV 25(3) 2006]

#### **Geometric means**

- □ Covariance matrices in computer vision [Forstner TechReport 1999]
- □ Math. properties [Moakher SIAM J. Matrix Anal App 26(3) 2004]
- □ Geodesic Anisotropy [Batchelor MRM 53 2005]

#### **Homogeneous Embedding**

 $\square$   $\beta$ =-1/(n+1) [Lovric & Min-Oo, J. Multivar Anal 74(1), 2000]

### First statistical tools: moments

#### Fréchet / Karcher mean:

□ Minimize the variance  $\sigma^2(y) = E[\operatorname{dist}(y, \mathbf{x})^2] = \frac{1}{n} \sum_i \operatorname{dist}(y, x_i)^2$ □ Optimum:  $E[\overrightarrow{\mathbf{x}\mathbf{x}}] = \frac{1}{n} \sum_i \overrightarrow{\mathbf{x}x_i} = 0$  [P(C) = 0]

Algorithm: Geodesic marching

$$\overline{\mathbf{x}}_{t+1} = \exp_{\overline{\mathbf{x}}_t}(v) \quad \text{avec} \quad v = \mathrm{E}\left[\overrightarrow{\mathbf{y}\mathbf{x}}\right] = \frac{1}{n} \sum_i \overrightarrow{\overline{\mathbf{x}}x_i}$$

**Covariance and higher orders** 

$$\Sigma_{\mathbf{x}\mathbf{x}} = \mathbf{E}\left[\left(\overrightarrow{\overline{\mathbf{x}}\mathbf{x}}\right)\left(\left(\overrightarrow{\overline{\mathbf{x}}\mathbf{x}}\right)^{\mathrm{T}}\right)\right] = \frac{1}{n}\sum_{i}\overrightarrow{\overline{\mathbf{x}}z}.\overrightarrow{\overline{\mathbf{x}}x_{i}}^{\mathrm{T}}$$

[Pennec, NSIP'99, JMIV06]

[Oller et Corcuera, AnnIs Stat 1995]



### A Statistical Atlas of the Cardiac Fiber Structure

[J.M. Peyrat, et al., MICCAI'06, TMI 26(11), 2007]

#### Database

- □ 7 canine hearts from JHU
- Anatomical MRI and DTI

#### Method

- Normalization based on aMRIs
- Log-Euclidean statistics of Tensors

Norm covariance

Eigenvalues covariance (1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>)

Eigenvectors orientation covariance (around 1st, 2nd, 3rd)





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### Tensor interpolation

 $\Sigma(t) = \exp_{\Sigma_1}(t \overrightarrow{\Sigma_1 \Sigma_2})$ 

 $\Sigma(x) = \min_{\Sigma} \sum w_i(x) \, dist(\Sigma, \Sigma_i)^2$ 

#### **Geodesic walking in 1D**



Weighted mean in general



## Gaussian filtering: Gaussian weighted mean $\Sigma(x) = \arg\min_{\Sigma} \sum_{i=1}^{n} G_{\sigma}(x - x_{i}) \ dist(\Sigma, \Sigma_{i})^{2}$





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### **PDE for filtering and diffusion**

#### Harmonic regularization

$$C(\Sigma) = \int_{\Omega} \left\| \nabla \Sigma(x) \right\|_{\Sigma(x)}^2 dx$$

Gradient: manifold Laplacian

$$\Delta \Sigma(x) = \sum_{i} \partial_{i}^{2} \Sigma - \sum_{i} (\partial_{i} \Sigma) \Sigma^{(-1)} (\partial_{i} \Sigma)$$

 Intrinsic numerical scheme thanks to exponential chart

$$\Delta \Sigma(x) = \sum_{u} \frac{\overline{\Sigma(x)\Sigma(x+u)}}{\|u\|^2} + O(\|u\|^2)$$

□ Integration with geodesic marching  $\Sigma_{t+1}(x) = \exp_{\Sigma_t(x)} \left(-\varepsilon \nabla C(\Sigma)(x)\right)$ 

