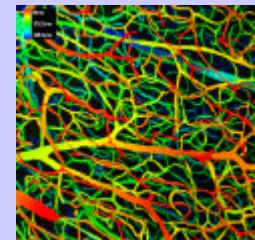
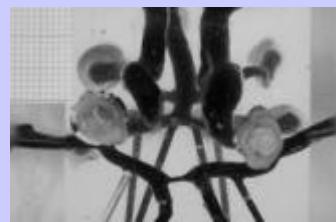


# *Medical Image Analysis*



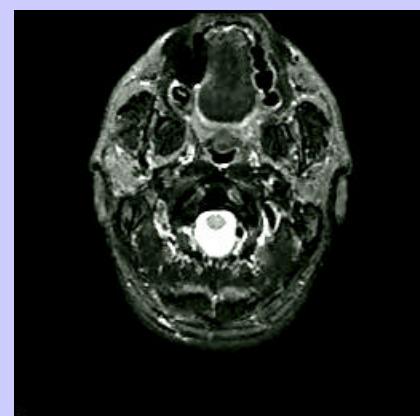
*Application to 3D angiography*

*Grégoire Malandain*



## *2D Visualisation*

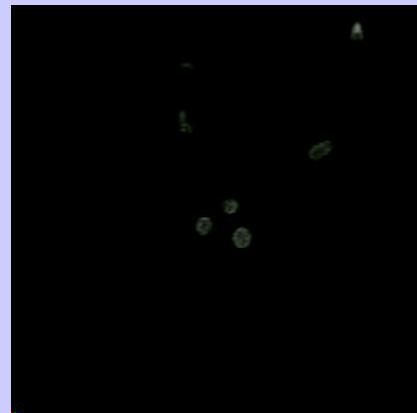
Adapted to large *volumes*



## 2D Visualisation

Adapted to large *volumes*

... but not to vessels



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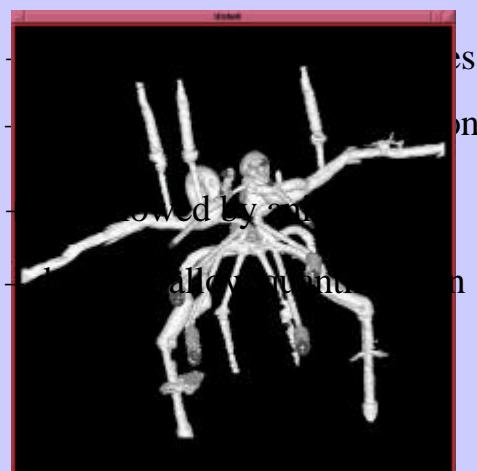


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## Maximum Intensity Projection



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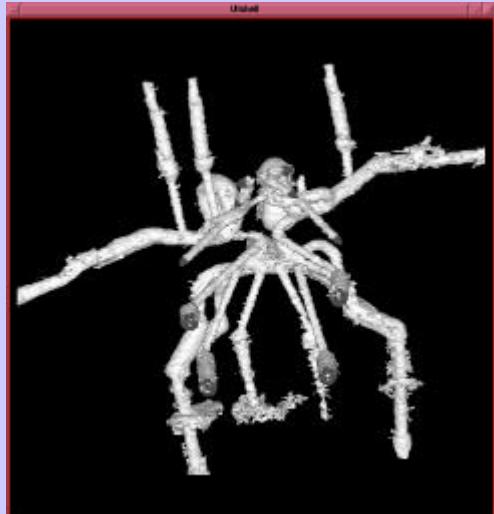


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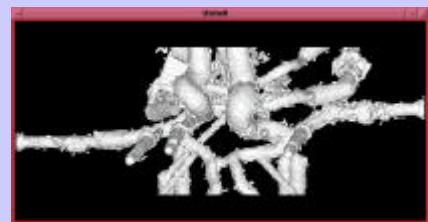
## Isosurfaces



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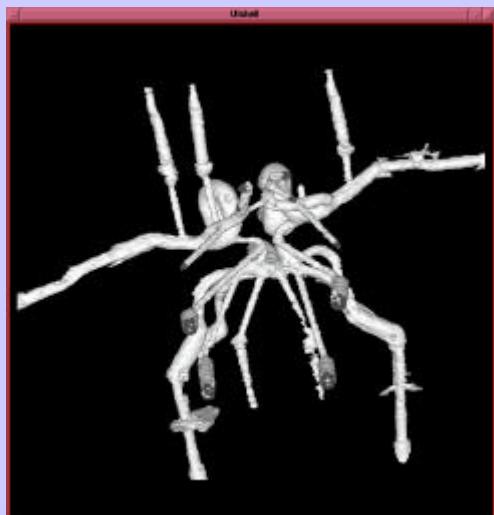


Threshold = 14



5

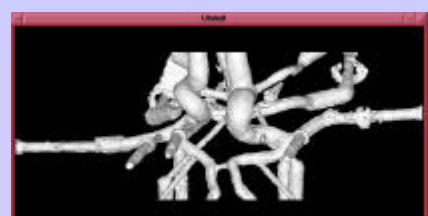
## Isosurfaces



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Threshold = 29



6

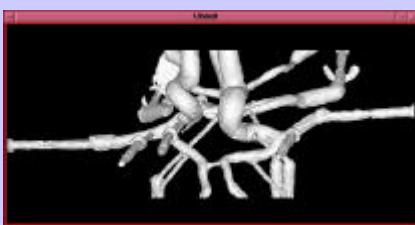
## Isosurfaces



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Threshold = 44

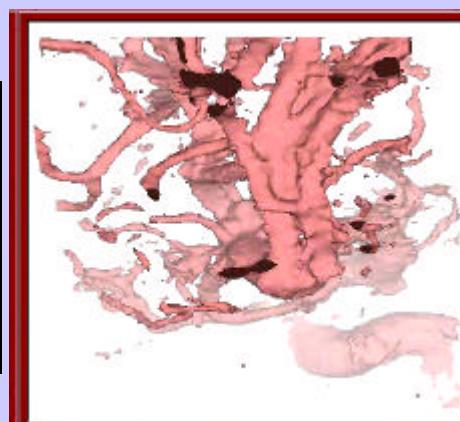
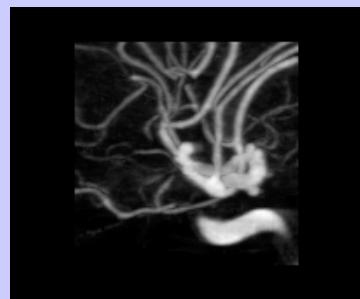


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## Isosurfaces

– Effectiveness of segmentation?

