3D ANATOMICAL VARIABILITY ASSESSMENT OF THE SCOLIOTIC SPINE USING STATISTICS ON LIE GROUPS

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ABSTRACT

We present a method to analyse the variability of the spine shape using rigid transforms. The spine was expressed as a set of rigid transforms that superpose local coordinates systems of neighbouring vertebrae. Those transforms were computed from anatomical landmarks reconstructed in 3D using two radiographs. Since rigid transforms do not belong to a vector space, conventional mean and covariance could not be applied. The Fréchet means and a generalized covariance computed in the exponential chart of the Fréchet mean were used instead. Those statistics were computed for each intervertebral transforms of a group of 307 untreated scoliotic patients. The variability of inter-vertebral transforms was found to be inhomogeneous (lumbar vertebrae were more variable than the thoracic ones) and anisotropic (with maximal rotational variability in the coronal plane and maximal translational variability in the axial direction).

1. INTRODUCTION

The shape of a healthy spine does not vary much from a healthy subject to another. However, there are pathological conditions that can induce a deformation of the spine. Such conditions greatly increase the variability of the spine shape. Idiopathic scoliosis is one of those conditions; it is generally diagnosed soon in the adolescence and its cause remains unknown.

In order to understand the disease and to document the morphology of the scoliotic spine, studies were conducted about the positions and orientations of vertebrae in scoliotic patients. For instance, Ghanem et al. [1] and Sawatzky et al. [2] studied the position and orientation of the vertebrae next to the spinal curve apex using optoelectronic measurements during a surgery. Petit et al. [3] studied the inter-vertebral rigid transforms modifications using radiographs taken before and after a corrective surgery. Those studies were mainly concerned with mean positions and orientations and did not study the variability of the spine shape. However, the variability is important because it is likely to be greater in some directions which might offer insights about the progression mechanisms of the disease and lead to the development of better orthopaedic treatments. Furthermore, the integration of a variability model of the whole spine would improve 2D-3D registration algorithms. Currently most methods use statistical models of isolated vertebrae [4] or *ad hoc* symbolic constraints on the whole spine shape [5]. A variability model (mean and dispersion) of the whole spine shape would allow 2D-3D registration algorithms to better cope with the aperture problem that limits the precision of those methods.

However, there are practical and theoretical limitations that had prevented variability studies of the spine. First of all, in order to compute statistics and to generalize those on the population of scoliotic patients, it is necessary to use a large sample of patients. Moreover, the 3D measurements have to be taken while the patient assumes his "natural" standing posture (because a large proportion of the deformation would be lost if the patient has to lie down [6]). Finally, mathematical and computational tools need to be developed to properly study the geometric variability of the spine because conventional statistical methods usually apply only in vector spaces, while rigid transforms naturally belong to a Lie group.

The contributions for this paper are: to introduce a new model of the variability for spine shapes based on statistics on Lie groups, to propose a 3D visualization method of this variability and, last but not least, to present the resulting variability model computed using a large group of scoliotic patients.

2. MATERIAL AND METHODS

2.1. Inter-Vertebral Transforms Computation from Bi-Planar Radiographs

Multi-planar radiography is a simple technique where two (or more) calibrated radiographs of a patient are taken to calculate the 3D coordinates of anatomical landmarks using a triangulation algorithm. It is one of the few imaging modalities that can be used to digitize the three-dimensional anatomy of the spine when the patient is standing up. Furthermore, biplanar radiography of scoliotic patients is routinely performed

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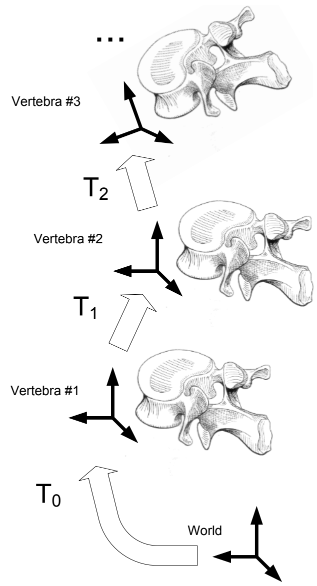

Fig. 1. The spine expressed as a rigid models assembly

at Sainte-Justine hospital. Thus, a sufficient amount of data is available for analysis.