### **Anisotropic regularization**

- □ [Perona-Malik 90 / Gerig 92]
- □ Phi functions formalism [ Deriche / Faugeras 1996 ]

### Isotropic vs. Anisotropic Diffusion

$$C(\Sigma) = \int \left\| \nabla \Sigma(x) \right\|_{\Sigma}^2 dx$$

$$C(\Sigma) = \int \phi \left( \left\| \nabla \Sigma(x) \right\|_{\Sigma} \right) dx$$
$$\phi(x) = \exp(-x^2 / \kappa^2)$$



Isotropic



Anisotropic

### Anisotropic Diffusion Euclidean vs Riemannian



**Extrapolation by Diffusion (Restoration)**  

$$C(\Sigma) = \frac{1}{2} \int_{\Omega} \sum_{i=1}^{n} G_{\sigma}(x - x_{i}) dist(\Sigma(x), \Sigma_{i})^{2} dx + \frac{\lambda}{2} \int_{\Omega} \left\| \nabla \Sigma(x) \right\|_{\Sigma(x)}^{2}$$

$$\nabla C(\Sigma)(x) = -\sum_{i=1}^{n} G_{\sigma}(x - x_{i}) \overline{\Sigma(x)\Sigma_{i}} - \lambda(\Delta \Sigma)(x)$$



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### Log Euclidean Metric on Tensors

### Exp/Log: global diffeomorphism Tensors/sym. matrices

- Vector space structure carried from the tangent space to the manifold
  - Log. product
  - Log scalar product
  - Bi-invariant metric

#### **Properties**

- $\Sigma_1 \otimes \Sigma_2 \equiv \exp(\log(\Sigma_1) + \log(\Sigma_2))$  $\alpha \bullet \Sigma \equiv \exp(\alpha \log(\Sigma)) = \Sigma^{\alpha}$
- $dist(\Sigma_1, \Sigma_2)^2 \equiv \left\| \log(\Sigma_1) \log(\Sigma_2) \right\|^2$
- Invariance by the action of similarity transformations only
- Very simple algorithmic framework

### Log Euclidean vs Affine invariant

- □ Both means are geometric (vs arithmetic for Euclidean)
- □ Log Euclidean slightly more anisotropic
- □ Speedup ratio: 7 (aniso. filtering) to >50 (interp.)



Euclidean Af**tig**eEiuovlædiæemt [ Arsigny, Fillard, Pennec, Ayache, MICCAI 2005, MRM'06 ]

### Log Euclidean vs Affine invariant

#### **Real DTI images: anisotropic filtering**

- □ Both means are geometric (vs arithmetic for Euclidean)
- Log Euclidean slightly more anisotropic but the difference is not significant
- □ Speedup ratio: 7 (aniso. filtering) to >50 (interp.)



Original Euclidean Log-Euclidean Diff. LE/affine (x100) [Arsigny, Fillard, Pennec, Ayache, MICCAI 2005, MRM'06]

### **Comparison of Metrics**

	Euclidean	Affine Invariant	Log- Euclidean
Null/Negative eigenvalues	Reachable	Unreachable!	Unreachable!
Invariance	Rotation	Affine transforms	Similarity
Swelling effect	Yes	No	No
Computation al burden	Low	Important	Low

## **A metric for all applications?** Structure tensor (guide for diffusion) $\Sigma_{\sigma}(x) = G_{\sigma} * (\nabla I \nabla I^{t})$



A null eigenvalue is physically OK (perfect straight edge) **Need to change the metric?** 

[Fillard, Arsigny, Ayache, Pennec, DSSCV'05]

### Geodesic shooting in tensors spaces



### Some references on other metrics

#### Log-Euclidean

□ [Arsigny, MICCAI 2005 & MRM 56(2), 2006]

#### **Square root metrics**

- □ Cholesky [Wang Vemuri et al, IPMI'03, TMI 23(8) 2004.]
- □ Size and shape space [Dryden, Koloydenko & Zhou, 2008]

