Articulated Spine Models for 3D Reconstruction from Partial Radiographic Data

Jonathan Boisvert, Farida Cheriet, Xavier Pennec, Hubert Labelle, Nicholas Ayache

***Abstract*—Three-dimensional models of the spine are extremely important to the assessment of spinal deformities. However, it could be difficult in practical situations to obtain enough accurate information to reconstruct complete 3D models. This paper presents a set of methods to rebuild complete models either from partial 3D models or from 2D landmarks. The spine was modeled as an articulated object to take advantage of its natural anatomical variability. A statistical model of the vertebrae and spine shape was first derived. Then, complete models were computed by finding the articulated spine descriptions that were consistent with the observations while optimizing the prior probability given by the statistical model. The observations used were the absolute positions, orientations, and shapes of the vertebrae when a partial 3D model was available. The reconstruction of 3D spine models from 2D landmarks identified on radiograph(s) was performed by minimizing the Mahalanobis distance and the landmarks re-projection error. The vertebrae estimated from partial models were within 2 mm of the measured values (which is comparable to the accuracy of currently used methods) if at least 25% of the vertebrae were available. Experiments also suggest that the reconstruction from posterior-anterior and lateral radiographs using the proposed method is more accurate than the conventional triangulation method.**

***Index Terms*—Statistical shape model, 3D reconstruction, model registration, X-ray imaging, spine, scoliosis.**

I. Introduction

THREE-dimensional models of the spine are widely used in applications related to spinal deformities. They are necessary since spinal deformities are three-dimensional and cannot be evaluated thoroughly using only 2D images (i.e. radiographs). They can be used to diagnose and evaluate the severity of those deformities. The three-dimensional nature of the models enables analysis that would be impossible to perform directly on radiographs. For example, clinical indices such as the orientation of the plane of maximal curvature or the spine torsion [1] rely on the availability of 3D spine models. Furthermore, these 3D models are also used to plan and evaluate outcomes of orthopedic treatments [2], [3]. Finally, biomechanical studies of the spine also rely on personalized properties that are extracted from 3D spine models [4], [5].

In theory, 3D spine models can be obtained from a wide variety of imaging modalities. However, few modalities are flexible enough to image patients in different postures. Furthermore, spinal deformities often afflict children or adolescents and require multiple follow-up examinations, which means that exposure to ionizing radiation should be minimized. For these reasons, the most commonly used modality is roentgenography (radiographs).

Reconstructing 3D spine models from radiographs can be done using different methods. The most common methods generally involve identifying anatomical landmarks on more than one calibrated radiographs and then triangulating their 3D positions. The underlying calibration method can implicitly compute the calibration parameters by solving a system of linear equations [6], [7], [8], or it can explicitly optimize the calibration parameters to minimize the re-projection error [9]. Methods based on self-calibration algorithms were also proposed to avoid using a calibration object [10], [11]. These methods, however, do not cope very well with missing or unrecognizable landmarks. The 3D positions of anatomical landmarks not visible on at least two radiographs cannot be computed. The resulting incomplete models are generally useless for biomechanics or statistical studies.

However, incomplete models are common because radiographs are noisy by nature and because the superposition of structures and artifacts can be misleading. It is also common to observe vertebrae that are partially or completely hidden because of incorrect framing. The presence of orthopedic instrumentation (rods, screws, and hooks used to straighten the spine) can also occlude anatomical landmarks. Moreover, the time required to identify the anatomical landmarks makes the operation of reconstructing the spine in 3D time consuming and costly.

If a large proportion of the needed data is missing, then, most of the time, the whole experiment has to be cancelled. Moreover, if a small proportion is missing, the authors are faced with a difficult decision. They can either abandon the incomplete models or make educated guesses about the missing information. The former can reduce the statistical power of the experiment to the point where hypothesis testing is useless, while the precision and accuracy achieved with the latter strategy are questionable at best.

One possible solution is to identify only a small number of reliable landmarks that approximate the volume of the vertebral bodies and that are easy to identify on both lateral and posterior-anterior radiographs [12]. Another solution is to identify anatomical landmarks only on the radiograph(s) where they can be reliably identified. A three-dimensional mesh can then be deformed to fit the observations of anatomical landmarks, while minimizing the deformation energy [13] [14].

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Jonathan Boisvert and Farida Cheriet are affiliated with the École Polytechnique de Montréal, Montréal, H3G 1A4, Canada

Jonathan Boisvert, Xavier Pennec and Nicholas Ayache are affiliated with the INRIA, Sophia-Antipolis Cedex, BP93 06902, France

Jonathan Boisvert, Farida Cheriet and Hubert Labelle are affiliated with the Sainte-Justine Hospital, Montréal, H3G 1A4, Canada

However, this method cannot be used to estimate the shape of a vertebra if no landmarks are available, since relationships between adjacent vertebrae are not taken into account.

Another possibility is to use radiographs directly in order to register 3D vertebral templates [15], [16]. The possible deformations of the template, however, have to be constrained by a statistical shape model [15], [17]. These methods depend highly on the quality of the metric used to quantify the similarity between the 3D model and the radiographs. Current metrics cannot handle occlusions caused by surgical implants. Moreover, the method relies on the presence of a good initial estimate of the geometry, which in most cases mean a manual identification of anatomical landmarks. A method able to reconstruct complete 3D spine models from a small number of vertebrae would alleviate those problems.

II. Theoretical Background : Articulated Modeling of the Spine

Three-dimensional spine models are usually simple collections of 3D anatomical landmarks. Depending on clinical constraints and research objectives, six to fourteen landmarks are identified on each vertebra. This operation is performed by a trained technician, so there is a tradeoff between the level of detail and the cost of the study. The advantage of only using landmarks is simplicity, since the resulting models belong to a vector space in which all conventional statistics and analytical methods can be applied. The articulated nature of the spine is, however, completely discarded. No discrimination is made between landmarks belonging to the same vertebra and landmarks from two different vertebrae.

