A Hybrid Framework for Surface Registration and Deformable Models

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Abstract

In computer vision, two complementary approaches have been widely used to perform object reconstruction and registration. The deformable model framework locally applies internal and external forces to fit 3D data. The non-rigid registration framework iteratively computes the best global transformation that minimizes the distance between a template and the data. In this paper, first we show that applying a global transformation on a surface model, is equivalent to applying an external force on a deformable model without any regularizing force. Second we propose a hybrid framework that combines the registration framework and the deformable models framework. Our hybrid deformation approach allows us to control the scale at which the model is deformed. This is clearly beneficial for performing both reconstruction and registration tasks. We show examples of this approach on active contours and deformable surfaces. Furthermore, a global transformation based on axial symmetry is introduced.

1 Introduction

Deformable models introduced in [5] have been widely used in computer vision and image analysis for segmentation or object recovery. They provide a compact representation of objects with interesting geometric properties. Deformable models iteratively evolve under the action of internal and external forces.

Registration is a complementary approach that estimates a global transformation to apply on a template to fit the data. It has been proven [15] that a registration process can be achieved iteratively by composing successive estimated transformations. In this paper, we show that the registration of a model onto the data can be seen as the iterative application of a global deformation field derived from a global transformation.

By mixing the local and global deformation fields, we obtain a *hybrid deformation scheme* which yields better control of the model deformations. It provides segmentation, rigid or non rigid registration, and reconstruction of data using a single framework based on deformable models. The hybrid approach constrains model deformations. both by limiting the set of possible deformations and by enforcing regularity properties at each point of the model. It provides a continuous range of behavior from weak (local) to strong (global) constraints at low computation cost. It allows us to find a proper trade-off between model variability and its robustness to noise and outliers.

1.1 Context and Previous Work

Deformable models can be represented by meshes or parametric curves and surfaces. In this paper, we consider meshes approximating continuous curves that allow us to represent complex shapes with almost any topology. A model $\mathcal{M}$ is thus a set of 2D or 3D vertices $\{V_i\}_{1\leq i\leq n}$ connected by edges $\{E_i\}_{1\leq i\leq m}$. Two main approaches have been proposed to control model deformations. Our work proposes a hybrid a framework.

The registration framework [1] consists in applying a restricted class of transformations to a template or in limiting the template to a given mathematical representation (such as superquadrics or Fourier curves). This reduces the template's number of degrees of freedom. The use of complex global transformations can improve the model's generality but this leads to prohibitive computation time and numerical stability problems. In this paper, we consider the registration template as a surface model $\mathcal{M}$ to which a global transformation g is applied. The deformed model is thus $g(\mathcal{M}) = \{g(V_i)\}_i$.

The deformable models [7] framework gives many more degrees of freedom to the model by applying a deformation field acting independently on each vertex. Data driven and regularizing forces are applied on the model. They result in a deformation field (a set of displacement vectors) $\mathcal{L} = \{l_i\}_{1\leq i\leq n}$. We note the application of the deformations onto $\mathcal{M}$ as $\mathcal{L} \star \mathcal{M} = \{V_i + l_i\}_i$. Deformable models are not robust if they are not initialized very close from the boundaries they should lock on.

A hybrid approach. Many approaches have been proposed to introduce more general transformations into the registration process [4, 13]. Similarly, researchers have improved the robustness of deformable models by applying more global constraints. For instance, Terzopoulos and Metaxas considered in [12, 8] the superposition of a rigid component with a finite element mesh. Another approach on deformable contours is proposed in [6]. Modal analysis [14] or Fourier representation [11] aim similarly at controlling the scale of deformation from global to local.

1.2 Contributions

In this paper, we propose a single hybrid deformation framework that integrates both global transformations, g, local deformations, $\mathcal{L}$, and where the user can specify the scale of deformation between those two ends. This approach is beneficial for performing both registration and shape recovery.

When extracting a shape from 3D data, this deformation scheme keeps a large number of degrees of freedom for the model thus allowing us to describe complex deformations. However, by introducing global regularizing forces, we greatly improve the robustness of image segmentation.

Furthermore, we demonstrate that our framework is well suited for the registration between two similar shapes. On the one hand, the additional flexibility introduced in the registration takes into account the shape variability better than just applying global transformations. On the other hand, because the deformation field is partially composed of global components, we show that our scheme keeps the geometric properties of the model (such as curvature) during the deformation.