#### **Non Riemannian distances**

- □ J-Divergence [Wang & Vemuri, TMI 24(10), 2005]
- □ Geodesic Loxodromes [Kindlmann et al. MICCAI 2007]

#### 4th order tensors

□ [Gosh, Descoteau & Deriche MICCAI'08]

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### **DTI Estimation from DWI**





 $S_i = S_0 \exp\left(-bg_i^T Dg_i\right)$ 

Stejskal & Tanner diffusion equation



**Diffusion Tensor Field** 

Baseline + at least 6 DWI

### **DTI Estimation from DWI**

#### **Maximum Likelihood Estimation**

- □ ML with Log-Gaussian noise:
  - linear system on the log-Images
- ML with Gaussian noise on the MRIs:
  - non-linear optimization

#### **Actual MRI Noise**

- □ Gaussian on the complex signal
- □ Rician on the amplitude
- This leads to a bias for low SNRs [Sijbers, TMI 1998]

$$S_i = S_0 \exp\left(-b \mathbf{g}_i^T \Sigma \mathbf{g}_i\right) + Noise$$

$$\arg\min_{D}\sum_{i} \left(\log(S_{i}/S_{0}) + bg_{i}^{T}Dg_{i}\right)^{2}$$

$$\arg\min_{D}\sum_{i} \left(S_{i} - S_{0} \exp(-bg_{i}^{T}Dg_{i})\right)^{2}$$

$$\hat{S}_i = \sqrt{(S_i + N_1(0, \sigma))^2 + N_2(0, \sigma)^2}$$

$$E[\hat{S}_i] \approx E[S_i] + \frac{\sigma^2}{2S_i}$$

### **DTI Estimation: A Few References**

#### **Standard log-Gaussian estimation:**

> [Westin et al., MedIA 2002]

#### Robust estimation on the log of the signal:

> [Tschumperlé et al., ISBI'04]

#### **Robust estimation on the signal itself:**

> [Chang et al., MRM'05]

#### Joint Estimation / Regularization from the complex DW signal:

> Anisotropic diffusion on Choleski factors [Wang & Vemuri, TMI'04]

#### **Estimation with a Rician noise model:**

- Smoothing DWI before estimation [Basu & Fletcher, MICCAI 2006]
- ML (MMSE) [Aja-Fernández, Alberola-López, Westin, TMI 2008]
- > MAP with log-Euclidean prior [Fillard et al., ISBI 2006, TMI 2007]

### MAP Estimation with a Rician Noise Model

Minimize an energy functional:

$$E(L) = \underbrace{Sim(L)}_{+\lambda} \underbrace{Reg(L)}_{+\lambda}$$

Data fidelity Smoothing term term

Maximum Likelihood estimator for Rician noise:

$$Sim(L) = -\sum_{i=1}^{N} \log\left(p\left(\hat{S}_i / S_i\right)\right) \qquad p\left(\hat{S}_i / S_i\right) = \frac{\hat{S}_i}{\sigma^2} \exp\left(-\frac{\hat{S}_i^2 + S_i(L)^2}{2\sigma^2}\right) I_0\left(\frac{S_i(L)\hat{S}_i}{\sigma^2}\right)$$

Anisotropic Log-Euclidean spatial prior

 $Reg(L) = \int \Phi\left( \left\| \nabla \Sigma(x) \right\|_{\Sigma(x)}^2 \right) dx$ 

#### **Gradient descent in Log-Euclidean space**

[Fillard, Arsigny, Pennec, Ayache ISBI'06, TMI 26(11) 2007]

