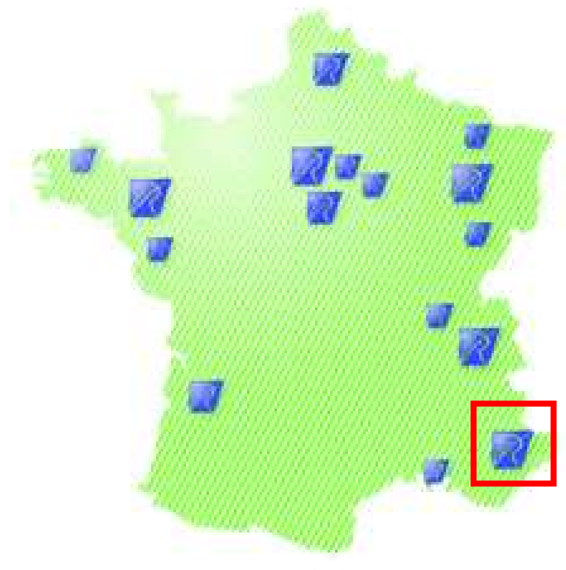
Hervé DELINGETTE



INRIA Sophia-Antipolis

French National Research Institute in Computer Science and Automatic Control

- Asclepios : 3D Segmentation, Simulation Platform, Soft Tissue Modeling

  http://www.inria.fr/asclepios

- CardioSense3D : Cardiac Simulation,

  http://www.inria.fr/CardioSense3D

- SOFA : Open Software Plateform for Medical Simulation

  http://www.sofa-framework.org/

Towards the virtual physiological human

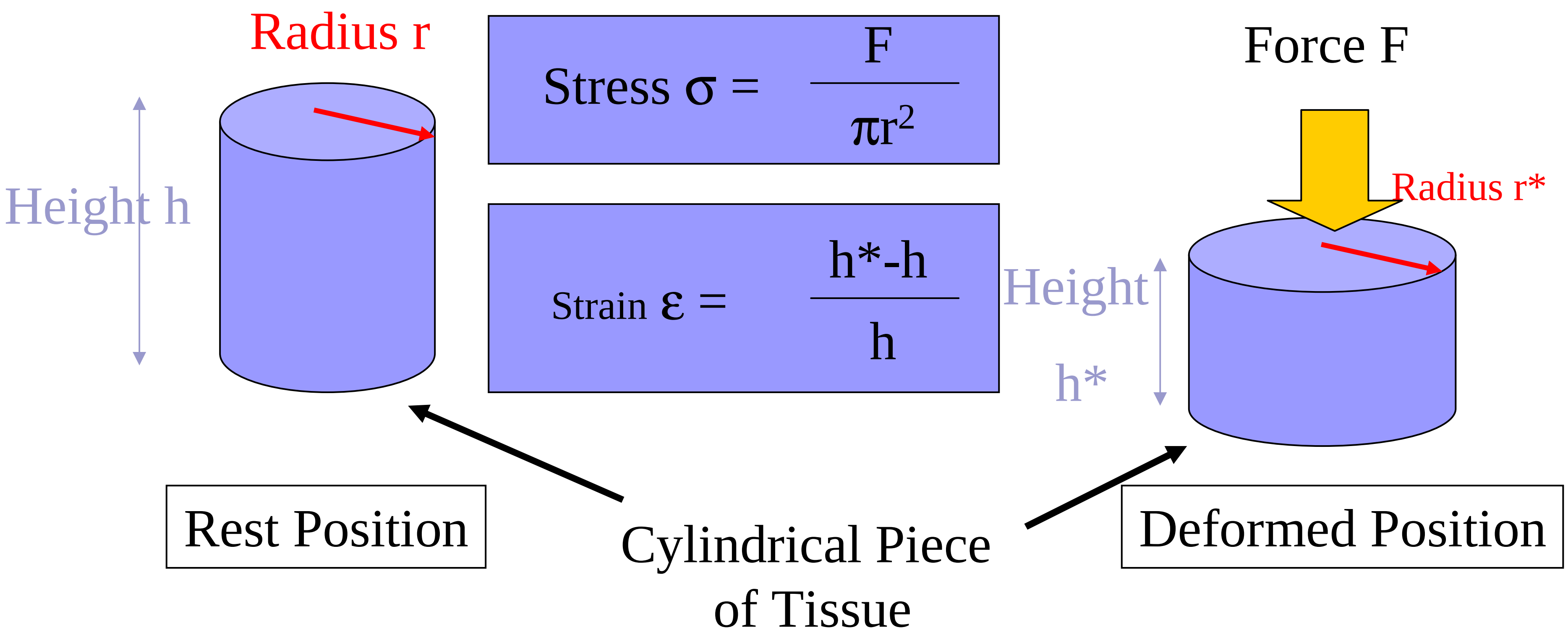
Physically-based simulation of biological tissues

