A Near-Incompressible Poly-Affine Motion Model for Cardiac Function Analysis

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Abstract. Understanding the motion of the heart through the cardiac cycle can give useful insight for a range of different pathologies. In particular, quantifying regional cardiac motion can help clinicians to better determine cardiac function by identifying regions of thickened, ischemic or infarcted tissue. In this work we propose a method for cardiac motion analysis to track the deformation of the left ventricle at a regional level. This method estimates the affine motion of distinct regions of the myocardium using a near incompressible non-rigid registration algorithm based on the Demon's optical flow approach. The global motion over the ventricle is computed by a smooth fusion of the deformation in each segment using an anatomically aware poly-affine model for the heart. We apply the proposed method to a data-set of 10 volunteers. The results indicate that we are able to extract reasonably realistic deformation fields parametrised by a significantly reduced number of parameters compared to voxel-wise methods, which better enables for statistical analyses of the motion.

1 Introduction

Better understanding the motion of the heart through the cardiac cycle is crucial in aiding diagnosis and therapy planning for patients with heart defects and in particular for those that are known to have deformed ventricular shape. However, tracking cardiac motion from 3D images is a difficult task due to the complex movement of the myocardium through the cardiac cycle.

The clinical challenge is to capture the apparent cardiac motion from the available data (i.e 3D cine MRI sequences), and for this we can apply non-rigid registration algorithms. In this case we require methods that are not only fast but also reproducible and robust (able to handle noisy and low resolution images). In order to compare the heart beat motion of a number of patients, we also require the tracked motion deformation to be characterised by a small number of parameters. Rigid-body or affine motion would do this but is not sufficient in capturing the observed dynamics. Therefore we are interested in finding a compromise between rigid (or affine) and non-rigid deformations.

A recently proposed method for tracking cardiac motion using cine MRI is the incompressible log-domain Demons algorithm (iLogDemons for short) [1].

This method has the nice advantage of ensuring near incompressibility in the myocardial region; a realistic constraint for the heart given that the myocardial muscle volume changes by around 5-10% during the cycle. However, the motion is highly localised since the deformation is considered on a voxel-by-voxel basis making the method sensitive to image noise and constrained by a high number of degrees of freedom. Therefore we are interested in tracking the motion in a more regional manner to capture a more realistic global deformation as well as including some anatomical priors in the regional regularisation process.

For that purpose, an interesting regional regularisation method was proposed in [2] to register mandibles by using the log-domain Demons algorithm [3] and in each region estimating the affine transformation from the resulting deformation field and fusing to a global deformation using the poly-affine model proposed by Arsigny et. al [4]. A poly-affine model was applied in cardiac imaging in [5] for 2D+t multi-modal images. In this last work the poly-affine model is based on an adaptive grid to determine the poly-affine regions. When going to 3D, interpretation of the results could be made easier with a lower number of regions that are anatomically grounded. In [5] the regions are determined on the fly by the images, and are thus without inter-subject reproducibility. A 3D combination of locally affine transformations was used in [6] as an initialisation step to a free-form deformation for cardiac image segmentation. This approach could be improved by coupling the poly-affine deformation with the non-rigid deformation rather than using only as an initialisation.

Inspired by the method of Seiler et. al [2], we propose in this paper to track cardiac motion by estimating an affine transformation in given regions of the left ventricle (LV) myocardium from a computed Log-Demons velocity field [3], with added penalisation to control the compressibility of the tissue. We apply the proposed method to estimate the left-ventricular motion of a 3D data-set of 10 volunteers from the STACOM 2011 MICCAI workshop motion tracking database [7]. We compare the results to the iLogDemons algorithm to deduce that we are able to obtain similar results at a significantly lower degree of parametrisation, which enables statistical analyses to be applied directly to the reduced parametrisation rather than the full iLogDemons velocity field.

2 Cardiac Motion Tracking with a Near-Incompressible Log-Domain Poly-Affine Model

We propose an algorithm for regional cardiac motion tracking that utilises the log-domain Demons algorithm (LogDemons) to estimate the motion of the left ventricle at a local level, in a given set of physiologically meaningful regions. This way, we can define a diffeomorphic transformation from one image to another. From this deformation, we estimate the affine parameters in each region to determine a global affine transformation to give a more regional based motion for each segment. The regional deformation fields are fused in a smooth manner using the poly-affine model. The key contribution of this work is an added penalisation term to the affine parameter estimation to control the amount of

compressibility we allow in each region, as well as an added regularisation term to control the similarity between regions, both formulated as efficient quadratic criteria.