\_ \_ \_

#### Standard estimation Standard + LE spatial Initial

Synthetic Data

#### **Rician ML**

**Rician MAP** 

prior

### **Results on Clinical DTI of the Brain with 7 directions**



[Fillard, Arsigny, Pennec, Ayache ISBI'06, TMI 26(11) 2007 ] <sup>37</sup>

### Impact on Fiber Tracking



### **Clinical DTI of the spinal cord**

FA Estimated tensors Standard ML Rician MAP Rician

[Fillard, Arsigny, Pennec, Ayache ISBI'06, TMI 26(11) 2007 ] <sup>39</sup>

### Clinical DTI of the spinal cord: fiber tracking



Standard

**MAP** Rician

[Fillard, Arsigny, Pennec, Ayache ISBI'06, TMI 26(11) 2007]

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### **DTI Registration Challenges**

#### **Similarity metric:**

□ Tensor comparison (distance)

 $C(\phi) = \int dist^2 \left( \Sigma_1(x), (\phi * \Sigma_2)(x) \right)$ 

### **Deforming tensor images**

- Tensor interpolation (resampling)
  - Local linear approximation using the Jacobian:  $\phi(x+v) \approx \phi(x) + J(x).v$  with  $J(x) = D\phi(x)$
  - Affine action:  $J^*\Sigma = J.\Sigma.J^t$  does not preserve eigenvalues
- □ Tensor re-orientation [Alexander TMI 20(11) 2001]:  $J^*\Sigma = R(J).\Sigma.R(J)^t$ 
  - Finite-Strain (FS): Clostest rotation  $R(J) = (J. J^t)^{-\frac{1}{2}} J$
  - Preservation of Principal Directions (PPD)



### **References on Tensor registration**

#### Image metric based on Transformation Invariant Features

□ No reorientation [Guimond 02, Leemans 05, Park 05, Ziyan'07]

#### **Euclidean metric on the full tensor**

Preservation of Principal Directions

- Elastic, PPD with approx grad (R(J) not differentiated) [Alexander and Gee CVIU 77(2), 2000]
- LDDMM with exact PPD differential [Cao et al MMBIA 2006]
- □ Finite-Strain
  - Parameters are locally affine transformation [Zhang et al. MedIA 10(5) 2006 & TMI 26(11) 2007]

### DT-REFinD: Diffusion Tensor Registration with Exact Finite-Strain Differential

[Thomas Yeo, et al. DTI Registration with Exact Finite-Strain Differential. ISBI'08]

#### **Tensor interpolation/metric**

□ Euclidean and Log-Euclidean (Arsigny '06)

#### **Tensor reorientation**

□ Finite Strain with Exact Differential

#### **Deformation model**

- □ Iterated one parameter diffeomorphisms
- Diffeomorphic Demons [Vercauteren MICCAI'07]

#### **Fast and accurate**

- □ 15 minutes,128x128x60, Xeon 3.2GHz
- □ Better tensor alignment

### Exact vs approximate FS gradient



\* DTI data courtesy of Denis Ducreux, MD, Bicêtre Hospital, Paris, France





Some results

T1 + Activation map + fibers



Corpus callosum + cingulum

Corticospinal tract and thalamo cortical connections



## **Conclusion & Perspectives**

### The Riemannian computing framework

□ Integral or sum in M: minimize an intrinsic functional

- Fréchet / Karcher mean
- Filtering, convolution through weighted means
- □ The exponential chart is the basic tool
  - Gradient descent is geodesic walking
  - Intrinsic numerical scheme for Laplace Beltrami

#### **Choice of the metric**

- □ Log-Euclidean (faster than affine invariant) for tensors
- Cholesky or Procruste for rank deficient tensors?
- Design new metrics for HARDI and higher order tensors?

### **Conclusion & Perspectives**

#### A consistent set of tools for

- Tensor Estimation
- Tensor Registration
- Statistical Atlas building

#### **Computational anatomy challenges**

Probing the information highways of the brain with DMRI
 From tensors / HARDI to fiber statistics?

# **Thank You!**





Riemannian DT-MRI processing with MedINRIA: http://www.inria.fr/sophia/asclepios/software/MedINRIA.

Special thanks to Pierre Fillard for most of the illustrations!

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