### Topology changes



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## Segmentation issues

- **Topological correctness**

Extraction of the central line

Separation of adjacent structures

Retrieval of junctions

- **Geometrical correctness**

Measure of length, diameter, area, ...

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## Principles

- **Vessel modelling**
- **Vessel characterisation**
- **Central line extraction**
- **Radius estimation**

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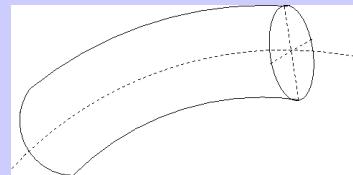


10

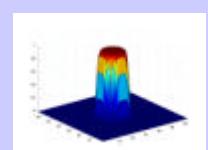
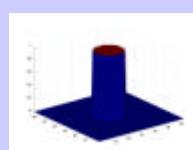
## 3D vessels modelling

- **Tubular-like structures**

- Shape of the section?



- Intensity distribution inside the section?



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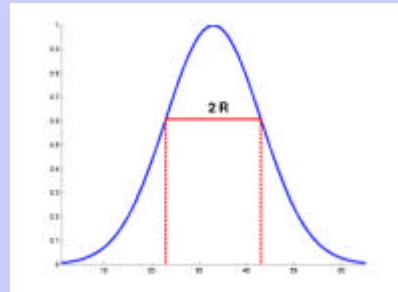


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## Intensity profile: 1D derivatives



$$I(M) = C e^{-(OM^2/2R^2)}$$

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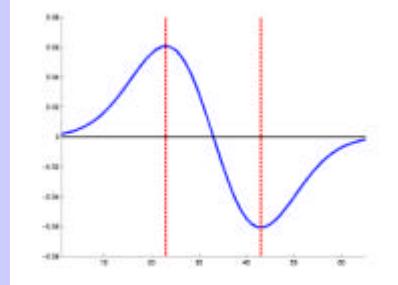


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## Intensity profile: 1D derivatives



$$I(M) = C e^{-(OM^2/2R^2)}$$



$$\frac{\partial I}{\partial x}$$

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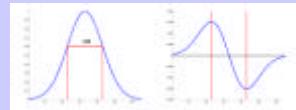


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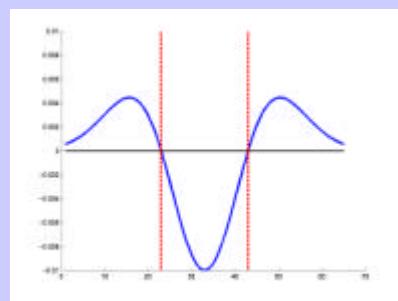


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## Intensity profile: 1D derivatives



$$I(M) = C e^{-(OM^2/2R^2)}$$



$$\frac{\partial^2 I}{\partial x^2}$$

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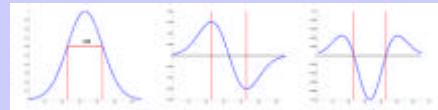


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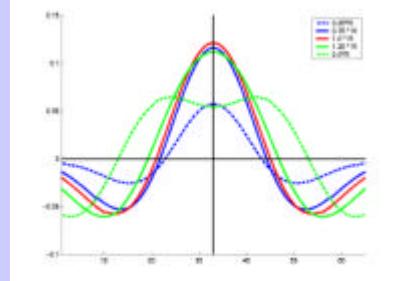


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## Intensity profile: 1D derivatives



$$I(M) = C e^{-(OM^2/2R^2)}$$



Maximal response

$$\Leftrightarrow Resp(M) > Resp(M \pm \epsilon)$$

$$Resp(M) = \frac{\partial I}{\partial x}(M - r) - \frac{\partial I}{\partial x}(M + r)$$

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## Intensity profile: 3D derivatives

- Matrix of second derivatives: Hessian

$$\mathbf{H} \mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} = [\mathbf{V}_1 \ \mathbf{V}_2 \ \mathbf{V}_3] \begin{bmatrix} \mathbf{I}_1 & & \\ & \mathbf{I}_2 & \\ & & \mathbf{I}_3 \end{bmatrix} \begin{bmatrix} {}^T \mathbf{V}_1 \\ {}^T \mathbf{V}_2 \\ {}^T \mathbf{V}_3 \end{bmatrix} \quad \mathbf{I}_i = I_{v_i v_i}$$

- Shape characterisation (white structure on dark background):

Sheet-like structure:  $|I_1| \gg |I_2|, |I_3| \quad I_1 < 0 \quad I_2, I_3 \approx 0$

Tubular structure:  $|I_1|, |I_2| \gg |I_3| \quad I_1, I_2 < 0 \quad I_3 \approx 0$

Cylinder direction:  $\vec{\mathbf{V}}_3$

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## Intensity profile: 3D response

- Vector of first derivatives: gradient  $\nabla I = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}$
- Cylinder direction:  $\vec{V}_3$
- Directions in the orthogonal section:  $\vec{V} = \cos(\alpha) \vec{V}_1 + \sin(\alpha) \vec{V}_2$

- 3D response

$$R(M) = \int_0^{2p} -\left( \vec{\nabla} I(M + r \vec{v}) \cdot \vec{v} \right) d\alpha$$

Maximal response  $\Leftrightarrow R(M) > R(M \pm e \vec{v}_i), i = 1, 2$

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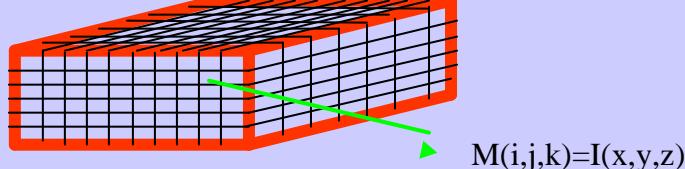
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## Derivatives computation

- Discrete signal: 3D lattice of voxels



$$\frac{\partial}{\partial x} I(x, y, z) ?$$

- Convolution with a low-pass filter

$$\frac{\partial}{\partial x} I(x, y, z) \approx \frac{\partial}{\partial x} (G(x, y, z) * I(x, y, z)) = \left( \frac{\partial G}{\partial x} \right) * I$$

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## Practical issues

- Vessel of unknown radius  $\sigma_0$   
Gaussian intensity profiles
- Derivatives computed with a Gaussian  $\sigma$   
Response computed at a distance  $\theta\sigma$
- Multi-scale extraction: several  $\sigma$ 's  
How to compare scales?