Finally, the hybrid deformation scheme also allows us to introduce domain specific constraints. We show a vessel segmentation example using cylindrical models with axial constraints.

2 Hybrid Deformations

2.1 Deformable Models Framework

Deformable models evolve under the action of forces usually resulting from an energy minimizing criterion. At vertex V_i, the external force f_i^{ext} is computed from the data and the regularizing force f_i^{int} is computed from the geometric properties of the model. It is possible to control the internal and external forces relative effect through weight coefficients α and β.

According to Newtonian dynamic laws applied at vertex V_i, $m\frac{d^2V_i}{dt^2} = F_i - \delta\frac{dV_i}{dt}$ where δ is a damping factor and $F_i = \alpha f_i^{int} + \beta f_i^{ext}$ is the total force applied on V_i. By discretizing time and approximating the derivatives by finite differences we get

$$V_i^{t+1} = V_i^t + (1-\delta)(V_i^t - V_i^{t-1}) + \alpha f_i^{int} + \beta f_i^{ext} \quad (1)$$

where V_i^t is the position of V_i at time t. The deformation field $\mathcal{L} = \{l_i = V_i^{t+1} - V_i^t = (1-\delta)(V_i^t - V_i^{t-1}) + F_i\}_i$ therefore apply on the model at time t.

2.2 Registration Framework

The registration process requires the computation of a global transformation which can be evaluated iteratively using an *Iterative Closest Point* algorithm [15]. When registering the model $\mathcal{M}$ with an object, each vertex V_i is matched with a closest point W_i. Let G be the group of transformations allowed by the registration process, at each step the ICP algorithm finds $g \in G$ which satisfies the minimization of the least square error criterion

$$g = \arg\min_{g \in G} \left\{ \sum_{i=1}^{n} \|g(V_i) - W_i\|^2 \right\} \quad (2)$$

2.3 Registration of Deformable Models

To integrate the registration process in the deformable models framework, we need to evaluate the global transformation g from the forces computed at each vertex. Since the registration process does not involve regularizing forces we only consider external forces $\{f_i^{ext}\}_i$. The best global transformation that registers $\mathcal{M}$ with $\{f_i^{ext}\}_i \star \mathcal{M}$ is evaluated by choosing $W_i = V_i + f_i^{ext}$.

The registration of the model iteratively computes a global transformation g such that $g(\mathcal{M}) = \{g(V_i)\}_i$. Let $g_i^{ext} = g(V_i) - V_i$, the application of g on the model can be seen as the application of a deformation field $\mathcal{G}$: $g(\mathcal{M}) = \{g(V_i)\}_i = \{V_i + g_i^{ext}\}_i = \mathcal{G} \star \mathcal{M}$ with $\mathcal{G} = \{g_i^{ext}\}_i$. At each vertex V_i, g_i is the solution of a Newtonian equation similar to (1): $V_i^{t+1} = V_i^t + g_i^{ext}$. In the ICP algorithm, there is no first order (inertial) term and the registration does not involve regularizing forces. g_i^{ext} can thus be assimilated to the external force f_i^{ext} of equation (1).

2.4 Hybrid Deformation Scheme

Using the deformable models framework we can apply free deformation fields to the model. The registration approach limits the possible deformations to a given group of transformations G. We now propose to mix these two frameworks to take advantages of both.

Consider a deformable model submitted to an external forces field $\mathcal{F} = \{f_i^{ext}\}_i$. From $\mathcal{F}$, we can estimate a global deformation field $\mathcal{G} = \{g_i^{ext}\}$. The hybrid force h_i^{ext} acting on vertex V_i is the weighted sum of the local external force and the global transformation (Fig. 1): $h_i^{ext} = \lambda f_i^{ext} + (1-\lambda)g_i^{ext}$ where λ is a locality factor which weighs the local and global components.

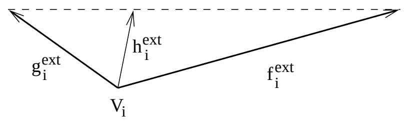

Figure 1. Global, local, and hybrid forces

Using Newtonian mechanics, the hybrid deformation field is $\mathcal{H} = \{h_i\}_i$ where $h_i = (1-\delta)(V_i^t - V_i^{t-1}) + \alpha f_i^{int} + \beta h_i^{ext}$ and $\mathcal{H} \star \mathcal{M} = \{V_i + h_i\}_i$. The hybrid deformation scheme allows us to control the degrees of freedom left to the deformable model through the single locality parameter. The following example illustrates its advantages.