Soft Tissue Characterization

- Biomechanical behavior of biological tissue is very complex
- Most biological tissue is composed of several components :
  - Fluids : water or blood
  - Fibrous materials : muscle fiber, neuronal fibers, …
  - Membranes : interstitial tissue, Glisson capsule
  - Parenchyma : liver or brain

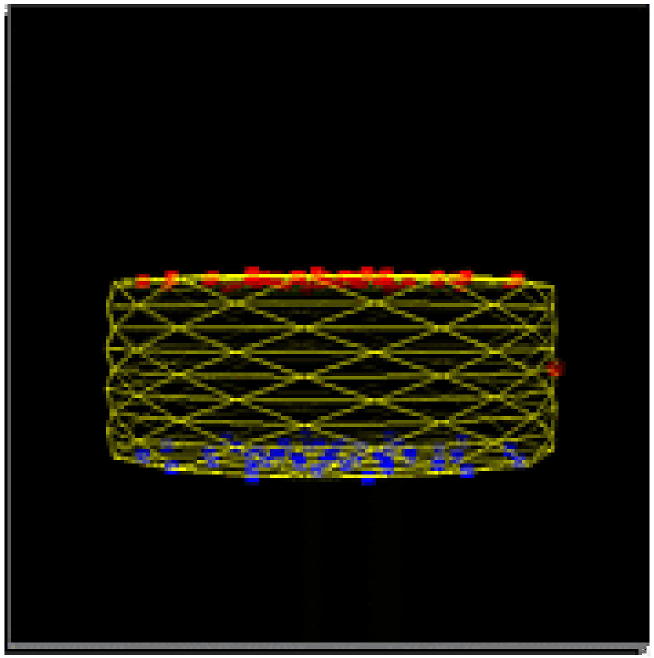
Soft Tissue Characterization

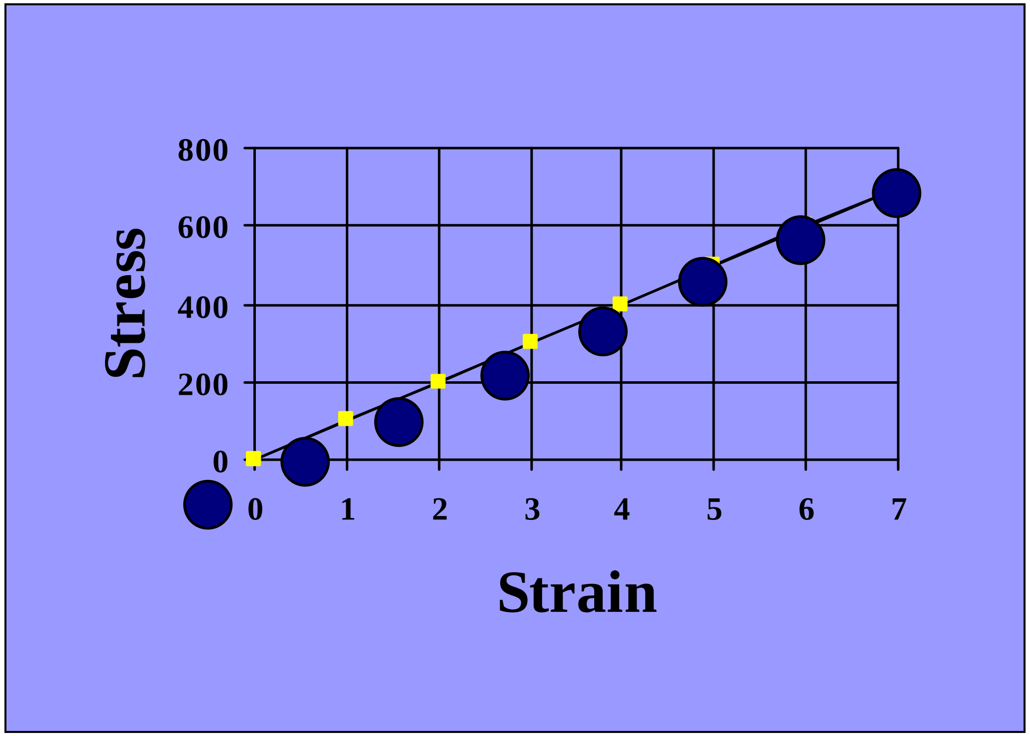
- To characterize a tissue, its stress-strain relationship is studied