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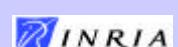
## Multiscale issues

- Comparing scales
  - $\gamma$ -derivatives:  $\partial_{x,g} = s^g \partial_x$
  - invariance w.r.t. scaling:  $\gamma=1$
- Response extrema in space and in scale
- The profile modelling by a Gaussian
  - Optimal value of  $\theta$
  - Estimation of  $\sigma_0$

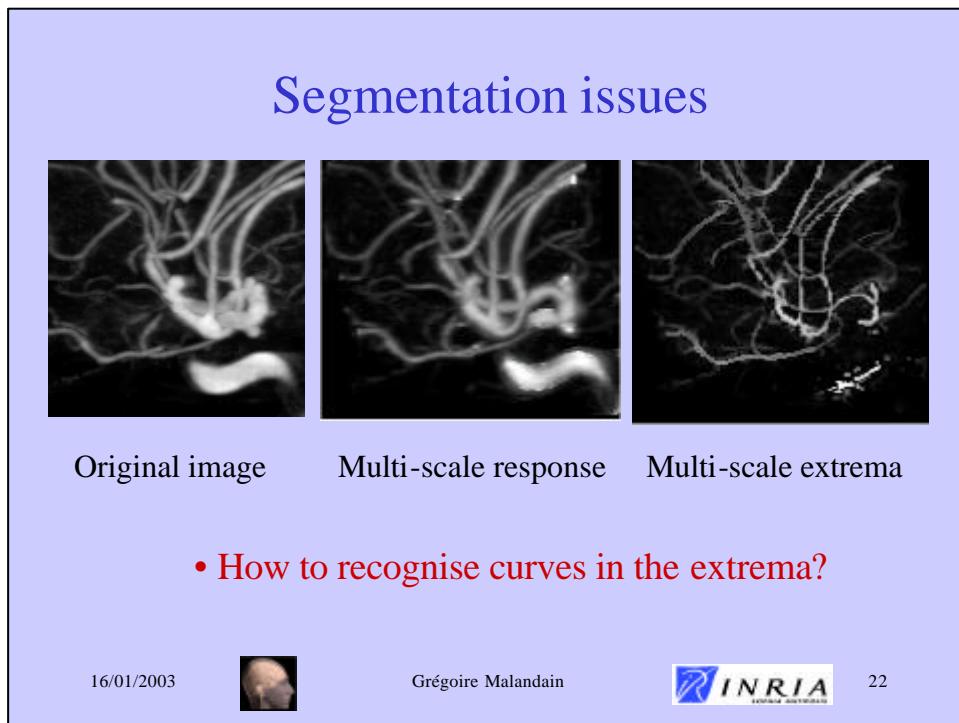
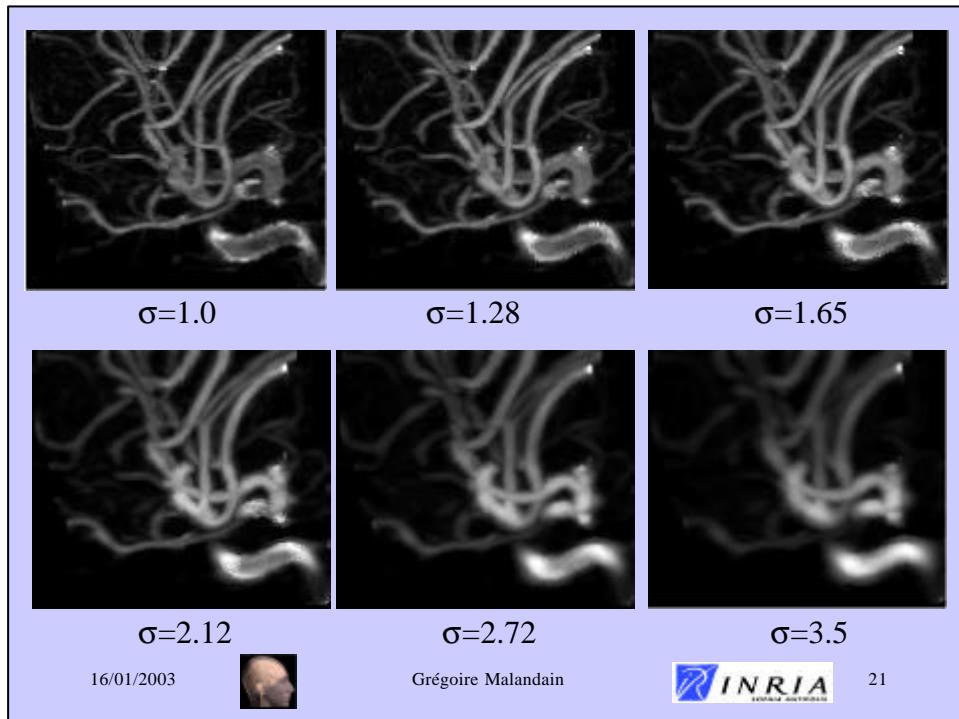
16/01/2003



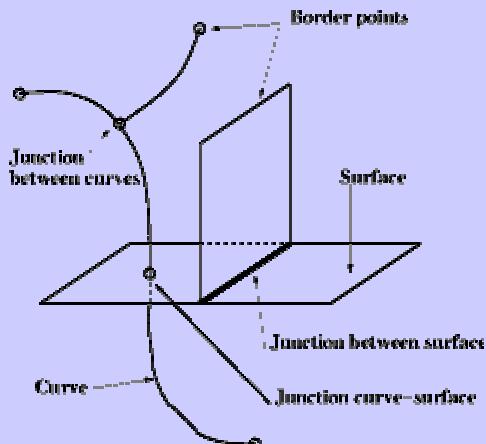
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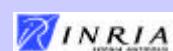
## 3-D Digital Topology



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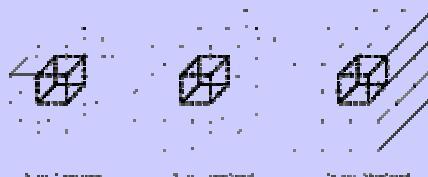


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## 3-D Digital Topology



$C^*$  Number of (26-)connected components of the foreground in the (26-)neighborhood

$\bar{C}$  Number of (6-)connected components of the background in the (18-)neighborhood, (6-adjacent to the centre)

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## 3-D Digital Topology

- Thinning the extrema

Iterative removal of simple points  $(C^* = \bar{C} = 1)$

- Removing small pieces of curve

Simple curves  $(C^* = 2, \bar{C} = 1)$

are ended either by curves junctions  $(C^* > 2, \bar{C} = 1)$

or borders (i.e. simple points)