2.4.1 Registration Quality using Hybrid Deformations

The hybrid scheme makes the model more robust to geometric distortions while deforming. The following experiment illustrates this: consider a model $\mathcal{M}$ obtained from range data. We disturb $\mathcal{M}$ and then let it evolve a fixed number of iterations, attracted by the range data, so that it recovers its original shape. Running the deformation process with different deformation schemes, the quality of the reconstruction is evaluated as the pointwise distance between the reconstructed model $\mathcal{M}'$ and $\mathcal{M}$: $d = \sum_i \|V_i - V_i'\|^2$. Fig. 2 shows the values of d for a rigid registration (first point) an affine registration (second point) or a hybrid deformation affinely constrained following a first rigid fit (other points).

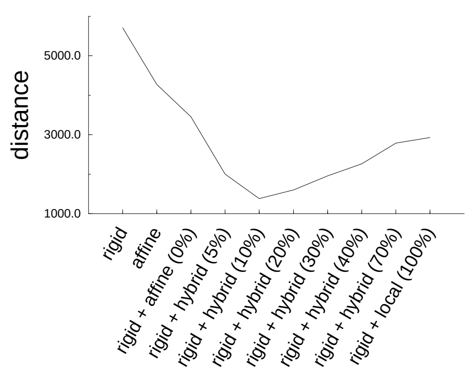

Figure 2. Distance of the deformed model

Neither the completely local deformation ($\lambda = 100\%$) nor the completely affine registration ($\lambda = 0\%$) gives the best fit. If the model is too constrained, the original shape cannot be retrieved. If it has too many degrees of freedom, the surface shows distortions while deforming, such that a point does not keep its geometric properties (a extremal curvature point does not fit a extremal curvature point after deformation).

3 Global Transformation Evaluation

Equation (2) provides a general criterion to evaluate the global transformation g. Its resolution depends on the group of transformations considered.

Best Rigid Transformation. A rigid transformation of a point P is $t_{rig}(P) = RP + T$ where T is a translation vector and R a rotation matrix. It can be shown [10] that $T = \bar{V} - R\bar{W}$ where $\bar{V}$ and $\bar{W}$ are the inertial centers of $\{V_i\}_i$ and $\{W_i\}_i$, respectively, and that R minimizes the criterion $\sum_{i=1}^n \|R\hat{V}_i - \hat{W}_i\|^2$ with $\hat{V}_i = V_i - \bar{V}$ and $\hat{W}_i = W_i - \bar{W}$.

Best Similarity. A similarity (rigid transformation plus a scale factor) can be written $t_{sim}(P) = SRP + T$ where S is a diagonal matrix whose diagonal terms are all equal to the scale factor s. The optimal rotation and translation are evaluated as for the rigid case. The scale factor is computed independently [10] $s = \frac{Tr(R\Sigma_{wv}^T)}{\sum_i \|V_i\|}$ with $\Sigma_{wv} = \sum_i W_i V_i^T$.

Best Affine Transformation. An affine transformation can be written in matrix form in homogeneous coordinates $t_{aff}(P) = AP$. It can be shown [10] that A is a closed form: $A = \Sigma_{wv}\Sigma_{vv}^{-1}$ with $\Sigma_{vv} = \sum_i V_i V_i^T$.

Best B-Spline Transformation. A B-spline transformation of P is $t_{spl}(P) = (f^x(P), f^y(P), f^z(P))^T$ where f^d is a piecewise polynomial function defined as a tensor product of B-spline base functions of a given order. The best spline transformation, given a set of pair of points (V_i, W_i), should minimize a criterion sum of residual distances and a smoothness term $C(T_{spl}) = C_{dist}(t_{spl}) + \rho C_{smooth}(t_{spl})$ where ρ is a weight. Details on solving this system using a conjugate gradient method can be found in [2].

Other Global Transformations. Many other global transformations could be investigated. The use of transformations with more degrees of freedom (such as high degree B-splines or radial basis functions) would allow to generate a wider range of shapes, however, at a great computation cost.