AHA Left Ventricle Segmentation Using the American Heart Association (AHA) standardised myocardial segmentation, we can define anatomically meaningful regions of the ventricle [8]. The recommendation given by the AHA is to divide the left ventricle of the heart into 17 regions with six regions for the basal area (1-6), six regions for the mid area (7-12) and five for the apical areas (13-17).

Log-Domain Demons Registration We are interested in tracking the motion of the heart from a reference time point (in this case we use end diastole as the reference) to the remaining time points along the cardiac sequence. To do this we want to estimate the transformation ϕ that minimises the distance between the reference image R and the target image T. For this we employ the Log-Demons algorithm which has the key property that the transformations are constrained to be diffeomorphic (therefore don't allow folding and are invertible), as well as enabling efficient computation in the log-domain by integrating stationary velocity fields using the exponential scaling and squaring method [3].

Poly-Affine Registration The poly-affine registration algorithm proposed in [4] and extended further in [9] allows to fuse locally affine transformations into a global diffeomorphism using weight functions. The method is suitable for cardiac motion tracking due to the fact that the deformations are computed in the log-Euclidean framework and therefore has the advantage that the transformations are invertible (and the inverse is also a poly-affine transformation).

Poly-Affine LogDemons In Seiler et.al [2], the authors propose a method to estimate a poly-affine model from a log demons deformation field. Using homogeneous co-ordinates, the parameters of the poly-affine model can be defined for points x in Cartesian co-ordinates as

$$\log(T) \stackrel{\text{def}}{=} \log \begin{pmatrix} A & t \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} M \\ 0 \end{pmatrix} \tag{1}$$

where log is a principal matrix logarithm, A is the linear part of the transformation, t its translation, and M a 3×4 matrix. For each segment the affine deformation fields parameterised by the M_i matrices are fused to a global deformation field using the poly-affine model:

$$\boldsymbol{v}_{poly}(x) = \sum_i \omega_i(x) M_i x, \tag{2}$$

where ω_i is a parameter controlling the weight of the i^{th} region for each voxel x. Eqn. 2 can be estimated by a linear least squares problem with the least squares

error with respect to the observed velocity field $\boldsymbol{v}(x)$ (in this case computed using the LogDemons algorithm) given by:

$$C(M_1, M_2, \cdots, M_N) = \int_\Omega \parallel \sum_i \boldsymbol{v}_{poly}(x) - \boldsymbol{v}_{obs}(x) \parallel^2 dx. \tag{3}$$

Ω defines the mask to restrict the estimation within the myocardium (1 inside the binary mask of the myocardium, 0 outside). As shown in [2] the log affine parameters M_i can be estimated by the least-squares minimisation problem given in Eq. 3 to give

$$M = B\Sigma^{-1}, \tag{4}$$

where $M = [M_1 M_2 \cdots M_3]$, $B_i = \int \omega_i(x) \cdot \boldsymbol{v}(x) \cdot x^T dx$ and $\Sigma_{ij} = \int \omega_i(x) \cdot \omega_j(x) \cdot x \cdot x^T dx$. Equivalently, the least-squares solution can be written in terms of vectors:

$$\bar{M} = (\Sigma \otimes I_3)^{-1} \cdot \bar{B}, \tag{5}$$

where $\bar{M}$ (resp. $\bar{B}$) is the standard matrix vectorisation of M (resp. B), $\otimes$ is the Kronecker Product.

2.1 Left-ventricle Poly-Affine Model

The weights $\omega_i(x)$ can be defined by a simple Gaussian function as

$$\omega_i(x) = -\exp\left(\frac{1}{2}(x - \bar{x}_i)^T \phi_i^{(-1)} (x - \bar{x}_i)\right), \tag{6}$$

with $\bar{x}_i$ the barycentre (centre point) of zone i and ϕ_i the corresponding covariance matrix as in [2].