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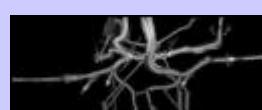
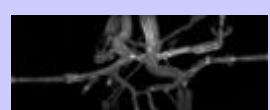
Original image



Multiscale response



Multiscale extrema



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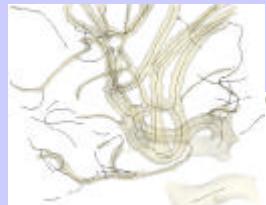


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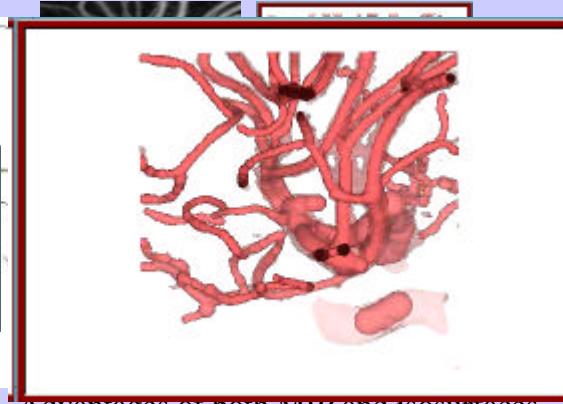


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## Reconstructions



- Simple curves smoothing



Advantages of both MIP and isosurfaces

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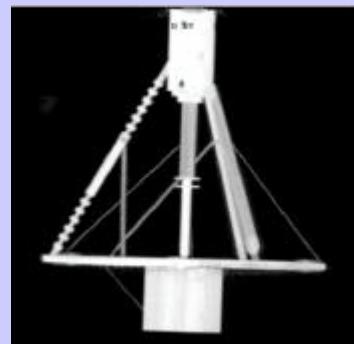
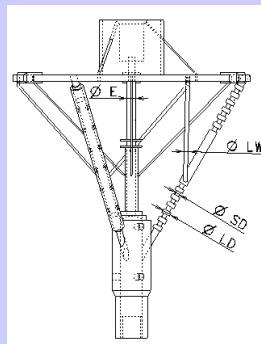


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## About radius estimation



LW = 2 mm E = 4 mm

Voxel = 0.267 mm<sup>3</sup>

SD = 2 mm LD = 4 mm

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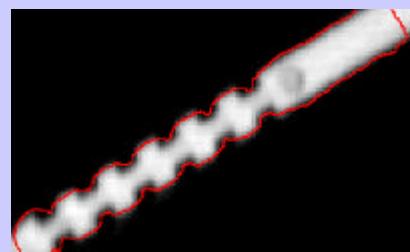
## About radius estimation

- **Estimation**

LW: 2.018 mm E: 3.884-3.968 mm

SD: 2.14 mm LD: 3.6 mm

- is disturbed by 3-D computation



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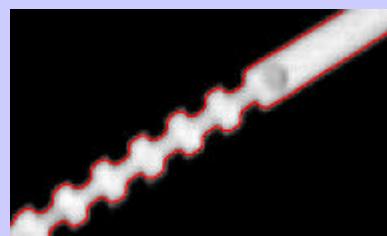
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## About radius estimation

- Estimation in 2D resampled sections



- Better but computationally more expensive

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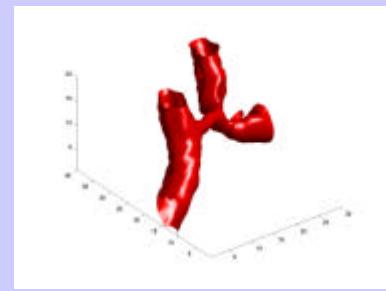
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## 2D orthogonal sections resampling

- Subvolume



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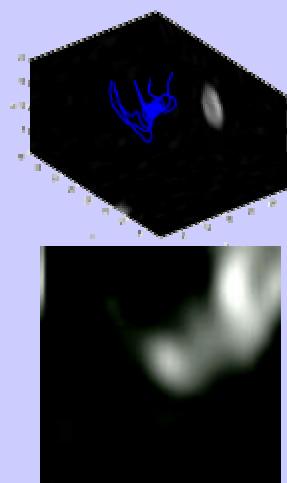
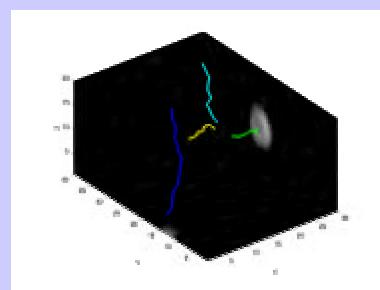
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## 2D orthogonal sections resampling

- Subvolume (scanner)



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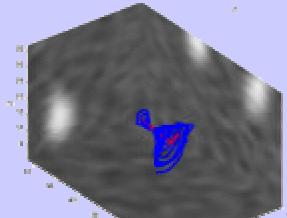
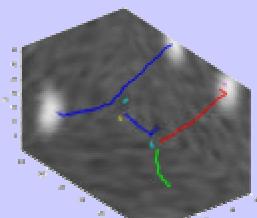
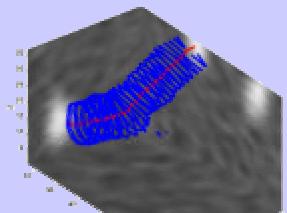
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## 2D orthogonal sections resampling

- Subvolume (MRA)



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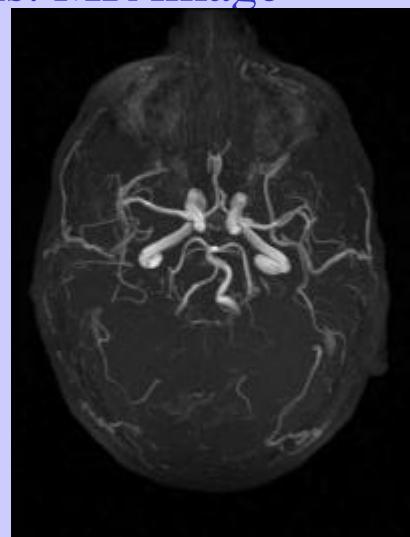


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## 2-D cross-sections: MR image