Linear Elastic Material

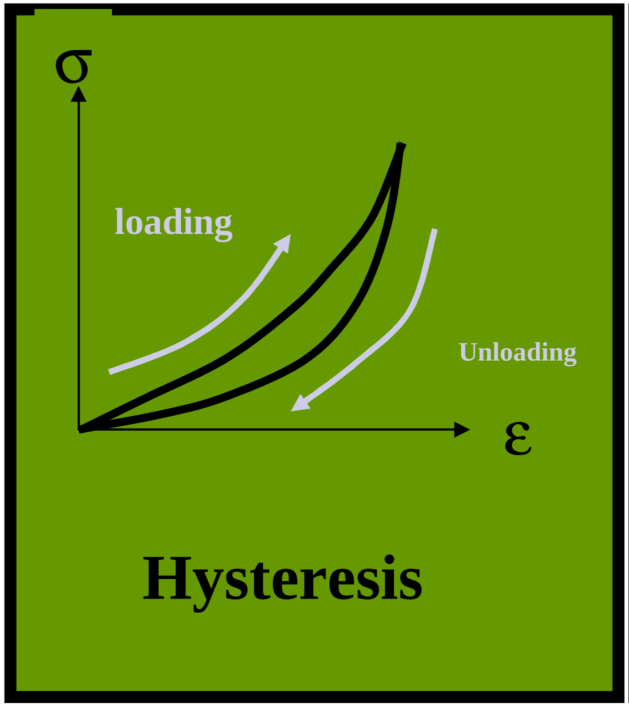
- Simplest Material behaviour
- Only valid for small deformations (less than 5%)