However, there is a fundamental distinction that should not be ignored. Vertebrae are bony structures, which can be considered as rigid bodies, whereas the spine is flexible. The spine can be deformed in complex ways because it is a collection of multiple rigid bodies (the vertebrae) linked together by soft tissues such as joint capsules, ligaments, intervertebral discs, and muscles. Thus, it is logical to model the shape of the spine using relative rigid transformations between neighboring vertebrae. This enables us to take into account the variability of the “inter-vertebral articulations” state.

In addition to the shape of the spine, the shapes of vertebrae can also be modeled in the context of an articulated model by using the local anatomical landmarks. These landmarks are expressed in the local coordinate system of the vertebra to which they belong. Any consistent local coordinate system can be used. However, good guidelines to establish such consistent local coordinate systems were provided in Stokes et al. [1] and in Wu et al. [18]. In our case, the origin of the coordinate system is the middle point between the centers of the upper and lower endplates. The local Z axis passes through the centers of the upper and lower endplates. The local Y axis is parallel to the line joining the centers of the left and right pedicles. Finally, the local X axis complete an orthonormal base.

In summary, the articulated description of the spine used throughout this paper is the combination of the *inter-vertebral rigid transformations* $T_0, T_1, T_2, \ldots, T_N$ (see Figure 1) and the *local anatomical landmarks* $p_{1,1}, p_{1,2}, \ldots p_{1,M} \cdots p_{N,M}$ (where N is the number of vertebrae studied and M is the number of landmarks digitized for each vertebra). In this paper $N = 17$, because thoracic and lumbar vertebrae are considered.

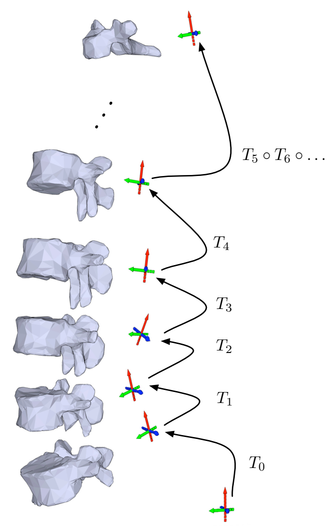

Fig. 1. Inter-vertebral rigid transformations used to describe the spine shape

Absolute rigid transformations (transformations between the general frame of reference and a vertebra's local reference frame) can be easily computed by composing all the *inter-vertebral rigid transformations* up to the vertebra of interest. This operation is summarized by the following equation (where $\circ$ is the composition operator):

$$T_i^{absolute} = T_0 \circ T_1 \circ \ldots \circ T_i$$

Consequently, transforming the local anatomical landmarks into absolute landmark coordinates can be performed using the following equation (where $\star$ is the operator that applies a transformation to a point):

$$p_{i,j}^{absolute} = T_i^{absolute} \star p_{i,j}$$

Important statistical tools can be adapted to work work with articulated models. First, the mean of a group of articulated spine descriptions can be defined simply by averaging the individual components of these descriptions, i.e., the local anatomical landmarks and the inter-vertebral rigid transformations. The local anatomical landmarks are three-dimensional vectors ($\Re^3$), thus their mean is well defined. However, the conventional mean cannot be applied to inter-vertebral rigid transformations since scalar multiplication and addition, are not defined on rigid transformations. Nonetheless, a left-invariant distance ($d(T_1, T_2) = d(T_3 \circ T_1, T_3 \circ T_2)$) exists. In this context, the mean is defined as the rigid transformation that minimizes the sum of the distances to the rigid transformations that need to be averaged [19]. In practice, this mean can be computed by performing a gradient descent on the distance summation, which is summarized by the following equation:

$$\vec{\mu}_{n+1} = \vec{\mu}_n \circ \left(\frac{1}{N} \sum_{i=0}^{N} \overrightarrow{\mu_n^{-1} \circ x_i} \right) \tag{1}$$

The inter-vertebral rigid transformations are usually well localized and the convergence is usually reached within five iterations [20]. Moreover, the starting point (μ_0) can be selected from the set of rigid transformations that are averaged. The arrow symbol ($\rightarrow$) indicates that the summation have to be performed on a vectorial representation of rigid transformations compatible with the selected distance. This suitable vectorial representation is a vector containing the translation vector and the axis of rotation scaled by the angle of rotation. (Numerical details of the conversion to/from this representation and a matrix representation of rigid transformations can be found in Pennec and Thirion [19].)

The mean inter-vertebral transformations $\bar{T}_i$ (computed using Equation 1) and the mean local anatomical landmarks $\bar{p}_{i,j}$ can then can then be used to compute the departure from the mean spine shape. This departure can be expressed by a vector S such as :

$$\delta_i = \bar{T}_i^{-1} \circ T_i$$

$$s_i = \left(\vec{\delta}_i, p_{i,1} - \bar{p}_{i,1}, p_{i,2} - \bar{p}_{i,2}, \ldots, p_{i,M} - \bar{p}_{i,M} \right)$$

$$S = (s_1, s_2, \ldots, s_N) \tag{2}$$

Using the departure vector S, it is now possible to quantify the dispersion of the articulated models around their mean. To do so, one can use the covariance matrix, which is given by :

$$\Sigma = \frac{1}{N} \sum_{i=1}^{N} S_i^T S_i$$

III. Novel 3D Reconstruction Methods

A. Estimation from Partial 3D Spine Models

The articulated description of the spine presented in the last section is well adapted to the estimation of 3D spine models from incomplete data since it captures efficiently the variability of the spine's shape (with the inter-vertebral rigid transformations) and of the local anatomy of the vertebrae (with local anatomical landmarks). In many situations, researchers and clinicians are confronted with incomplete 3D spine models. Anatomical landmarks not digitized because of a lack of resources, anatomical landmarks hidden by surgical instrumentation, or vertebrae located outside the radiographs' field of view are just a few examples.