3.1 Axial Constraint

Another advantage of the hybrid scheme is the possibility to introduce domain specific constraints in the deformation process. We have developed an axial constraint [9] to deform 3D models based on an axial symmetrical topology such as vessels. As shown in section 5.4 it gives encouraging results in segmenting vessels from angiographic images.

A 3D model $\mathcal{M}$ with a cylindrical topology can be attached to an axis $\mathcal{A}$ so that it is constrained to bend along that axis. $\mathcal{A}$ is discretized and represented by a polygonal line. Fig. 3 shows a deformed cylinder without axial constraint (left) and with axial constraint (center left).

The axis is a deformable contour. The external force applied to each vertex on the axis, corresponds to the mean

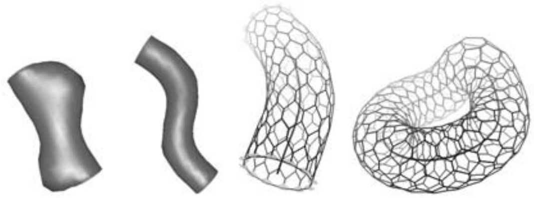

Figure 3. Deformable cylinders

external forces applied to vertices of the surface model. Similarly the global transformation g applied on the surface model is derived from the displacement of the underlying axis. Using the hybrid scheme, the rigidity of the model can be tuned through the locality factor.

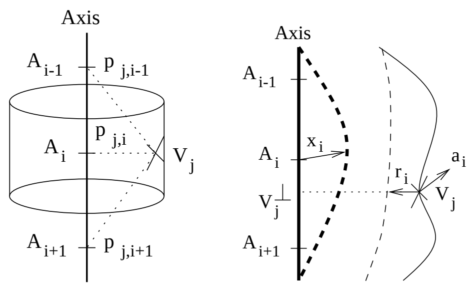

Figure 4. axial forces

Each vertex V_j of the surface model is attached to 3 axis vertices with the weight coefficients $p_{j,i}$ depending on the distance between the vertex V_j and the axis vertices A_{i-1}, A_i and A_{i+1}. Fig. 4 (left) illustrates this link. Thus each axis vertex A_i is linked to a set of vertices $\mathcal{V}_i$. The mean force acting to A_i is $x_i^{ext} = \frac{\sum_{V_j \in \mathcal{V}_i} p_{j,i} f_j^{ext}}{\sum_{V_j \in \mathcal{V}_i} p_{j,i}}$. Regularizing constraints $\{x_i^{int}\}_i$ apply to the axis thus the total force is $x_i = \alpha_x x_i^{int} + \beta_x x_i^{ext}$.

Inversely, each vertex V_j is submitted to a global force composed of an axial force a_j and a radial force r_j. The axial force results from the average displacement of the axis : $a_j = \sum_{k=i-1}^{i+1} p_{j,k} x_k$. The radial force is oriented perpendicular to the axis and tends to keep the vertices the same distance from the axis. Let $V_j^{\perp}$ be the projection of V_j onto the axis and $n_j = V_j^{\perp} V_j / \|V_j^{\perp} V_j\|$ be the radial direction. The mean radius at vertex A_i is $R_i = \sum_{V_j \in \mathcal{V}_i} p_{j,i} \|V_j^{\perp} V_j\|$ and the radial force experienced by V_j from axis point A_i is $r_{j,i} = V_j^{\perp} + \left((1-\rho)\|V_j^{\perp} V_j\| + \rho R_i\right) n_j - V_j$ where ρ is a radial stiffness parameter. The total radial force is then $r_j = \sum_{k=i-1}^{i+1} r_{j,k}$.

In the case of a toroidal topology, the axis is a closed polygon. For a cylindrical object, it owns contours which are bound to extremities of the axis. Fig. 3 shows a torus (right) and a cylinder (center right) deformed using the axial constraint. The global transformation applied onto $\mathcal{M}$ can be written as the deformation field $\mathcal{G} = \{g_i = a_i + r_i\}_i$.

4 2D Case: Snakes

First we implemented the hybrid deformation scheme in the 2D case using the active contours also known as *snakes*. Snakes have been widely used to perform tracking or segmentation in 2D images.