Incompressibility penalisation In order to control the compressibility of the myocardium to be within physiological ranges, an added penalisation term is needed. Given that a transformation is incompressible if its Jacobian determinant is equal to one, for an infinitesimal transformation $T = I + \boldsymbol{v}_{poly}$ with $\nabla T = I + \nabla \boldsymbol{v}_{poly}$ we have

$$\det(\nabla T) = \det(I + \nabla \boldsymbol{v}_{poly}) = Tr(\nabla \boldsymbol{v}_{poly}) + \mathcal{O}(\parallel \nabla \boldsymbol{v}_{poly}) \parallel^2). \tag{7}$$

Here $\mathcal{O}(\cdot)$ represents higher order terms. Therefore the velocity field $\boldsymbol{v}_{poly}$ is locally incompressible if the trace of $\nabla \boldsymbol{v}_{poly}$ is zero. A penalisation term can then be derived as:

$$\alpha \int_\Omega Tr(\nabla \boldsymbol{v}_{poly})^2 dx. \tag{8}$$

The parameter α is used to control the strength of the penalisation. Incorporating this term into the least squares minimisation (3) gives the penalised least squares formula:

$$C(M_1, M_2, \cdots, M_N) = \int_\Omega \parallel \sum_i \omega_i(x) \cdot M_i \cdot x - \boldsymbol{v}_{obs}(x) \parallel^2 dx + \alpha \int_\Omega Tr(\nabla \boldsymbol{v}_{poly})^2 dx \tag{9}$$

To incorporate the new term into the least squares computation, (8) needs to be re-formulated to obtain a quadratic form of $\bar{M}$. Taking the partial derivative of the poly-affine velocity field with respect to x gives

$$\frac{\partial \boldsymbol{v}_{poly}(x)}{\partial x} = \sum_i \left(\omega_i(x) M_i \begin{bmatrix} I_3 \\ 0 \end{bmatrix} + M_i \cdot x \cdot \frac{\partial \omega_i(x)}{\partial x} \right). \tag{10}$$

Using $T = \text{vect}[I_3; 0])$ to extract the diagonal elements from the matrix, we have

$$Tr(\nabla \boldsymbol{v}_{poly}(x)) = \sum_i \left(\omega_i(x) \cdot T^T \cdot \text{vect}(M_i) + g_i(x)^T \cdot \text{vect}(M_i) \right), \tag{11}$$

with $g_i(x) = \text{vect}(\nabla \omega_i(x) \cdot x^T)$. A penalisation term can then be derived as:

$$\alpha \int_\Omega Tr(\nabla \boldsymbol{v}_{poly})^2 dx = \alpha \sum_{i,j} \text{vect}(M_i)^T \cdot V_{ij} \cdot \text{vect}(M_j) \tag{12}$$

with $V_{ij} = \int_\Omega (\omega_i(x) \cdot T + g_i(x))(\omega_j(x) \cdot T + g_j(x))^T dx$. Seemingly, this could be simplified to consider only the first order terms: $V_{ij} = \int_\Omega (\omega_i(x) \cdot T)(\omega_j(x) \cdot T)^T dx$. This is sufficient to penalise the trace per region, but does not take into account the directional information meaning that neighbouring regions can have high deformations in opposing directions, causing problems in the overlap.

Regularisation term We can also define a regularise term to control how neighbouring regions influence one another. The weights $\omega_i(x)$ control how smooth the transition is between two regions, however we would also like to control how similar the affine matrices are, as an addition regularisation. To do this we can add an additional term:

$$\sum_{ij} \pi_{ij} dist(M_i, M_j). \tag{13}$$

Defining a matrix Q such that $Q = \left[\begin{array}{c|c} I_3 & 0 \\ \hline 0 & \mu \end{array}\right]$ allows to account for the different scaling between the rotation/sheering part of the affine matrix and the translation part. The distance term can be written as:

$$\begin{aligned} dist^2(M_i, M_j) &= Tr[(M_i - M_j)^T \cdot Q \cdot (M_i - M_j)] \\ &= Tr(M_i^T Q M_i) + Tr(M_j^T Q M_j) - 2Tr(M_i^T \cdot Q \cdot M_j), \end{aligned} \tag{14}$$

with

$$Tr(M_i^T Q M_j) = \text{vect}(M_i^T Q)^T \cdot \text{vect}(M_j) = \text{vect}(M_i)^T \cdot (Q \otimes I_3) \cdot \text{vect}(M_j). \tag{15}$$