The proposed approach is to estimate the most likely articulated description of the spine that matches available 3D measurements. This is achieved by minimizing the Mahalanobis distance of the estimated model in the tangent plane of the mean articulated description. The minimization, however, has to be constrained so that known vertebrae's positions, orientations, and shapes match available 3D measures.

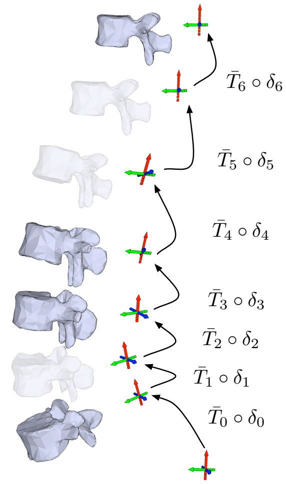

Fig. 2. Combination of the mean inter-vertebral rigid transformations $\bar{T}_i$ and the departures δ_i to produce actual inter-vertebral rigid transformations on a partial spine model (semi-transparent vertebrae are assumed to be missing).

1) Rigid Constraints: The articulated models presented in the last section are based on relative rigid transformations and local anatomical landmarks. Partial spine models, however, provide us with information in absolute coordinates. Additional precautions must therefore be taken to ensure that the estimated model preserves the absolute poses of known vertebrae. An elegant solution is to use constrained optimization.

Finding the most likely articulated spine description given a partial model can be performed by solving the following constrained optimization problem:

$$\tilde{S} = \arg\min_{S} S \Sigma^{-1} S^T \tag{3}$$

Subject to:

$$\tilde{T}_i^{absolute} = T_{i-1}^{absolute} \circ \bar{T}_i \circ \delta_i \quad \forall i \in K \tag{4}$$

$$\tilde{p}_{i,j} = p_{i,j} \quad \forall \quad (i,j) \in L, \tag{5}$$

where $\tilde{T}_i^{absolute}$ are known vertebrae' absolute poses, $\tilde{p}_{i,j}$ are known anatomical landmarks, K is the set of all known vertebrae, and L is the set of known landmarks.

It is important to stress that $T_i^{absolute}$ depends on $\delta_0, \delta_1, \ldots, \delta_i$, which are variables that must be estimated. For example, Figure 2 depicts a situation in which $K = \{0, 2, 3, 6\}$ (unknown vertebrae are semi-transparent). In this case, the constraints on vertebrae poses are:

$$\begin{aligned}
\tilde{T}_0^{absolute} &= \bar{T}_0 \circ \delta_0 \\
\tilde{T}_2^{absolute} &= \bar{T}_0 \circ \delta_0 \circ \bar{T}_1 \circ \delta_1 \circ \bar{T}_2 \circ \delta_2 \\
\tilde{T}_3^{absolute} &= \bar{T}_0 \circ \delta_0 \circ \bar{T}_1 \circ \delta_1 \circ \bar{T}_2 \circ \delta_2 \circ \bar{T}_3 \circ \delta_3 \\
\tilde{T}_6^{absolute} &= \bar{T}_0 \circ \delta_0 \circ \bar{T}_1 \circ \delta_1 \circ \bar{T}_2 \circ \delta_2 \circ \bar{T}_3 \circ \delta_3 \circ \\
&\quad \bar{T}_4 \circ \delta_4 \circ \bar{T}_5 \circ \delta_5 \circ \bar{T}_6 \circ \delta_6
\end{aligned}$$

The number of degrees of freedom varies based on the number of local anatomical landmarks used, the number of vertebrae considered, and the number of constraints. Typically, the number of degrees of freedom ranges between 350 and 700. Analytical derivatives of the cost function and of the constraints can therefore substantially decrease the computational requirements of the optimization process.

The cost function (Equation 3) and the constraints presented in Equation 5 are simple to differentiate since they are linear or quadratic functions of S. However, the constraints that preserve the absolute poses of known vertebrae (introduced in Equation 4) involve the multiple compositions of rigid transformations, which are non-linear functions over the rotation and translation vectors.

By using the definition of T_i^{abs} and the chain rule, the derivative of Eq. 4 with respect to the j^{th} inter-vertebral rigid transformation can be expressed by:

$$\frac{\partial}{\partial \delta_j} T_{i-1}^{abs} \circ \mu_i \circ \delta_i = \frac{\partial}{\partial \delta_j} T_i^{abs} = \frac{\partial}{\partial T_j^{abs}} T_i^{abs} \frac{\partial}{\partial \delta_j} T_j^{abs} \tag{6}$$

with:

$$\frac{\partial}{\partial T_j^{abs}} T_i^{abs} = \frac{\partial}{\partial T_{i-1}^{abs}} T_i^{abs} \frac{\partial}{\partial T_{i-2}^{abs}} T_{i-1}^{abs} \cdots \frac{\partial}{\partial T_j^{abs}} T_{j+1}^{abs} \tag{7}$$

These equations are valid only if $j < i$, otherwise T_i^{abs} does not depend on δ_j, and the derivative is zero. The intermediate values of Equation 7 can be reused in the computations of the derivatives of multiple constraints.