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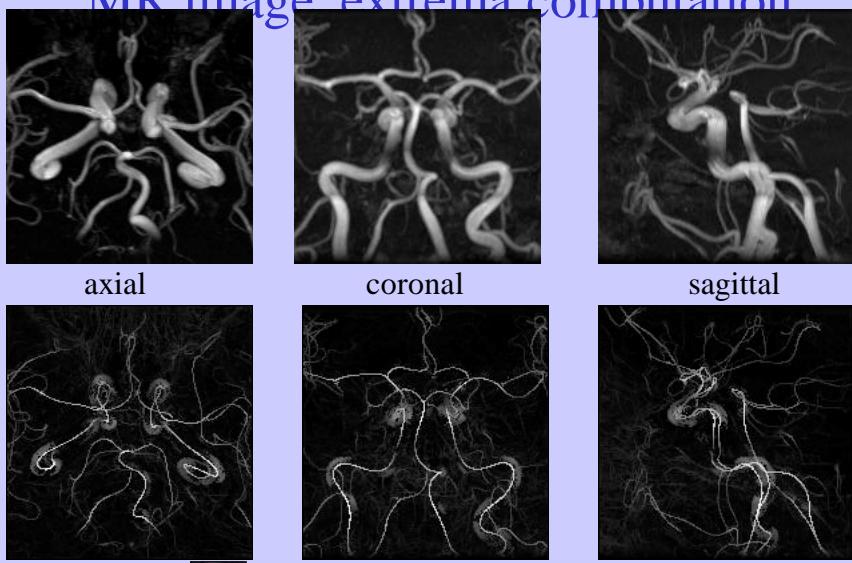


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## MR image: extrema computation



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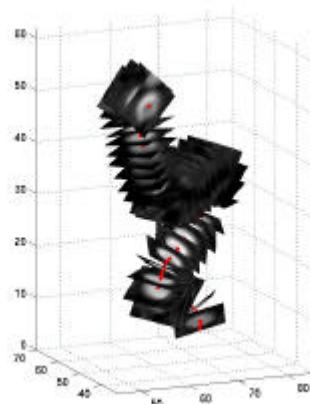


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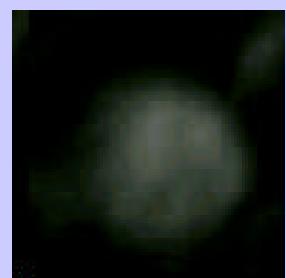
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## MR image: 2-D cross-sections



Resampling with  
cubic splines

Travelling inside the  
vessel



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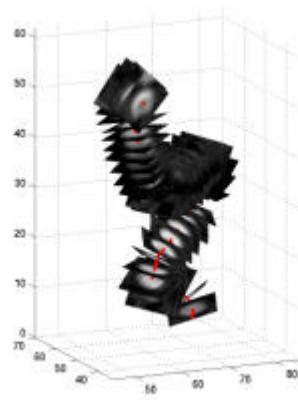
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## 2-D cross-sections: segmentation

- ✓ Thresholding: iso-contours
- ✓ Edges detection
  - ↳ zero-crossings

$$\Delta I = 0$$
$$\vec{\nabla}I \cdot H\vec{I} \cdot \vec{\nabla}I = 0$$

- ✓ Sub-pixel accuracy



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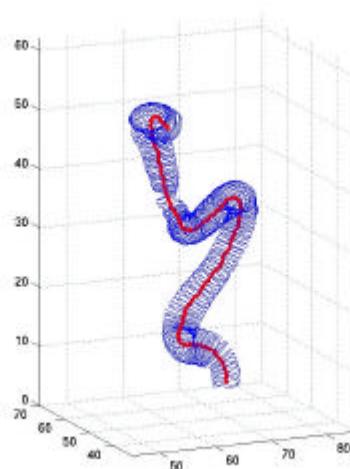


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## 2-D cross-sections: segmentation



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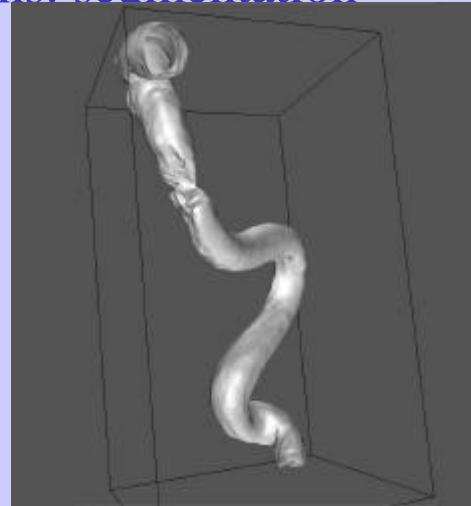
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## 2-D cross-sections: segmentation

- ✓ allows to build a 3-D mesh  
(computational geometry, Prisme team)
- ✓ numerical simulation



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## Synthetical model



Known vessels radii

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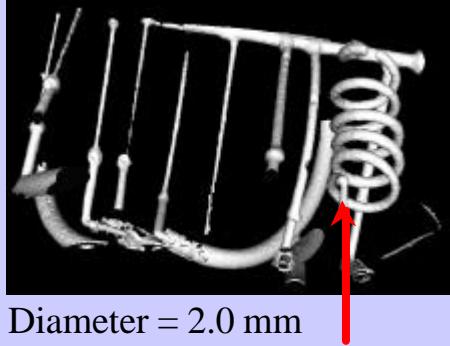
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menu

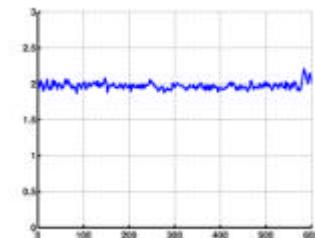
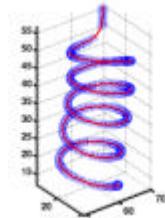
## 2-D cross-sections: segmentation



Diameter = 2.0 mm

$$\vec{\nabla}I \cdot H I \cdot \vec{\nabla}I = 0$$

derivatives with gaussian of  
 $\sigma = 0.5$  voxel



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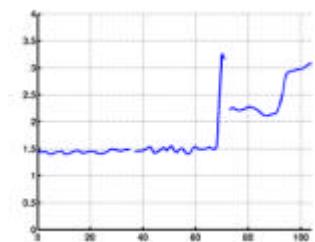
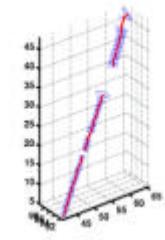
## 2-D cross-sections: segmentation