4.1 Example

We illustrate the hybrid deformation scheme by tracking the lips in a sequence of video frames. These images are textured and low-contrast. The active contour is initialized by hand around the lips in the first frame. It then deforms under the action of edge-based forces to fit the lips. The contour obtained after deformation in one image is used as the initialization of the following one. We performed an experiment using a standard local deformation scheme and a hybrid deformation scheme affinely constrained ($\lambda = 20\%$). Fig. 5 shows the active contour around the lips at the sixth (left column) and twelfth (right column) video frame. Local results appears in the top row, while hybrid results are shown in the bottom row.

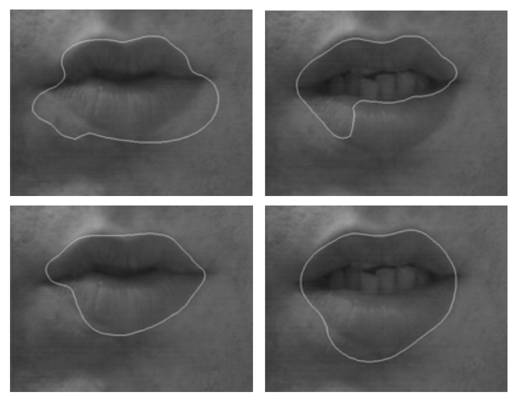

Figure 5. Active contour example

The hybrid scheme keeps better track of the lips than the local one. It performs an affine registration (since the lips globally move inside the image) while adapting the contour shape to the non-rigid deformations of the mouth.

5 3D Case: Simplex Meshes

5.1 Simplex Meshes

Simplex meshes [3] are meshes with constant vertex adjacency and interesting geometric properties. 2-simplex meshes are a natural extension of active contours in 3D and they provide a powerful framework to express regularizing constraints. A 2-simplex mesh of $\mathbb{R}^3$ is a 3-connected mesh. Each vertex V_i can be expressed as a function of its neighbors V_i^1, V_i^2 and V_i^3, its *metric parameters* ϵ_i^1, ϵ_i^2, ϵ_i^3 and its *simplex angle* ϕ_i. The metric parameters and the simplex angle are intrinsic parameters that describe the *shape* of a mesh with a given topology up to a similarity.

The simplex mesh framework is computationally very efficient. Local force computation does not require a minimization step. The example given below requires a few minutes of deformation on a middle range unix workstation.

5.1.1 Shape Constraint

The metric parameters and simplex angles of simplex meshes allow us to define smoothness as well as shape regularizing forces. Without external forces, a simplex mesh submitted to the shape constraint converges toward its reference shape. An example of a face model iteratively returning to its reference shape is given in Fig. 6.

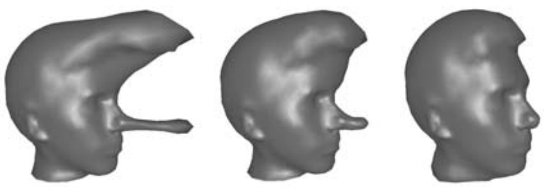

Figure 6. Shape constraint

5.2 Segmentation

Deformable models are used to segment organs in 3D medical images. The geometric representations of anatomical structures extracted are suitable for surgery simulation or the study of pathologies' evolution. Fig. 7 shows complex topology simplex meshes: a liver (left) and brain ventricles (right) models that have been deformed to match an abdominal CT-scan and head MRI data.

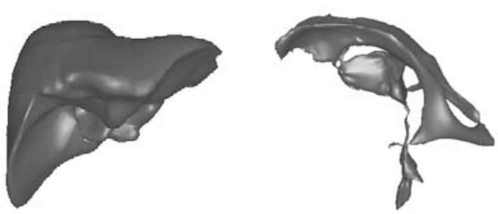

Figure 7. Liver and ventricles models

CT-scan images are low-contrast and the model must be robust enough to deform smoothly when data is noisy or lacking. Moreover the model must be deformable enough to fit the drastic inter-patient variability of abdominal organs.

We performed the segmentation of the liver using the hybrid model combined with the shape constraint mechanism. The model is first registered rigidly then affinely (locality parameter $\lambda = 0$). As the fit gets better and the model is less sensitive to outliers, degrees of freedom are added by smoothly increasing λ up to 40%. Segmentation of the brain ventricles is performed in a similar way (λ is released up to 30%). The λ boundaries are a trade-off between the ability of the model to deform locally and the applied constraints. They are domain dependent.