In the case of bi-planar radiography of the spine, six anatomical landmarks are identified on each vertebra from T1 (first thoracic vertebra) to L5 (last lumbar vertebra) on a posterior-anterior and a lateral radiograph. The 3D coordinates of the landmarks are then triangulated. Moreover, the deformation of a high-resolution template using dual kriging yields 16 additional reconstructed landmarks. The accuracy of this method was previously established to 2.6mm [7].

Once the landmarks are reconstructed in 3D, each vertebra is rigidly registered to its first upper neighbour and the resulting rigid transforms are recorded. By doing so, the spine is represented by a set of rigid transforms (see the figure 1). It is this set of inter-vertebral transforms that will be used to compute the mean and covariance of the spine shape.

2.2. Centrality and Dispersion Measures in Lie Groups

Scalar multiplication and addition are not defined on rigid transforms therefore traditional mean and covariance cannot be computed. However, Riemannian geometry can be used to generalized those concepts. This approach was previously used to perform PCA on m-reps (medial axis representations) [8] and to propose a framework for probability and statistics on Riemannian manifolds [9]. It turns out that the generalisation of the classical mean and covariance can be computed if one knows the exponential and logarithmic map associated with a Riemannian metric on a manifold. This framework can be applied to rigid transforms since they belong to a Lie group (which is differentiable manifold).

2.2.1. Fréchet means

When given a distance, a generalization of the usual mean can be obtained by defining the mean as the element μ of a manifold $\mathcal{M}$ that minimizes the sum of the distances with a set of elements $x_{0\ldots N}$ of the same manifold $\mathcal{M}$.

$$\mu = \arg\min_{x\in\mathcal{M}} \sum_{i=0}^{N} d(x, x_i)^2$$

This generalization of the mean, called the Fréchet mean, can be computed using a simple iterative scheme on a Lie group provided with a left invariant distance. This procedure is obtained by performing gradient descent on the distance sum and is expressed by the following recurrent equation :

$$\mu_{n+1} = \mu_n \mathrm{Exp}\left(\frac{1}{N}\sum_{i=0}^{N} \mathrm{Log}(\mu_n^{-1} x_i)\right) \quad (1)$$

The functions Exp and Log are respectively the exponential map and the log map associated with the distance $d(x, y)$. The exponential map projects an element of the tangent plane $T_x\mathcal{M}$ on the manifold $\mathcal{M}$ and the log map is the inverse function.

2.2.2. Generalized Variance and Covariance

The variance (as it is usually defined on vector spaces) is the expectation of the L_2 norm of the difference between the mean and the measures. An intuitive generalization of the variance on Riemannian manifolds is thus given by the expectation of a distance.

$$\sigma^2 = \frac{1}{N}\sum_{i=0}^{N} d(\mu, x_i)^2 \quad (2)$$

In addition to the variance, we would like to have a dispersion measure that is directional because the anatomical variability is expected to be greater in some directions. The covariance is usually defined as the expectation of the matricial product of the vectors from the mean to the elements on which the covariance is computed. A similar definition for Lie groups would be to compute the expectation in the tangent plane of the mean using the log map.

$$\Sigma = \frac{1}{N}\sum_{i=0}^{N} \mathrm{Log}(\mu^{-1}x)\mathrm{Log}(\mu^{-1}x)^T \quad (3)$$

2.2.3. The case of Rigid Transforms

A rigid transform is the combination of a rotation and a translation. The action of a rigid transform on a point is usually written as $y = Rx + t$ where $R \in \mathcal{SO}^3$ and $x, y, t \in \Re^3$. Thus, a simple representation of a rigid transform would be

$T = \{R, t\}$. Using this representation composition and inversion operations have simple forms (respectively, $T_1 \circ T_2 = \{R_1 R_2, R_1 t_2 + t_1\}$ and $T^{-1} = \{R^T, -R^T t\}$).