Setting

$$l_{ij} = \begin{cases} -\pi_{ij} = -\int_\Omega \omega_i(x)\omega_j(x)dx & \text{for } i \neq j \\ \sum_{k \neq i} \pi_{ij} = \sum_{k \neq i} \int_\Omega \omega_i(x)\omega_k(x)dx & \text{for } i = j \end{cases}$$

we can account for the correlation between regions. Thus we obtain

$$\sum_{i,j} l_{ij} Tr(M_i^T \cdot Q \cdot M_j) = \bar{M}^T \cdot L \otimes (Q \otimes I_3) \cdot \bar{M}. \quad (16)$$

For $R = L \otimes (Q \otimes I_3)$ the penalised least squares error is given by:

$$C(M) = \bar{M}^T (\Sigma \otimes I_3) \bar{M} - \bar{M}^T \cdot \bar{B} + \alpha \cdot \bar{M}^T \cdot V \cdot \bar{M} + \beta \cdot \bar{M}^T \cdot R \cdot \bar{M}, \quad (17)$$

where β controls the strength of the regularisation. We want to find the optimum by solving $\nabla C_M = 0$.

$$\nabla C_M = (\Sigma \otimes I_3 + \alpha V + \beta R) \bar{M} - \bar{B}. \quad (18)$$

Therefore the solution for M is given by:

$$\bar{M} = (\Sigma \otimes I_3 + \alpha V + \beta R)^{-1} \cdot \bar{B} \quad (19)$$

Algorithm 1 Heart Poly-Affine Near-Incompressible Log-Domain Demons (Regional iLogDemons)

- Segment LV into 17 AHA zones
- Let $\boldsymbol{v}_{poly}(x) = I$ (identity transformation)

Require: Let $\boldsymbol{v}^0 = v_{poly}(x)$

loop {*over n until convergence*}

- Compute the update velocity: $\delta \boldsymbol{v}^n$ given $\boldsymbol{v}^{n-1}$.
- Update the correspondence velocity field: $\boldsymbol{v}^n \leftarrow Z(\boldsymbol{v}^{n-1}, \delta \boldsymbol{v}^n)$.
- Estimate affine transformation of each segment from $\boldsymbol{v}^n$ by solving (2) under the incompressibility penalisation (19).
- Let $\boldsymbol{v}_{poly}(x) \leftarrow \sum_i \omega_i(x) M_i x$

return $\boldsymbol{v}$, $\phi = \exp(\boldsymbol{v})$ and $\phi^{-1} = \exp(-\boldsymbol{v})$.

3 Left-Ventricular Motion Tracking in Healthy Volunteers

Patient Data and Preparation We illustrate these tools on 10 volunteers (3 females, mean age $\pm$ SD $= 28 \pm 5$) obtained from the STACOM 2011 MICCAI cardiac motion tracking challenge database [7]. Steady-state free precision cine MRI were acquired using a 3T scanner (Philips Achieva System, Philips Healthcare) in the short axis view covering entirely both ventricles (12-15 slices; isotropic in-plane resolution:1.21x1.21mm to 1.36x1.36mm; slice thickness: 8mm; 30 frames).

Myocardium Mask and AHA Segment Delineation We extract a binary mask image to define the left ventricle myocardium where the least squares minimisation is computed. To do this we to define a surface mesh of the myocardium by

annotating the boundary of the ventricle directly on the given patient images, and create a surface mesh (and related binary mask) from these annotations. This was done with a 3D interactive segmentation tool based on implicit variational surfaces and provided within the CardioViz3D package[3]. Each LV mesh was then divided into 17 regions according to the AHA recommendations using a semi-automatic C++ segmentation tool that required just the input of four landmarks to define the base, apex, LV-RV junction on anterior and LV-RV junction on posterior.

3.1 Results

In order to determine a suitable range of parameters for α and β a set of simulations were run for one patient fixing α (resp. β) and ranging β (resp. α) (see Fig. 1). From this analysis, the values for α and β were set for all patients as $\alpha = 1$ and $\beta = 10$. Higher values of β give better values for the Jacobian determinant, but result in over regularisation of the field, converging towards a single affine transformation and thus restricting the global motion. Values of α greater than 10 (towards an incompressibility constraint rather than projection) result in numerical instabilities in the matrix inversion of Eq. 19.