The derivatives contained in Equations 6 and 7 are the derivatives of the composition of two rigid transformations with respect to one of the two transformations. From the definition of the composition of two rigid transformations, the derivatives of $T_2 \circ T_1$ can be expressed as follows:

$$\frac{\partial}{\partial T_2} T_2 \circ T_1 = \begin{bmatrix} \frac{\partial}{\partial r_2} r_1 \circ r_2 & 0 \\ \frac{\partial}{\partial r_2} r_2 \star t_1 & I_3 \end{bmatrix},$$

$$\frac{\partial}{\partial T_1} T_2 \circ T_1 = \begin{bmatrix} \frac{\partial}{\partial r_1} r_1 \circ r_2 & 0 \\ 0 & R_2 \end{bmatrix}.$$

The symbols r_1, r_2, represent the rotation vectors associated with the rigid transformations T_1 and T_2. Moreover, t_1 is the translation vector associated with the rigid transformation T_1, R_2 is the rotation matrix equivalent to r_2 and I_3 is a $3x3$ identity matrix. The values of $\frac{\partial}{\partial r_1} r_1 \circ r_2$, $\frac{\partial}{\partial r_2} r_1 \circ r_2$ and $\frac{\partial}{\partial r_2} r_2 \star t_1$ can be obtained using the Rodrigues formula or using unit quaternions as an intermediary representation. Detailed descriptions of those computations, as well as linear developments around numerical instabilities, can be found in Pennec and Thirion [19].

2) Optimization Method and Initialization: The minimization problem presented in Equations 4 and 5 can be solved using standard constrained optimization methods. The method selected for the experiments presented in this paper is sequential quadratic programming [21]. This method was selected because the cost function is quadratic and the constraints are close to linear constraints in most solutions. Sequential quadratic programming is a generalization of Newton's method for unconstrained optimization that iteratively solves a quadratic model of the problem with linear approximations of the constraints. Like Newton's method, it is a local optimization method, and it is subject to entrapments in local minima.

A good starting point is thus necessary. Preliminary experiments have shown that the most critical characteristic of a good starting point is the satisfaction of the constraints introduced in Equations 4 and 5. A simple method to obtain such initial estimates is to subdivide the difference of orientation and position between two known vertebrae into equal rigid transformations. More precisely, the initial estimates Δt of the inter-vertebral rigid transformations located between two known vertebrae (with index j and k) needs to satisfy:

$$(\Delta t)^{k-j} = \left(T_j^{abs}\right)^{-1} T_k^{abs},$$

where rigid transformations are expressed as $4x4$ transformation matrices.

The initial estimates can thus be computed using the following equation:

$$\Delta t = \exp\left(\frac{\log(\left(T_j^{abs}\right)^{-1} T_k^{abs})}{k-j}\right),$$

where $\exp$ and $\log$ represent respectively the matrix exponential and the logarithm of a matrix.

B. Articulated Spine Model Reconstruction from Radiograph(s)

We now introduce a new general 3D reconstruction method that let researchers (or clinicians) extract 3D spine models from landmarks on any number of radiographs. The key difference with current methods is that a prior knowledge of the spine shape encoded by a statistical articulated model of the spine is used. This permits us to restrain the search for a 3D spine model that fits the measurements made on radiographs to anatomically plausible configurations.

Let $p_{2D}^{i,j,k}$ be the image coordinates of an anatomical landmark identified in a radiograph. The index i associates a landmark with a vertebra. The index j indicates the position of the anatomical landmark within the set of landmarks used for the i^{th} vertebra. Finally, k denotes the index of the radiograph on which the coordinates were measured. Now let I be the set of all anatomical landmarks identified on radiographs. More formally, we have:

$$I = \{p_{2D}^{i,j,k} | 0 \le i < n, 0 \le j < m, 0 \le k < o\}, \tag{8}$$

where n is the number of vertebrae considered, m the number of anatomical landmarks by vertebra, and o the number of radiographs used.

A simple but effective way to combine the similarity between I and S with prior knowledge of possible spine shapes is to sum the Mahalanobis distance and the quadratic error on the anatomical landmarks. The following equation summarizes this operation:

$$C(S, I) = S\Sigma^{-1}S^T + \alpha \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=0}^{o} \|p_{2D}^{i,j,k}(S) - \hat{p}_{2D}^{i,j,k}\|^2, \tag{9}$$

where α is the relative weight of the landmark error with respect to the prior spine shape knowledge.

If one assumes that S follows a normal distribution, then the cost function $C(S, I)$ leads to a *maximum a posteriori* estimation. This assumption is generally justified since the Ricci curvature of manifold to which S belongs is inconsequential, given the level of dispersion of the inter-vertebral rigid transformations.

The image coordinates of the anatomical landmarks $p_{2D}^{i,j,k}(S)$ are computed by first computing the absolute 3D coordinates of all given anatomical landmarks. This is done by composing the inter-vertebral rigid transformations to obtain the absolute pose of the vertebrae and then by applying the associated rigid transformation to the local anatomical landmark.

$$[X_{abs}, Y_{abs}, Z_{abs}]^T = \bar{T}_0 \circ \delta_0 \circ \bar{T}_1 \circ \delta_1 \circ \ldots \circ \bar{T}_i \circ \delta_i * p_{3D}^{i,j} \quad (10)$$

Then the 3D absolute coordinates of the anatomical landmark can be projected on the radiograph image plane by a simple linear transformation in homogenous coordinates.

$$\begin{aligned} [x, y, z]^T &= M\,[X_{abs}, Y_{abs}, Z_{abs}, 1]^T \\ p_{2D} &= [x/z \quad y/z]^T \end{aligned} \quad (11)$$

This linear projection model assumes that the measured coordinates $\hat{p}_{2D}^{i,j,k}$ were already corrected for geometric distortion (if necessary). The projection matrix M is computed from a calibration object visible on the radiographs using a linear method [6].