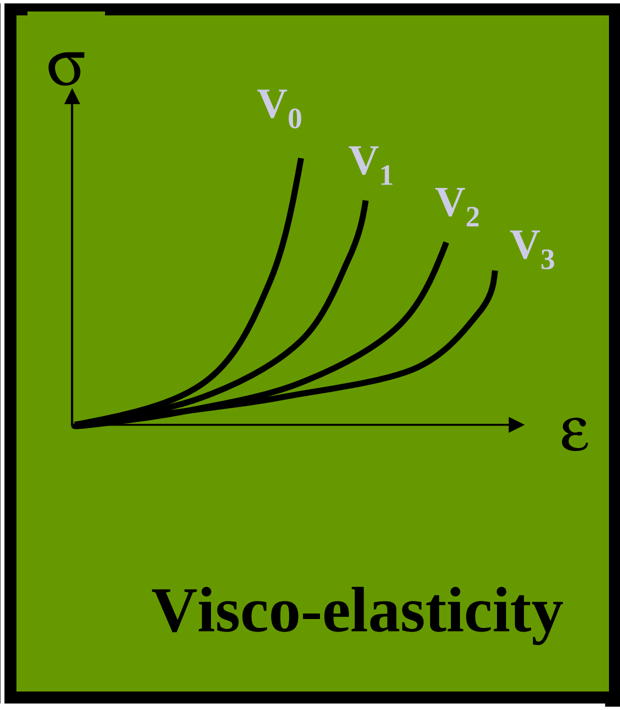


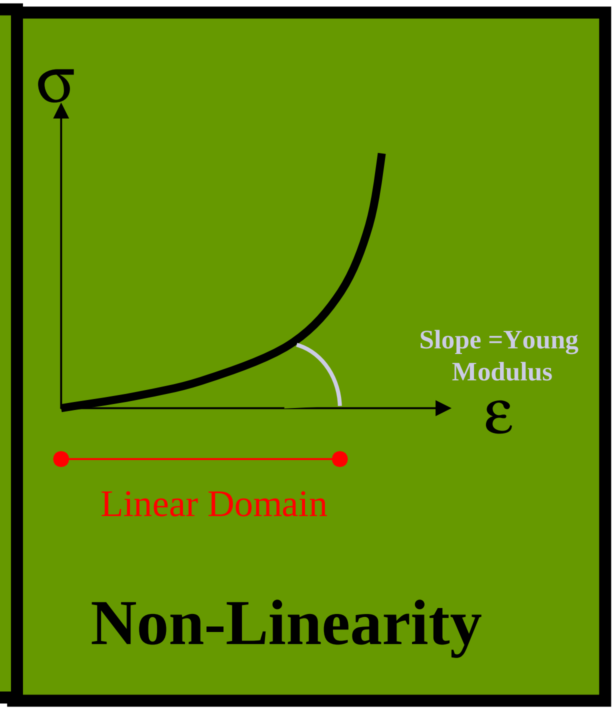


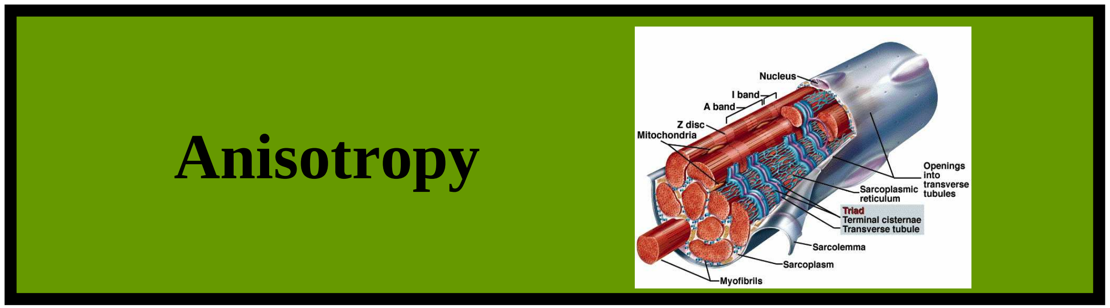
Biological Tissue

- Many complex phenomena arises

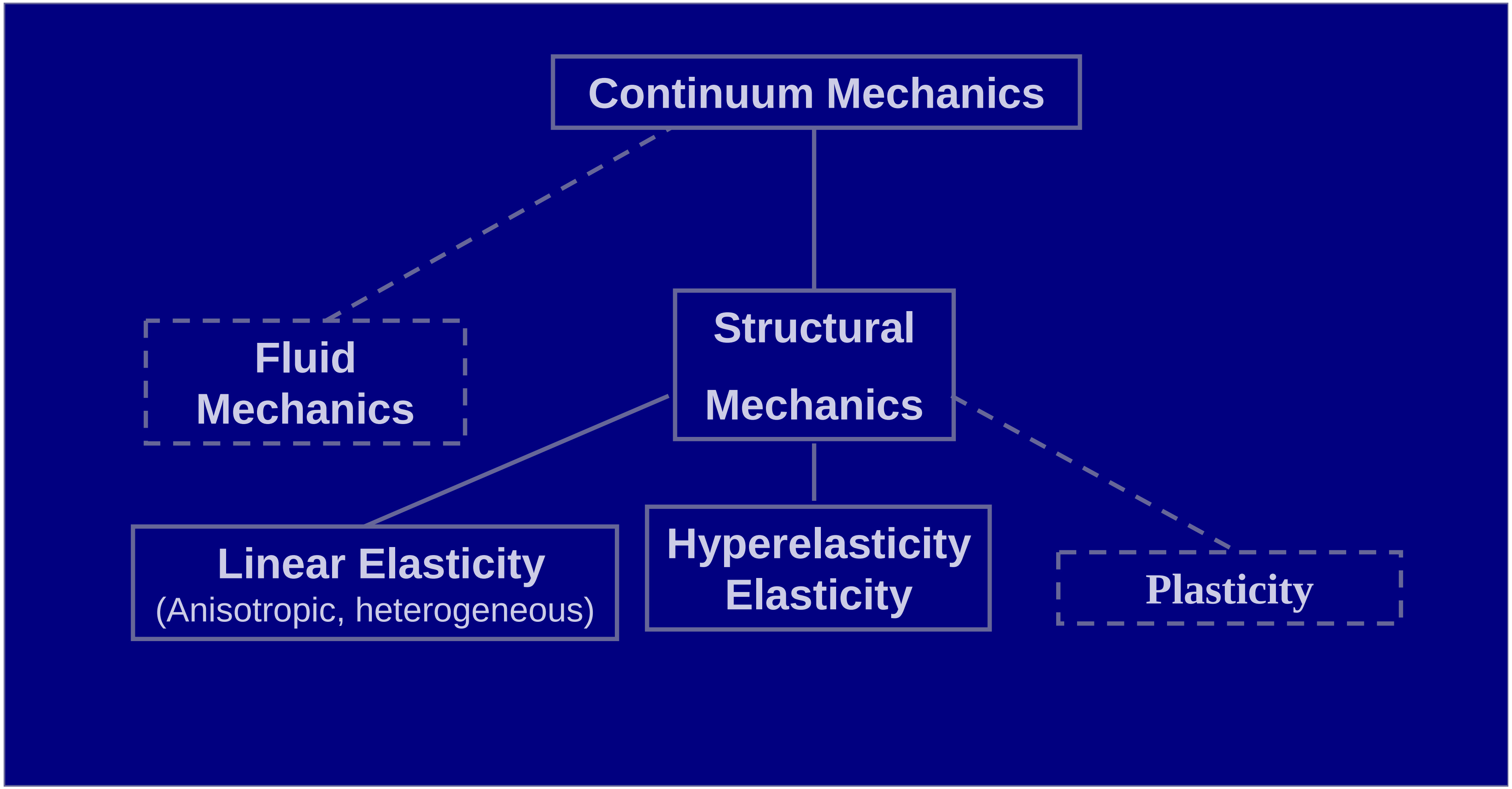