Diameter < 1.0 mm

$$\vec{\nabla}I \cdot H I \cdot \vec{\nabla}I = 0$$

derivatives with gaussian of  
 $\sigma = 0.5$  voxel



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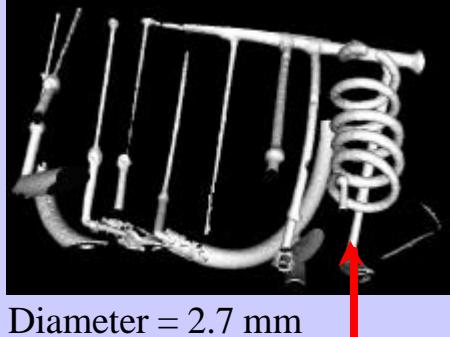


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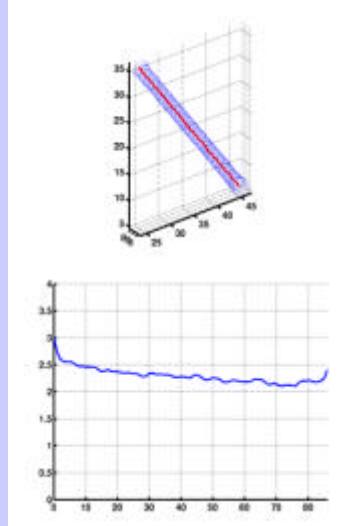
## 2-D cross-sections: segmentation



Diameter = 2.7 mm

$$\vec{\nabla}I \cdot H I \cdot \vec{\nabla}I = 0$$

derivatives with gaussian of  
 $\sigma = 0.5$  voxel



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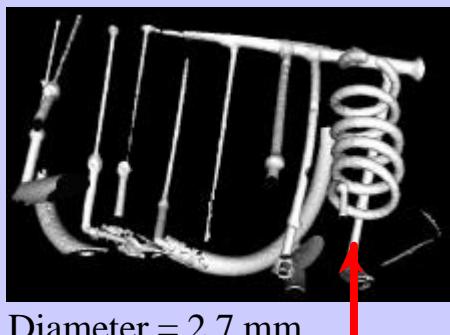


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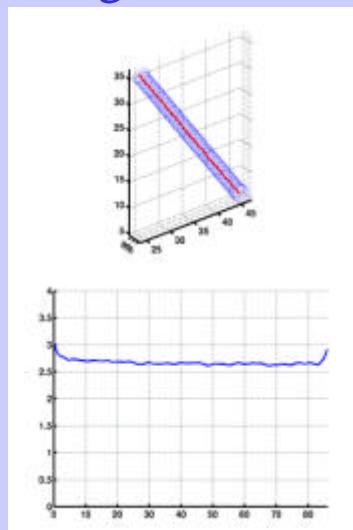
## 2-D cross-sections: segmentation



Diameter = 2.7 mm

$$\vec{\nabla}I \cdot H I \cdot \vec{\nabla}I = 0$$

derivatives with gaussian of  
 $\sigma = 1.0$  voxel



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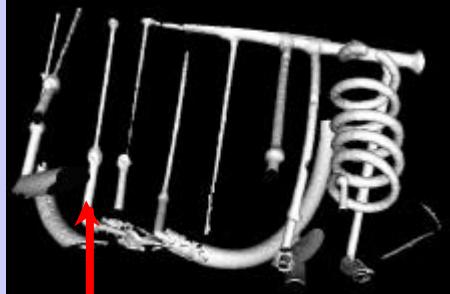


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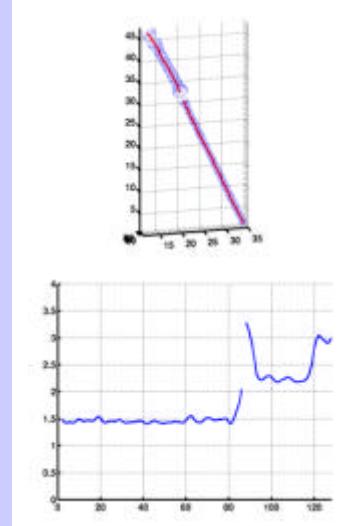
## 2-D cross-sections: segmentation



Diameter < 1.0 mm

$$\vec{\nabla}I \cdot H I \cdot \vec{\nabla}I = 0$$

derivatives with gaussian of  
 $\sigma = 0.5$  voxel



16/01/2003

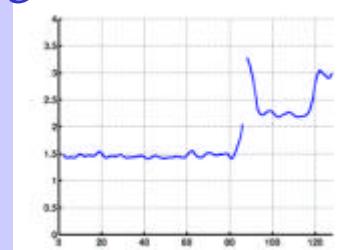
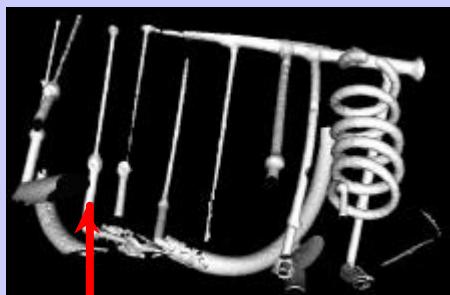


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## 2-D cross-sections: segmentation



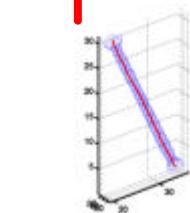
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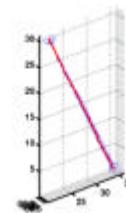
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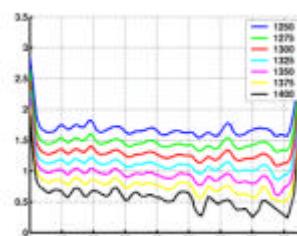
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1250



1400



23

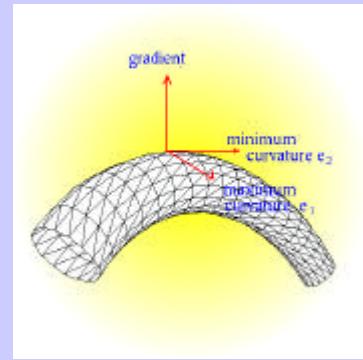
## Pre-processing?

- Isotropic diffusion

$$\frac{\partial I}{\partial t} = \Delta I = \operatorname{div}(\nabla I) \Rightarrow I(t) = G_t * I(0)$$

- Anisotropic diffusion

$$\begin{aligned}\frac{\partial I}{\partial t} &= \operatorname{div}(g(|\nabla I|)\nabla I) \\ &= f_1 I_{\nabla I \nabla I} + f_2 (\Delta I - I_{\nabla \nabla I})\end{aligned}$$



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## Pre-processing?