The cost function presented in Equation 9 can be analytically differentiated using the equations introduced in Section III-A1 to differentiate rigid constraints. These analytical derivatives lead to significant improvements in computational requirements in comparison to numerical derivatives because of the scale of the optimization problem. The Mahalanobis distance regularizes the cost function and reduces the number of local minima. Theoretically, there is no guarantee that the optimization will not be trapped by a local minimum. One could choose a robust optimization method, such as simulated annealing, because of the local minima. In practice, however, a simple gradient descent procedure was sufficient.

The proposed method has only one free parameter, α, which controls the relative weight of the Mahalanobis distance and the re-projection error. If one assumes identical independent normal distributions for the landmark localization errors and a normal distribution for the articulated description of the spine, then the optimal value would be $\alpha = \frac{1}{\sigma_{pix}^2}$ (where σ_{pix}^2 is the variance of the landmark localization errors). Unfortunately, the distribution of the noise is rarely known and the actual value of α has to be adjusted manually ($\alpha = \frac{1}{\sigma_{pixels}^2}$ can however be used as an initial guess).

IV. Experiments and Results

The validation of anatomical model estimation methods is challenging since two important but contradicting factors must be managed. First, the ground truth against which the results are compared must be as accurate and precise as possible. Second, the realism of the experiments is also very important. Realism is even more important when the estimation method relies on statistical models of the anatomy, since those models are sensitive to posture and pathologies.

Simulation studies can be appealing since the ground truth is known with absolute certainty. Such studies are very useful to investigate intrinsic limitations of the method and to test the sensitivity of the method to different error sources. However, the realism of simulation studies is limited since it is not possible to take into account all error sources.

Highly accurate, three-dimensional measures of the spine (sub-millimeter accuracy) cannot generally be achieved on living patients, and the shape of spine phantoms are unlikely to follow the same statistical distribution as the spines of living patients, because ligaments, discs, joint capsules and neuromuscular tonus may influence the 3D-shape of the spine. Moreover, the patients' postures have to mirror closely the posture used in real clinical applications because two postures (*e.g.*, standing up and lying down) will be associated with two different statistical distributions.

The *de-facto* standard in three-dimensional evaluation of scoliosis is stereo-radiography. It can be performed on living patients and its accuracy is more than adequate for a wide variety of applications (diagnostic, surgical planning, braces design, biomechanics research, *etc.*).

The validation of the proposed method was thus conducted using a group of 291 patients from the Sainte-Justine Hospital (Montreal, Canada) diagnosed with adolescent idiopathic scoliosis (AIS). The mean age of the patients was 13.5 years old (with a standard deviation 1.8 years) and 89 % were females (AIS mainly afflicts young adolescent women). These patients were selected because they had a stereo-radiographic examination where a standardized posture and imaging protocol were used. Vertebrae from L5 to T1 were digitized using six anatomical landmarks. For each experiment, $\frac{5}{6}$ (242) of the patients were randomly selected to compute the statistical distribution of articulated spine description. The remaining $\frac{1}{6}$ (49 patients) was then used to compute articulated spine description using the proposed methods. In other words, no patient was ever used both for the estimation of the statistical distribution of the articulated spine descriptions and for validation. Leave-one-out cross-validation could be used to increase the number of patients used to estimate the mean and covariance. However, the distribution estimation and validation sets are large enough for practical purposes and the computational requirements of cross-validation outweigh its benefits in our application. The errors were always measured in absolute coordinates (with respect to a global frame of reference).

A. Estimation from Partial 3D Spine Models

The estimation of complete 3D spine models from partial models can be influenced by two important factors: the distribution of the missing vertebrae in the input partial model and the sensitivity of the method to the accuracy of the known vertebral shapes.

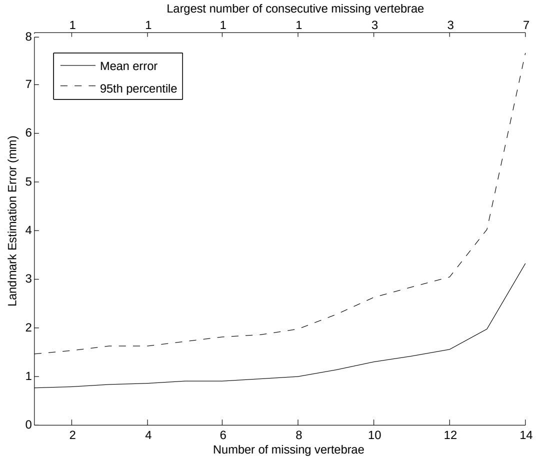

Fig. 3. Error on the estimated vertebrae with respect to the number of vertebrae missing (uniformly distributed along the spine). The largest gap between two known vertebrae for each considered missing vertebrae configuration is also provided on top of the graph.

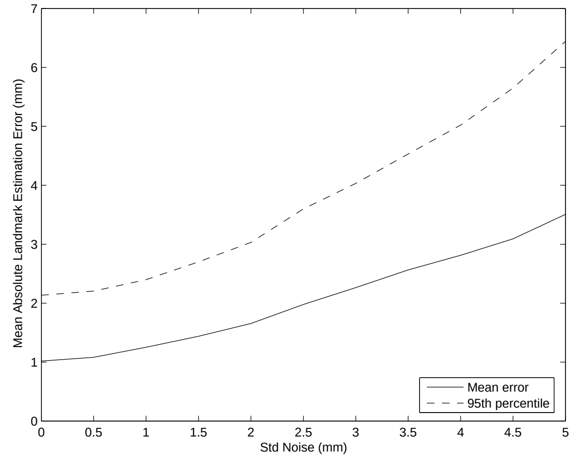

Fig. 4. Effect of simulated noise applied to known 3D coordinates on the estimated vertebrae.