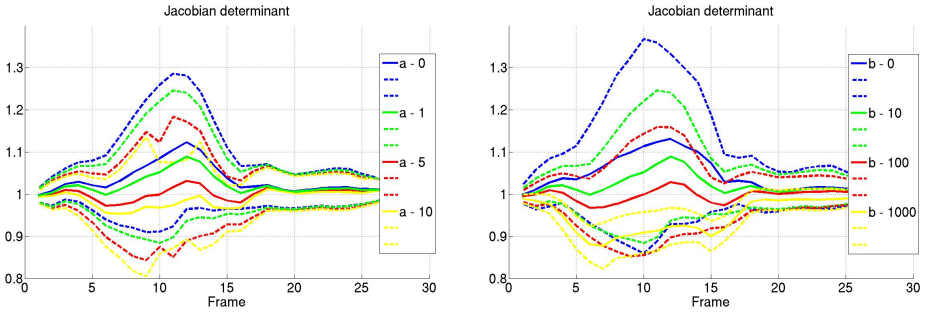

Fig. 1. Mean and standard deviation of the Jacobian determinant for one patient computed as an average over each AHA zone with varied values of α (left) and β (right). A reasonable trade-off between the range and smoothness of the Jacobian determinant is given for $\alpha = 1$, $\beta = 10$.

From each of the computed frame-to-reference deformation fields, the corresponding Jacobian determinant images were computed. The average value within each of the AHA regions was calculated and the average per region for all patients is shown in Fig 2 (left), to show the amount of regional compression (or expansion) in the myocardium. The strain was computed over the cycle in each of the circumferential, radial and longitudinal directions, and averaged over each of the AHA regions. In Fig. 2 we show the average strain in each direction of all patients per region.

[3] *http://www-sop.inria.fr/asclepios/software/CardioViz3D/*

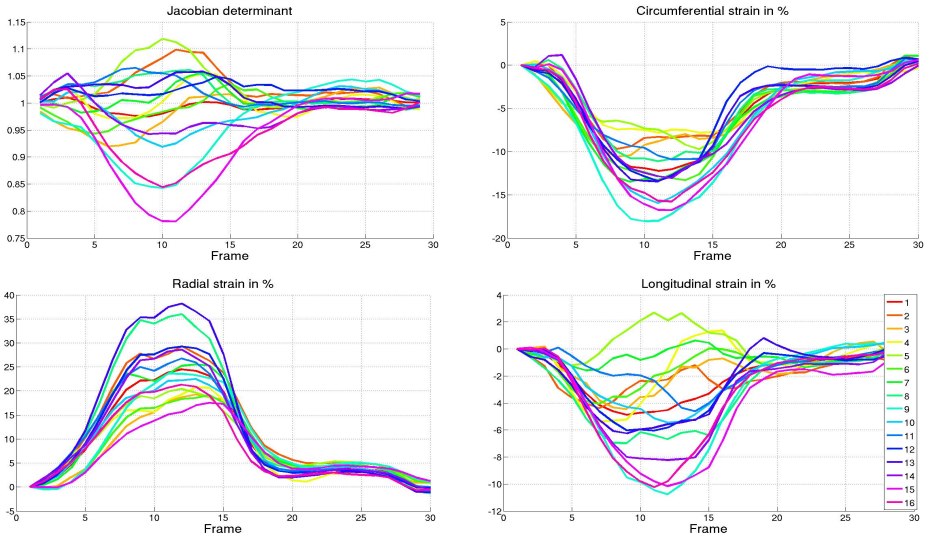

Fig. 2. Left: Plot of the average Jacobian determinant per AHA region averaged for all patients, shown at each frame of the cycle. The plot shows that the volume change of the left ventricle is maintained with 15% over the cycle (each region shown in a different colour). Regional strain curves computed in the circumferential (centre-left), radial (centre-right), and longitudinal (right) directions for each of the 17 AHA regions then averaged over the patients.

The values of the strain, Jacobian determinant and the apparent registration accuracy based on the figures shown in Fig. 4, can be compared directly to the results obtained from the STACOM 2011 MICCAI workshop cardiac motion tracking challenge algorithms applied to the same data-set [7]. The left ventricular volume was compared between the iLogDemons algorithm and the proposed algorithm (see Fig. 3) showing an ejection fraction of 50% and 58% resp. (note the normal range is $55-70\%$ and typical value is 58% [10]). These results suggest that the method is able to obtain comparable registration, strain curves, and left ventricular volume to those for the iLogDemons algorithm [11].