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## Tomographie à transmission et mouvement : application à l'angiographie rayon X cardiaque

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## Motivations

- Contexte diagnostique et interventionnel léger : détection / quantification 3D de sténoses
- Contexte interventionnel lourd : modèle patient pour la chirurgie mini-invasive robotisée

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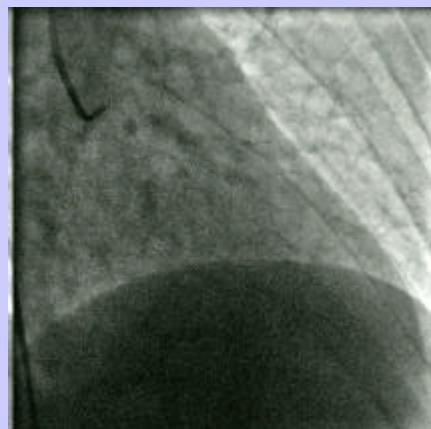


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# Angiographie cardiaque rotationnelle



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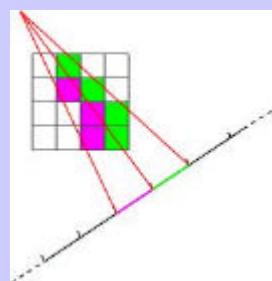
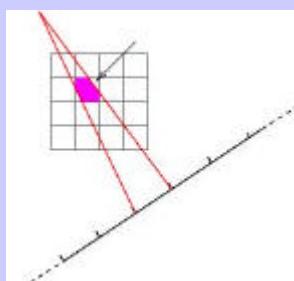
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## Problème

- Voxels  $v_i$
- Pixels  $p_j$
- $R_j$  vecteur des contributions des voxels au pixel  $p_j$



$$R_j \cdot v = p_j$$
$$R \cdot v = p$$

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## Résolution

- Système linéaire de grande taille,  
sous-déterminé (+bruit de mesure)
- Méthodes ( $R, p$ )  $\rightarrow v$  :
  - analytiques (FBP)
  - algébriques (ART, SIRT)
  - stochastiques (EM)
  - optimisation (CG)

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## Tomographie statique

- Observation en transparence (rayons-X)
- Plusieurs angles de vues
- ~~Objet statique~~



La matrice des contributions des voxels aux pixels est  
*modifiée par le mouvement* de l'objet observé

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## Tomographie dynamique

- Pour chaque vue :
  - Pour chaque voxel :
    - *on déforme son centre*
    - on projette le centre déformé
    - on l'attribue au pixel dans lequel la projection arrive

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## Tomographie dynamique

- Calcul de la déformation :
  - segmentation (analyse multi-échelle, intensité)
  - appariement
  - ajustement de faisceau
  - calcul de déformation 4D paramétrique

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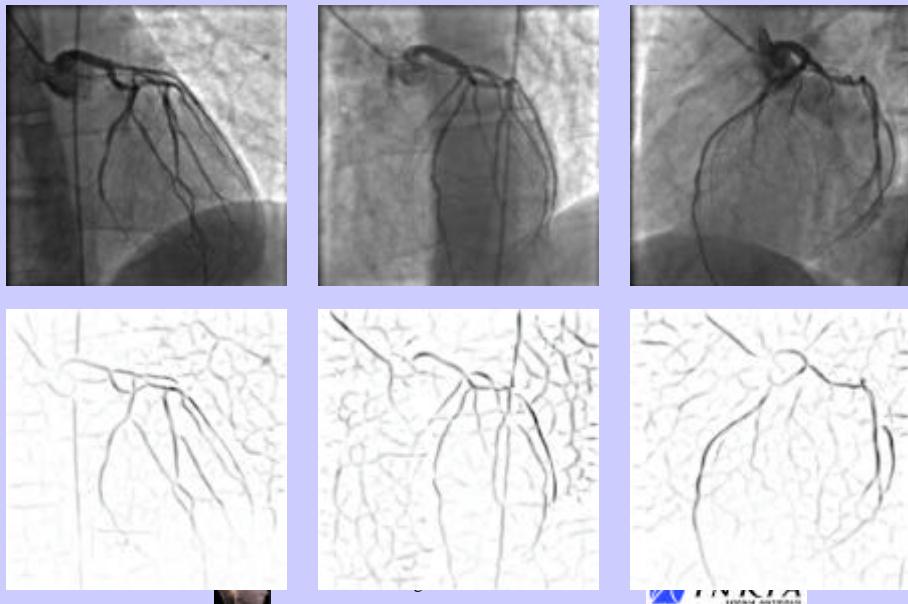


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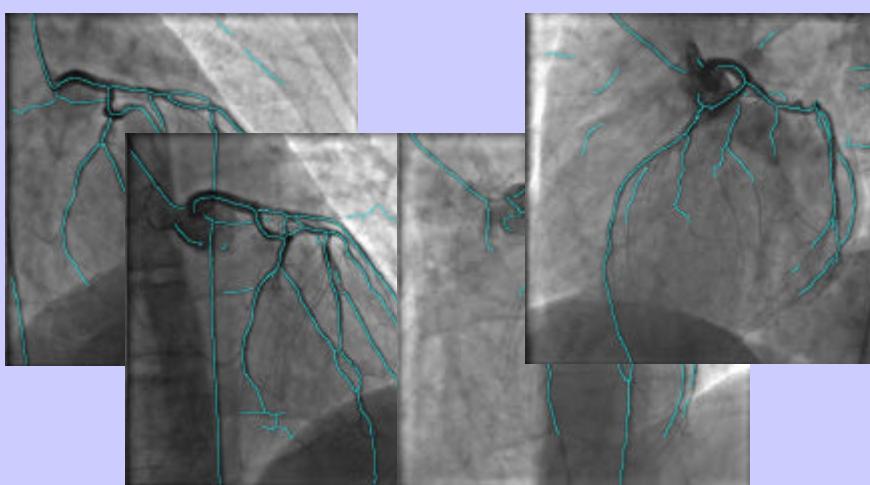