1) Distribution of the Missing Vertebrae: The number of possible configurations for the placement of missing vertebrae is too large to test every possibility. However, it is possible to select a smaller number of representative configurations for validation purposes. In this case, we chose to study configurations with one to fourteen missing vertebrae that are evenly spread across the spinal column.

The 49 randomly selected 3D spine models were thus successively altered to remove a given number of vertebrae. Then, the missing vertebrae were estimated using the method presented in section III-A. The resulting complete 3D spine models were compared to the original reconstructions. The obtained error with respect to the number of missing vertebrae is presented in Figure 3.

The mean difference range from $0.8mm$ with a single missing vertebrae to $3.25mm$ with fourteen missing vertebrae (out of seventeen vertebrae). The 95^{th} percentile of the error follows a curve similar to the mean error but ranges from $1.5mm$ to $7.5mm$. The error appears to be linearly dependent on the number of missing vertebrae until the proportion of missing vertebrae reaches about 75 % ; the performances degraded more rapidly after this point. This behavior is explained by the fact that the shapes and poses of the vertebrae are statistically related, thus a complete 3D spine model implicitly contains redundancies. These redundancies are used to rebuild a complete model when parts are missing. However, if the proportion of missing vertebrae is too great, then redundancy cannot counterbalance the missing data, which leads to a precipitous increase in the reconstruction error.

2) 3D Error Effect: An important concern with the estimation from partial models is whether errors on the partial models result in dramatic errors on the estimated vertebrae. In order to test the effect of errors in input data on the proposed method, we added artificially generated noise to the partial models before using them to estimate complete models. Artificially generated noise was added to the absolute landmarks coordinates. Standard deviations from 0 to 5 mm with 0.5 mm increments were tested. In this experiment, one out of two vertebrae starting from L5 were present in the partial model.

The results of this experiment are summarized in Figure 4. The mean error without noise was about 1 mm (same as Figure 3) and progressed linearly to 3.5 mm, when the standard deviation of the noise was 5 mm. This shows that the proposed method is tolerant of variations in input data.

B. *Reconstruction from Radiograph(s)*

We first compared the sensitivity to identification errors of the proposed method and of the standard linear triangulation algorithm used in previous 3D reconstruction methods [7]. In order to do so, the same 49 randomly selected 3D spine models were projected using known projection matrices and artificially generated noise was added to the obtained pixel coordinates. The projection matrices emulate a posterior-anterior and a lateral radiograph, which is the most common acquisition setup used in spinal deformity studies.

The resulting 2D coordinates were then used to reconstruct the 3D spine models. The mean differences between the original 3D models and the reconstructed models are illustrated by Figure 5. It can be observed that the proposed method is associated with smaller errors when the standard deviation of the noise is greater than 1 pixel and that the errors associated to both methods are similar when the noise standard deviation is less than 1 pixel.

We also used the same number of randomly selected patients to reconstruct 3D spine models from raw landmark coordinates recorded from radiographs (posterior-anterior and lateral) by a qualified technician. We compared the 3D spine models obtained using the proposed method to the 3D spine models obtained with the conventional stereo-radiographic method [7] and the mean difference between the reconstructions was $1.1mm$.

V. Discussion

A. *Estimation from Partial 3D Spine Models*

The proposed method takes advantage of a strong prior knowledge of the inter-vertebral rigid transformations. This

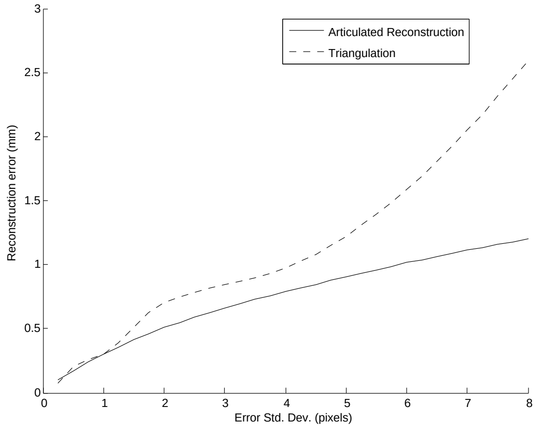

Fig. 5. Simulation of the noise effect on the 3D reconstruction of spine models from two radiographs (a lateral and a posterior-anterior) using the proposed method (articulated reconstruction) and using triangulation.

prior knowledge has to be completed by constraints on absolute positions and orientations. Otherwise, the accumulation of small errors on relative inter-vertebral estimations would result in an articulated spine description inconsistent with the known absolute coordinates of the vertebrae.

This combination of strong prior knowledge and constraints resulted in a method that was resistant to a reasonable amount of noise on the known vertebrae, as suggested by the experimental results presented in Figure 4. Thus, the method is likely to withstand the noise present in clinical data without dramatic failures.

The accuracy of the method with respect to the number of missing vertebrae is also very interesting (results illustrated by Figure 3). The difference between the estimated landmarks and the coordinates measured from stereo-radiography is below the accuracy of the stereo-radiographic reconstruction method when less than 13 vertebrae are unknown. We cannot conclude that the estimated landmarks are more accurate than the stereo-radiographically reconstructed landmarks in those cases, since we lack a more accurate (and realistic) ground truth. However, we can conclude that the accuracy of the estimated landmarks is close to the accuracy of a conventional stereo-radiographic reconstruction method when more than 25% of the vertebrae are available and uniformly spread along the spine.