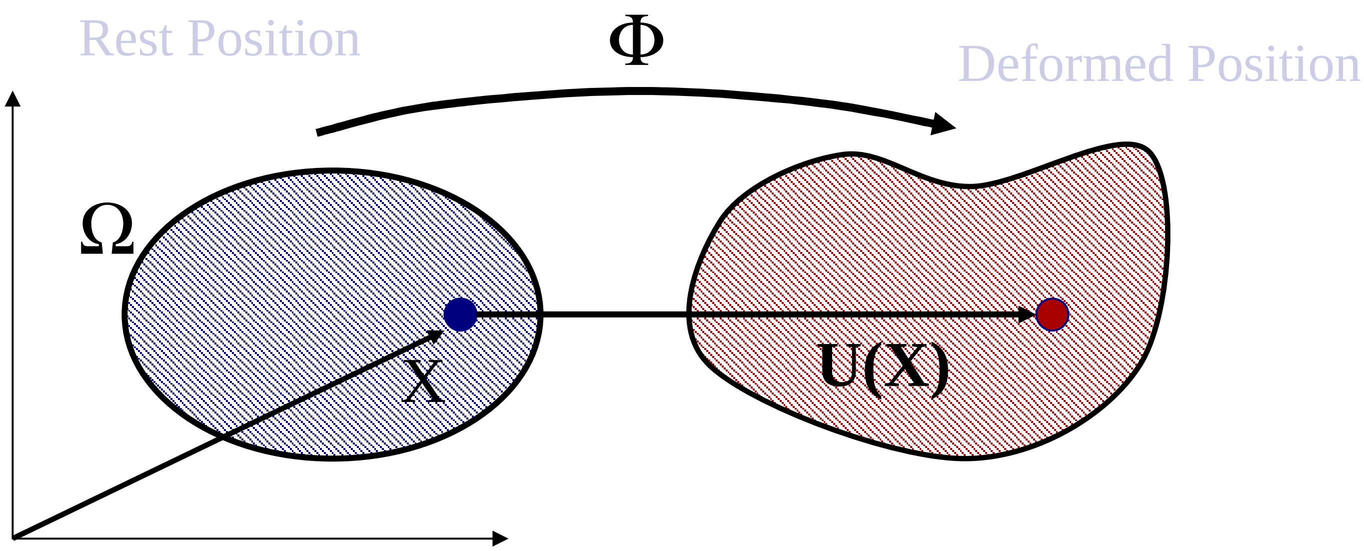
Continuum Mechanics



Basics of Continuum Mechanics

- Deformation Function
- $$X \in \Omega \mapsto \phi(X) \in \Re^3$$
- Displacement Function