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## Results - Matched views



## Segmentation



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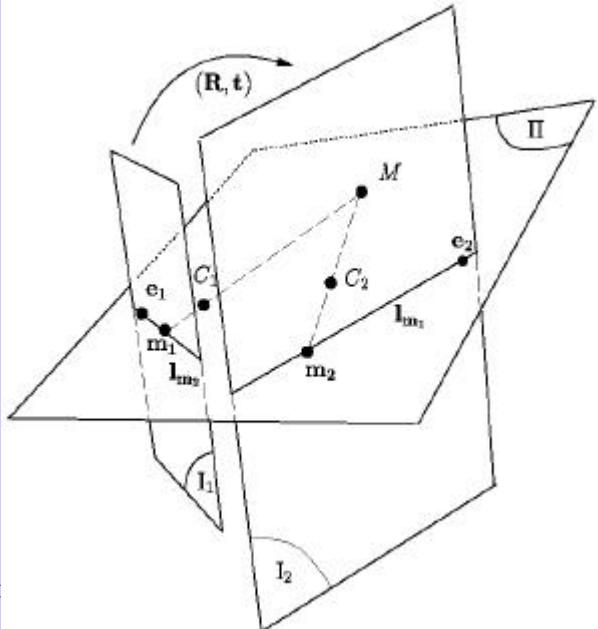


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## Principle of binocular stereovision

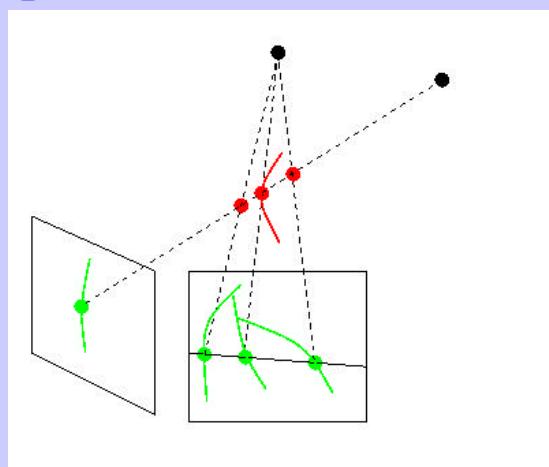


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RJA

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## Principle of binocular stereovision



Binocular matching may be ambiguous

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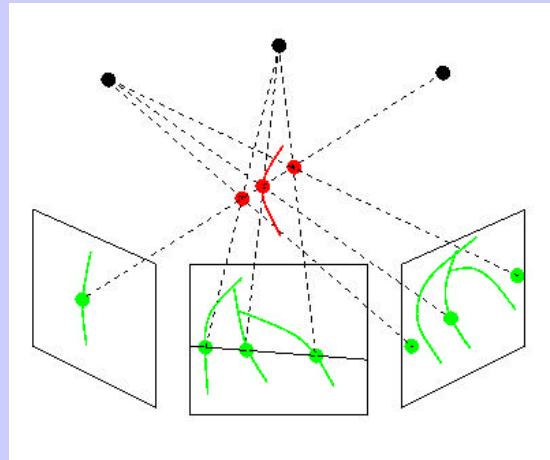


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## Principle of trinocular stereovision



The third view often desambiguates the matching

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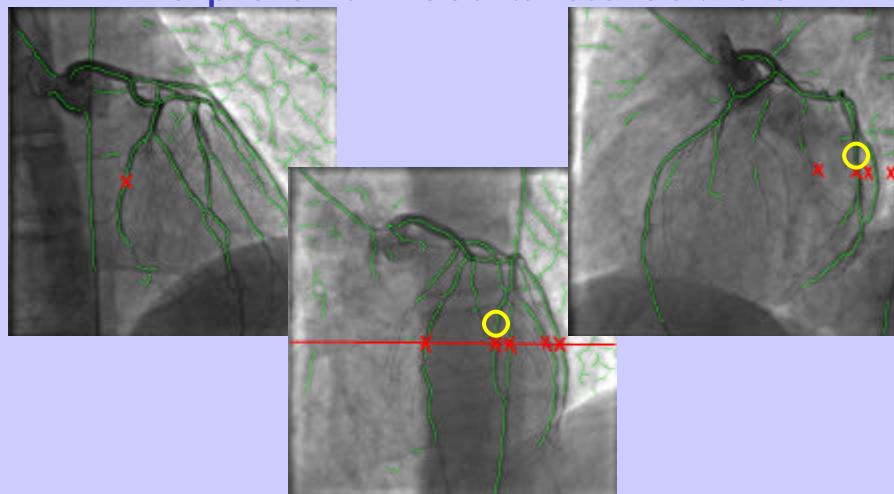


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## Principle of trinocular stereovision



The third view often desambiguates the matching

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## Appariement / Ajustement



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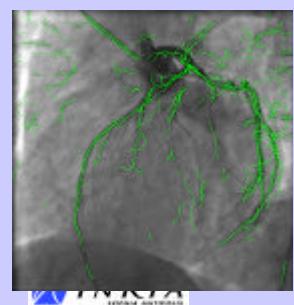
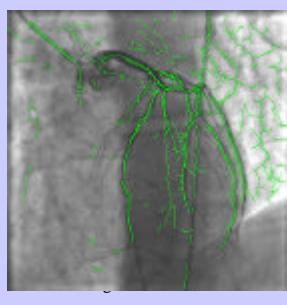
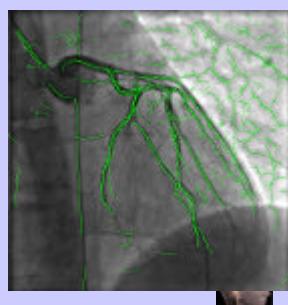


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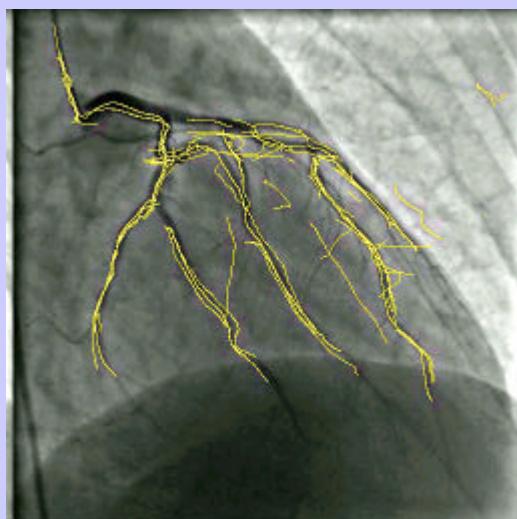


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## Results - 3D Model Reprojection



## Déformation 4D



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## Formation de l'image

- Intégrales des atténuations le long des rayons :  $I'(x) = -\mu I(x)$

$$I(x) = I(0) e^{- \int_R^x m} \quad \int_R^x m = \log(I(x)) - \log(I(0))$$

- On néglige :
  - l'incohérence du faisceau
  - le rayonnement diffusé primaire (collimation)
  - le rayonnement diffusé secondaire ( détecteur)
  - le post-traitement sur les images

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## Résultats : séquence 4D (3D+t)