These results were obtained with uniformly distributed missing vertebrae. In the case of different distributions, the disposition of the missing vertebrae might be even more important than their number. Thus, if one is not interested in uniformly distributed missing vertebrae, then the Figure 3 should be considered with caution. The largest gap between two known vertebrae might provide more guidance and enable a more conservative estimation of the expected reconstruction error. In any case, the results presented in Figure 3 show that gaps of different sizes and positioned in different locations can be successfully filled. Thus, we are confident that the proposed method can be used with a large variety of missing vertebrae dispositions.

From a practical perspective, the obtained results indicate that the method can be used to fill the small gaps that are the result of radiographic artifacts or surgical instrumentation. These results also demonstrate that it would be possible to use this method to digitize complete models from a small number of vertebrae, thus saving physicians and researchers a lot of time and money. The resulting complete spine models can be used in applications such as: diagnostic, surgical planning, and biomedical research.

Statistical studies based on the resulting complete models should, however, be undertaken with caution. The variability of the resulting models is likely to underestimate the true variability of the shapes of patients' spines since their estimation is not based on actual measurements for all vertebrae. This underestimation, if uncorrected, could bias statistical hypothesis tests (see [22] for more details).

B. *Estimation from 2D Landmarks*

Reconstruction of articulated descriptions from radiographs serves a different purpose than the estimation of complete models from partial ones. The latter is a general procedure to complete a 3D spine model regardless of imaging source used to build this partial model. The reconstruction of articulated spine descriptions from radiographs specializes in cases where projections of the anatomy are available.

The overall mean difference when two radiographs were used is $1.1mm$ (with real data). This difference is too close to the precision of the stereo-radiographic method [8] used for comparison to make strong claims about it. Furthermore, both stereo-radiographic reconstruction and the proposed method share the same calibration procedure, thus the errors of both methods are not independent. However, the calibration is performed using two grids of lead pellets, which can be accurately and precisely identified on radiographs. The anatomical landmarks are much more difficult to identify reliably. Landmarks identification is thus probably a greater source of error than the calibration process. Synthetic results (presented in Figure 5) indicate that the proposed method might be more accurate than the stereo-radiographic method currently used.

C. *General Remarks*

Articulated models of the spine are very versatile. They could be used to solve 3D reconstruction problems in many clinical applications with different constraints. We presented in this paper the most common and most important situations related to spinal deformities studies. However, there are other possibilities.

For example, we studied the case of missing vertebrae in section III-A but the method could be extended to also accommodate missing anatomical landmarks. Moreover, some anatomical landmarks are easy to identify on posterior-anterior radiographs, but are difficult to locate on lateral radiographs since ribs and lungs often hide parts of the thoracic vertebrae. This situation could be handled by the proposed method by allowing some anatomical landmarks to be reconstructed from one view and other from two views.

The reconstruction based on radiographs is not limited to the case in which a posterior-anterior and a lateral radiograph are available. The same method could be used with a different pair of radiographs or with a single radiograph. This could be especially useful in clinical tests where it is difficult to acquire more than one radiograph (for example, during bending tests).

The accuracy will however always be function of the quality and completeness of the input data. For instance, a reconstruction based on a single radiograph will be associated with an important error along the projection axis of the radiograph since no information is provided by the radiograph along that axis.

Clinical applications often depend on indices computed from 3D reconstructions. These indices can be local measures such as the shifts between adjacent vertebrae, or global measures such as the Cobb angle. The effect of the reconstruction errors on those clinical indices is not always linear and will therefore have to be quantitatively evaluated in the future. The proposed methods should yield acceptable results since they implicitly smooth the 3D reconstructions, which reduce the effect of noise. For comparable reconstruction errors, the proposed methods should be associated with equal or better estimates of the clinical indices than conventional reconstruction methods. However, for cases in which a large portion of information is missing, the resulting 3D spine models will be biased toward the mean. This situation might trend toward an unacceptable bias in some clinical indices (for example, if one wants to compute the lateral Cobb angle when only one posterior-anterior radiograph is available).

VI. Future Work and Conclusion

In this paper, we proposed methods to estimate 3D models based on an articulated description of the spine. A statistical model of the spine was proposed and used to leverage the implicit redundancy contained in three-dimensional spine models. This statistical model enabled us to reconstruct 3D models of the spine in cases where conventional methods could not be applied because of missing information.

The articulated description used includes inter-vertebral rigid transformations as well as 3D positions of anatomical landmarks measured with respect to vertebrae's local frame of reference. This description naturally captures the deformations of the spine shape, which are described by rigid transformations between adjacent vertebrae, and the variability of vertebrae's anatomy, which is characterized by landmark positions. Two different reconstruction problems were discussed.

First, the reconstruction of complete 3D spine models from partial 3D models was considered. These reconstructions were performed by minimizing the Mahalanobis distance of the estimated articulated spine model while constraining the absolute positions, orientations, and shapes of known vertebrae. The anatomical landmark estimates obtained by using partial models were within 1 mm of measured values if at least 50% of the vertebrae were available in the initial partial model. Moreover, the estimates were within 2 mm of measured values (*i.e.* equivalent to the conventional method with complete models) if at least 25% of the vertebrae were available.

The second reconstruction problem is the estimation of 3D spine models from radiographs. Three-dimensional reconstruction of the spine from two or more radiographs can be solved by using calibration and triangulation methods borrowed from the computer vision field. The proposed statistical shape model enables more accurate 3D reconstructions and is very flexible since it could be applied in spite of missing landmarks or missing vertebrae.

The proposed methods could be used in a large number of clinical applications, such as diagnosing problems, follow up, and surgical planning. More importantly, three-dimensional reconstructions of the spine will be available in situations where they were formerly impossible to compute.