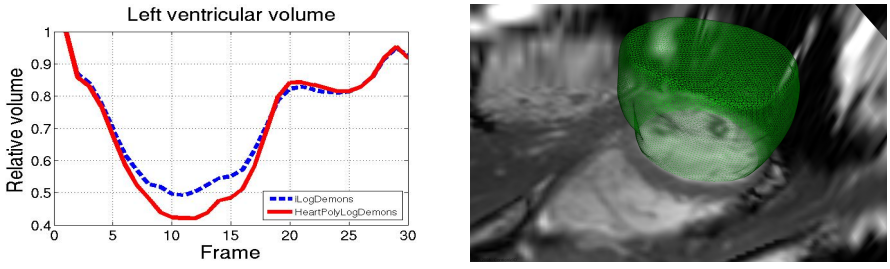

Fig. 3. Left ventricular volume along the cardiac cycle for the iLogDemons algorithm (blue dashed line), and proposed algorithm (red solid line) shown in relative measures to the end-diastolic volume (left), computed from deforming the 3D segmented end-diastolic volume mesh (right).

To exemplify the results of the registration for one individual, the mesh segmented at the reference frame (end diastole) is overlaid on the reference image (see Fig. 4 top row). The same mesh is deformed by the poly-affine deformation field computed from peak contraction to reference, and the resulting mesh is overlaid on the peak contraction image (see Fig. 4 bottom row). The results show good alignment between the deformed peak systole mesh and the image.

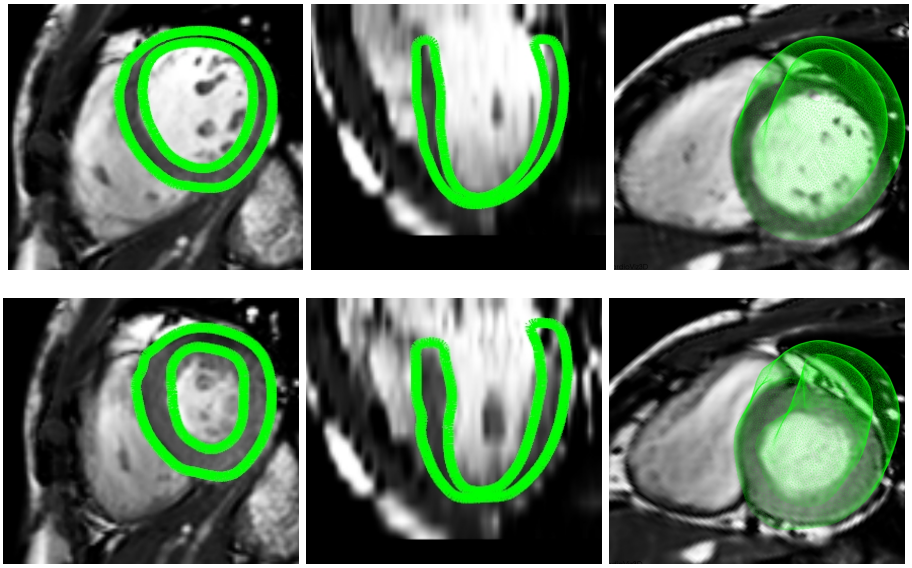

Fig. 4. Top row: Three views of the reference frame (end diastole) with segmented mask over-laid in green. Bottom row: Same views of peak systole with the segmented reference mask deformed by the computed poly-affine deformation field. The results in the bottom row indicate that the registration provides reasonable deformation fields that capture the motion of the heart.

4 Discussion and Future Work

The results suggest that the method is able to track the cardiac motion reasonably well and with less than 20% volume change in the myocardium for all patients and all regions. Moreover, we are able to parameterise the deformation by 204 parameters, as opposed to over 5 million parameters for the iLogDemons algorithm (or similarly for other registration algorithms parameterised at the voxel level). Full incompressibility can only be achieved in this model with a global affine incompressible deformation, thus with so few degrees of freedom, the volume change is penalised within reasonable ranges. Using more regions may possibly improve the results with respect to the incompressibility.

This paper describes a proof of concept of the method. More work is needed to better understand the weight functions, the choice of suitable regions, as well

as the optimal number of regions. Given that the definition of the regions is consistent in this work from subject to subject, we expect to obtain reproducible and powerful clinical scores to characterise different heart conditions. A long term objective is to use the computed parameters as clinical scores to aid in quantifying healthy heart motion and then to analyse the motion in the pathological case.

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