Another way to represent a rigid transformation is to use a rotation vector instead of the rotation matrix. The rotation vector representation is based on the fact that a 3D rotation can be fully described by an axis of rotation supported by a unit vector n and an angle of rotation θ. The rotation vector r is defined as the product of n and θ. So we have a representation $\vec{T} = \{r, t\} = \{\theta n, t\}$ that we called the rigid vector.

The conversion between the two representations is simple since the rotation vector can be converted into a rotation matrix using the Rodrigues equation :

$$\mathbf{R} = \mathbf{I} + \sin(\theta).\mathbf{S_n} + (1 - \cos(\theta)).\mathbf{S_n}^2$$

$$\text{with :} \quad \mathbf{S_n} = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & n_x \\ -n_y & n_x & 0 \end{bmatrix}$$

And the inverse map (from a rotation matrix to a rotation vector) is given by the following equations :

$$\theta = \arccos(\frac{Tr(\mathbf{R}) - 1}{2}) \quad \text{and} \quad \mathbf{S_n} = \frac{\mathbf{R} - \mathbf{R}^T}{2\sin(\theta)} \tag{4}$$

Using the rotation vector representation, we can easily define a left-invariant distance between two rigid transformations.

$$d(\vec{T_1}, \vec{T_2}) = N_\lambda(\vec{T_2}^{-1} \circ \vec{T_1}) \tag{5}$$
$$\text{with :} \quad N_\lambda(\vec{T})^2 = N_\lambda(\{r, t\})^2 = \|r\|^2 + \|\lambda t\|^2$$

The parameter λ is a real number that controls the relative weight of the translation and rotation. Because the rotation vector and the translation do not have the same units it can be though as a unit conversion constant.

It can be demonstrated that the exponential and log map associated with the distance of equation 5 are the mappings (up to a scale) between the rigid vector and the combination of the rotation matrix and the translation vector [10].

$$\mathrm{Exp}(\vec{T}) = \left| \begin{array}{l} \mathbf{R}(r) \\ \lambda^{-1} t \end{array} \right. \quad \text{and} \quad \mathrm{Log}(T) = \left| \begin{array}{l} r(\mathbf{R}) \\ \lambda t \end{array} \right.$$

Using the functions Exp and Log defined on rigid transforms it is possible to use equations 1 and 3 to compute the mean and covariance of the rigid transformations resulting from the registration of neighbouring vertebrae.

The generalized covariance matrix associated with a rigid transform is a six by six matrix. Thus, an intuitive visualization of the whole covariance matrix is difficult. However, the upper left and lower right quarters of this matrix are three by three tensors and can easily be visualized in 3D using an ellipsoid. The principal axes of these ellipsoids are the eigenvectors scaled by the corresponding eigenvalues. The extent of the first ellipsoid (associated with the rotation) in a given direction is then the angular variability in the plane perpendicular to the chosen direction and the extent of the second ellipsoid (associated with the translation) in a given direction is the translational variability along that direction.

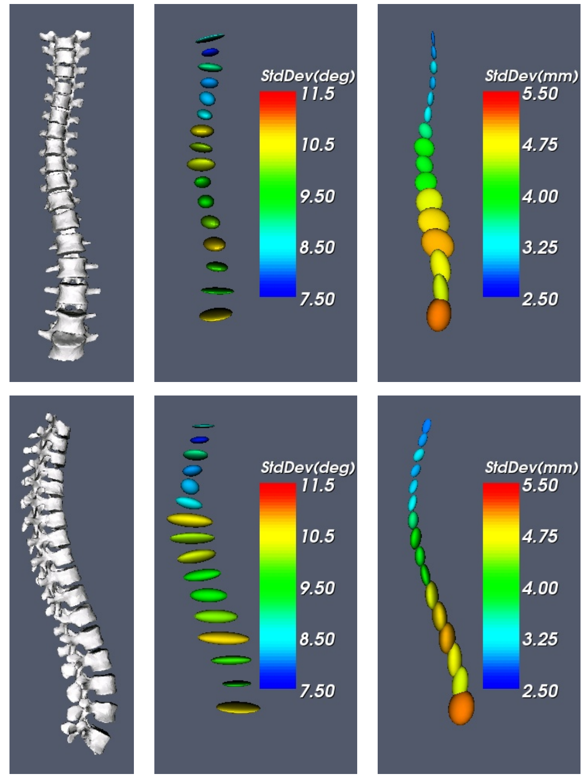

Fig. 2. Statistical spine model. From left to right: mean spine model, rotation and translation covariance. Top: frontal view. Bottom: sagittal view.