The ideas discussed in this paper could be integrated in an automated or semi-automated reconstruction system. This could be accomplished by using an automated method for the detection of landmarks, such as the one previously proposed by Deschênes et al. [23]. Another possible approach would be to integrate an image-based metric, but the main challenge of this approach would be to efficiently generate simulated radiographs from articulated spine descriptions and to compare those with actual radiographs.

Registration of soft-tissues surrounding rigid structures by combining multiple rigid transformations, using methods similar to the method previously proposed by Little et al. [24] or to the more recent method proposed by Arsigny et al. [25], will also be considered in the future. Registration of structures such as the spinal cord and the inter-vertebral discs could be greatly improved with applications ranging from the correction of spinal deformities to epidural injections of steroids in the treatment of back pain.

Finally, although the proposed methods are extraordinarily well suited for research on spinal deformities, they could also be applied to any other anatomical structures that can be divided into multiple parts. The added value of the articulated description will depend on the strength of the statistical relationships that exist between the positions, orientations and shapes of the individual parts. Articulations are ideal candidates, but statistical relationships also exist between the positions, orientations and shapes of softer structures.

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Jonathan Boisvert received the B.Eng in computer engineering from École Polytechnique de Montréal (Montréal, QC, Canada) with distinction in 2002. He then pursued doctoral studies in the field of medical image analysis and received Ph.D. degrees from École Polytechnique de Montréal and from the University of Nice-Sophia Antipolis (France) in 2008 for his work on statistical modeling of the spine geometry. He is now a postdoctoral fellow at Queen’s University (Kingston, ON, Canada). Dr. Boisvert research interests include computer graphics, computer vision, image processing and pattern recognition. However, he is primarily interested in biological and medical applications such as computer-assisted surgery, biomedical image analysis and modeling of biological structures.

Farida Cheriet was born in Algiers, Algeria, in 1961. She received the B.Sc. degree in computer science from the University USTHB, Algiers, in 1984, the D.E.A. degree in the field of languages, algorithms and programming from the University of Paris VI, France, in 1986, and the Ph.D. degree in computer science from the University of Montreal, QC, Canada, in 1996. She held a postdoctoral position at the Biomedical Engineering Institute, École Polytechnique de Montréal, from 1997 to 1999. Since 1999, she has been appointed in the Department of Computer and Software Engineering, École Polytechnique de Montréal, where she is currently a full Professor. Her research interests include, three–dimensional (3-D) reconstruction of bone structures from x-rays, calibration of x-ray imaging systems, noninvasive 3-D modeling of scoliosis deformities, 3-D Augmented Reality systems for minimally invasive surgery, 3-D reconstruction and visualization of vascular structures from multimodal images, and spatio-temporal registration of medical images.

Xavier Pennec was born on May 2nd, 1970. Dr Pennec is currently a Research Director at the ASCLEPIOS team at INRIA Sophia-Antipolis (France). He holds an Engineering degree from the French Ecole Polytechnique in 1992 and a PhD degree in Computer Science from the same institution in 1996. He was a post-doctoral associate at MIT AI Lab (Mass. USA) in 1997, before joining INRIA in 1998. His main research axes are about statistics on geometric data, in particular for medical image analysis, and biomedical image registration. Over the last years, these fields have gradually converged toward computational anatomy, which aims at statistically describing the normal and abnormal shape of organs across populations. X. Pennec co-authored more than 100 peer-reviewed papers in international journals and conferences in these fields. He is associate editor of the Medical Image Analysis Journal (Elsevier) and chairs the International Workshop on Mathematical Foundations of Computational Anatomy since its inception in 2006.

Hubert Labelle was born in Montreal, QC, Canada, in 1952. He received the M.D. degree and completed his residency in orthopaedics at University of Montreal, Montreal, Canada, followed by a two-year fellowship in pediatric orthopaedics at Sainte-Justine Hospital, Montreal, Rancho Los Amigos Hospital, CA, and the A.I. Du Pont Institute, DE. Since 1982, he has been appointed in the Department of Surgery at University of Montreal and at Ste-Justine Hospital where he is currently a Professor of surgery and Titular of the Motion Sciences Research Chair of Sainte-Justine Hospital and the University of Montreal. His clinical work is focused on the evaluation and treatment of scoliotic deformities in children and adolescents. He is head of the musculoskeletal research group at Sainte-Justine Hospital Research Center and Director of the Three-Dimensional Scoliosis Laboratory. His research interests include the three-dimensional (3-D) evaluation and treatment of scoliotic deformities, with a particular emphasis on computer assisted surgery, 3-D design and evaluation of braces for the treatment of idiopathic scoliosis, and 3-D evaluation and simulation of surgery for scoliotic deformities.

Nicholas Ayache received his Ph.D in 1983, and his "Thèse d'Etat" in 1988, both in computer science, from the University of Paris XI. He is a Research Director at INRIA (French Research Institute in Computer Science and Automatic Control), Sophia-Antipolis, France, where he was the scientific leader of the EPIDAURE research group on medical image analysis and robotics between 1993 and 2005, and has been the leader of the ASCLEPIOS research group on biomedical image analysis and simulation since November 2005. Dr. Ayache is the co-founder and co-Editor in Chief of the scientific journal Medical Image Analysis (Elsevier) and an Associate Editor of Transactions on Medical Imaging (IEEE). He has also been serving on the editorial board of major conferences in Medical Imaging, Computer Vision, Visualisation, and Robotics including MICCAI, ISBI, CVPR, ECCV and ICCV. His research interests include the analysis of medical and biological images with advanced geometrical, statistical, physical and functional models, the simulation of physiological systems with computational models built from biomedical images and other signals, and the application of these tools to medicine and biology to better assist prevention, diagnosis and therapy of diseases.