Fig. 8 (left) illustrates the quality of the segmentation by showing the intersection of the deformed models with one slice of the data.

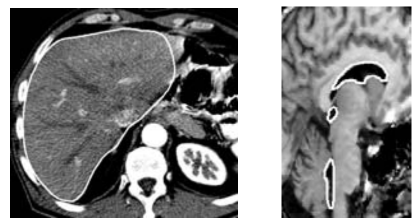

Figure 8. Recovered shapes

5.3 Reconstruction

Deformable models can also reconstruct an object from 3D data coming from range data, point clouds, stereo points, isosurfaces extracted from volumetric images, etc. We illustrate the ability of the hybrid deformation scheme to perform face reconstruction.

In Fig. 9 (left) appear the reference model with an adapted geometry and the range data of a face in a slightly different position. The idea is to use the geometry of our reference model to recover the shape of the other face. Thus, points of the nose must deform toward points of the nose, the eyes must fit the eyes, etc.

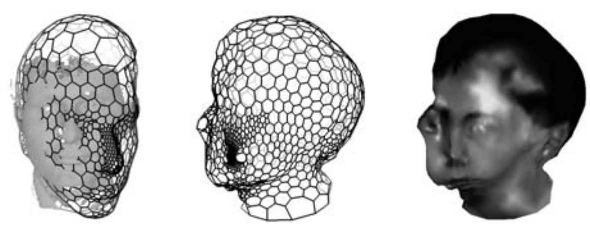

Figure 9. Local deformations

Fig. 9 (center and right) shows the model after local deformation. The shape of the face has been recovered but obviously the geometry was not preserved. The hybrid deformation scheme first applies global transformations to provide a better initialization state before introducing local deformations. Fig. 10 illustrates a few stages of the reconstruction. The textured view shows how the geometry of the face was preserved.

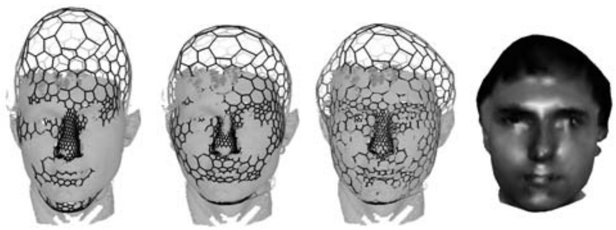

Figure 10. Hybrid deformation stages

5.4 Vessel Segmentation

Segmentation of vessels around an aneurism is important for diagnosis and pathological understanding. Physicians need to determine from 3D angiographic images, the connection between vessels and the aneurism. Using deformable axially constrained models we perform a segmentation of the aneurism and the surrounding vessels. The extracted model is easy to visualize and it allows one to perform quantitative measures such as volumetric evolution.

Axially constrained models are represented by 2-simplex meshes with a 1-simplex mesh axis. Fig. 11 (left) shows the initialization of deformable models inside a volumetric image. The aneurism is modeled by a sphere affinely constrained through the hybrid deformation scheme. Vessels are initialized as cylinders restricted by a hybrid axial constraint. The result of the deformation process is shown in Fig. 11 (center) and the final model obtained after topological operations is shown on the right.

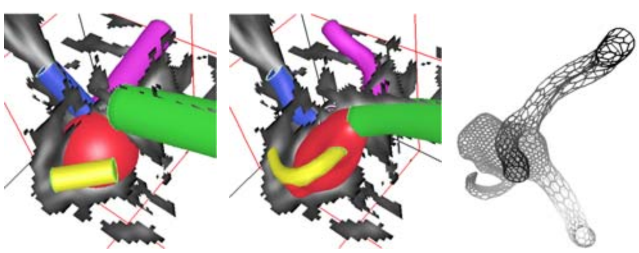

Figure 11. Vessel models

6 Conclusions

We introduced a hybrid deformation scheme that integrates in a single framework registration and local deformations of deformable models. Hybrid deformations allow one to deform meshes with good geometric property preservation. This is a fundamental hypothesis to perform statistical analysis of shape variations from 3D image databases. We expect to improve segmentation accuracy by adding to reference models an a priori knowledge of admissible deformations.

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