Because, the first tensor is the covariance of the rotation and the second tensor is the covariance of the translation this visualisation is quite intuitive and can be understood by people without strong mathematical backgrounds (such as physicians). The price of this visualisation is that the coupling between the rotation and the translation is lost during the visualisation process. In spite of that, preliminary tests indicated that, for the specific case of the inter-vertebral transforms, the amount of variance explained by this coupling is small compared to the one of the rotation and of the translation.

3. RESULTS AND DISCUSSION

3.1. Statistical Model of Scoliotics Patients

The method described in the previous section was applied to a group of 307 scoliotic patients of the Sainte-Justine Hospital. The selection of the patients of this group was based on the availability of the radiographs and on the absence of an underlying neuromuscular disease. The age, sex and growth stage were not used in the selection. The variability observed is predominantly associated with anatomical variability but it also includes variability caused by other factors such as posture and landmarks reconstruction error.

The mean spine shape and the variability are illustrated by figure 2, where it can be observed that the mean shape has curvatures in the lateral and frontal plane. The curvatures in the lateral plane correspond to healthy kyphosis and lordisis, but the light curve in the frontal plane is not part of the normal anatomy of the spine and is caused by scoliosis. It is also interesting to note that the curve is on the right side because there is more right thoracic curves than left thoracic curves among scoliotic patients. The variability is also inhomogeneous (it varies from a vertebra to another) and anisotropic (stronger variability in some directions). The strongest translational variability is found along the axial direction and one can also observe from figure 2 that the main extension of the rotation vector covariance ellipsoid is along the antero-posterior axis, which indicates that the main rotation variability is around this axis (as it could be expected for scoliotis).

3.2. Propagation of the Landmark Reconstruction Error

The anatomical landmarks reconstruction error induces variability on inter-vertebral transforms. However, we are only interested in the variability that is intrinsic to the patients. Therefore, we ran computer simulations to assess the relative effect of reconstruction error on the computed variability.

The 3D reconstruction method used to compute the 3D coordinates of the anatomical landmarks was previously evaluated and the mean squared error on the landmarks reconstruction was evaluated to 2.6 mm [7]. So, we simulated virtual spine models with this mean squared error and we computed the variance (see equation 2) of the resulting inter-vertebral transforms. We found a variance of 1.66 mm^2 in translation (setting $\lambda \to \infty$) and 2.0 x 10 $^{-3}$ rad^2 in rotation ($\lambda = 0$). Both simulated variances are well below the variability computed for scoliotic patients therefore the observed variability is mainly associated with spine geometry and not with the imaging system.

4. CONCLUSION

We presented a method to compute and visualize the geometric variability of the spine. We also successfully applied our method to a group of scoliotic patients. To our knowledge, it is the first time that experimental results quantifying the inter-vertebral transforms variability are published. Results presented in this paper suggest that highly relevant information about the geometry of the spine can be obtained by studying the variability using rigid transformations. From a medical perspective, this could lead to the optimisation of treatment strategies or diagnostic methods (by taking advantage of the strong variability in the coronal plane, for example).

Furthermore, the development of a variability model, like the one presented in this paper, offers many ways to improve image analysis algorithms because *a priori* insights could be easily introduced in the form of a variability model. Future directions include the analysis of global motions of the spine using joint covariance, the development of temporal variability models to assess the evolution of the pathology or the effect of orthopaedic treatments (such as braces and surgeries) and the integration of this model in registration algorithms.

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