$$U(X) = \phi(X) - X$$

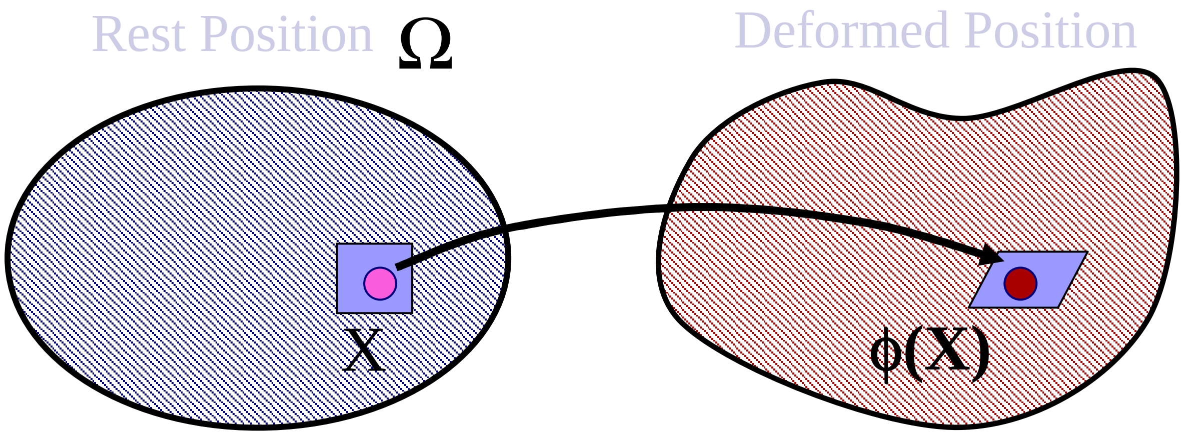


Basics of Continuum Mechanics

- The local deformation is captured by the deformation gradient :

$$\boxed{F = \frac{\partial \phi}{\partial X}}$$

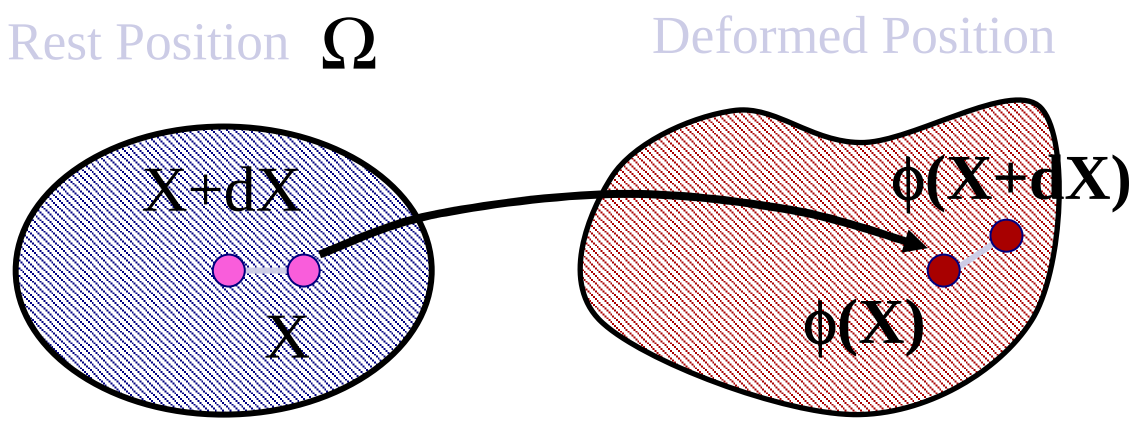
$$F_{ij} = \frac{\partial \phi_i}{\partial X_j} = \begin{bmatrix} \frac{\partial \phi_1}{\partial X_1} & \frac{\partial \phi_1}{\partial X_2} & \frac{\partial \phi_1}{\partial X_3} \\ \frac{\partial \phi_2}{\partial X_1} & \frac{\partial \phi_2}{\partial X_2} & \frac{\partial \phi_2}{\partial X_3} \\ \frac{\partial \phi_3}{\partial X_1} & \frac{\partial \phi_3}{\partial X_2} & \frac{\partial \phi_3}{\partial X_3} \end{bmatrix}$$



F(X) is the local affine transformation that maps the neighborhood of X into the neighborhood of φ(X)

Basics of Continuum Mechanics

- Distance between point may not be preserved



- Distance between deformed points

$$(ds)^2 = \|\phi(X + dX) - \phi(X)\|^2 \boxed{\approx dX^T \left(\nabla\phi^T \nabla\phi\right) dX}$$

- Right Cauchy-Green Deformation tensor

$$C = \nabla\phi^T \nabla\phi$$ Measures the change of metric in the deformed body

Basics of Continuum Mechanics

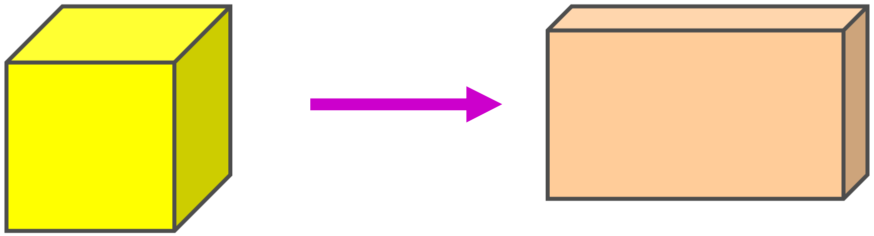
- Example : Rigid Body motion entails no deformation $\phi(X) = RX + T$
  $F(X) = \nabla\phi(X) = R$ $\quad C = R^T R = Id$

- Strain tensor captures the amount of deformation
  - It is defined as the “distance between C and the Identity matrix”

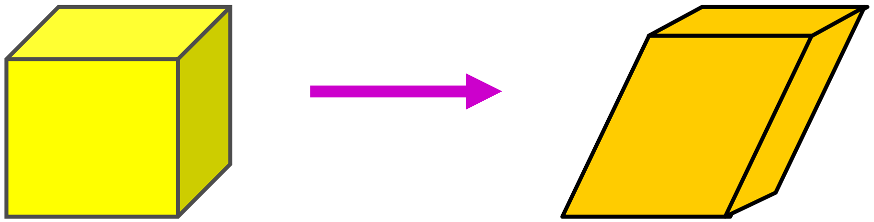
$$E = \frac{1}{2}\left(\nabla\phi^T \nabla\phi - Id\right) = \frac{1}{2}\left(C - Id\right)$$

Strain Tensor

- Diagonal Terms : ε_i
  - Capture the length variation along the 3 axis



- Off-Diagonal Terms : γ_i
  - Capture the shear effect along the 3 axis



$$E = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

Linearized Strain Tensor

- Use displacement rather than deformation

$$\nabla\phi(X) = Id + \nabla U(X)$$

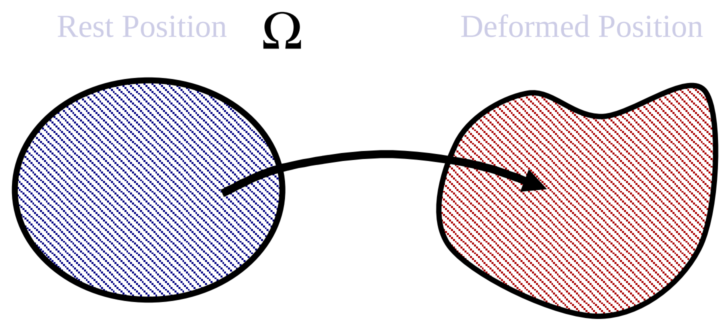
$$E = \frac{1}{2}\left(\nabla U + \nabla U^T + \nabla U^T \nabla U\right)$$

- Assume small displacements

$$E_{Lin} = \frac{1}{2}\left(\nabla U + \nabla U^T\right)$$

Hyperelastic Energy

- The energy required to deform a body is a function of the invariants of strain tensor E :
  - Trace E = I_1
  - Trace E*E= I_2
  - Determinant of E = I_3



$$W(\phi) = \int_{\Omega} w(I_1, I_2, I_3)\, dX$$

Total Elastic Energy

Linear Elasticity

- Isotropic Energy

$$w(X) = \frac{\lambda}{2}\left(tr\, E_{Lin}\right)^2 + \mu\, tr\, E_{Lin}^2$$

(λ, μ) **: Lamé coefficients**

Hooke's Law

$w(X)$ **: density of elastic energy**

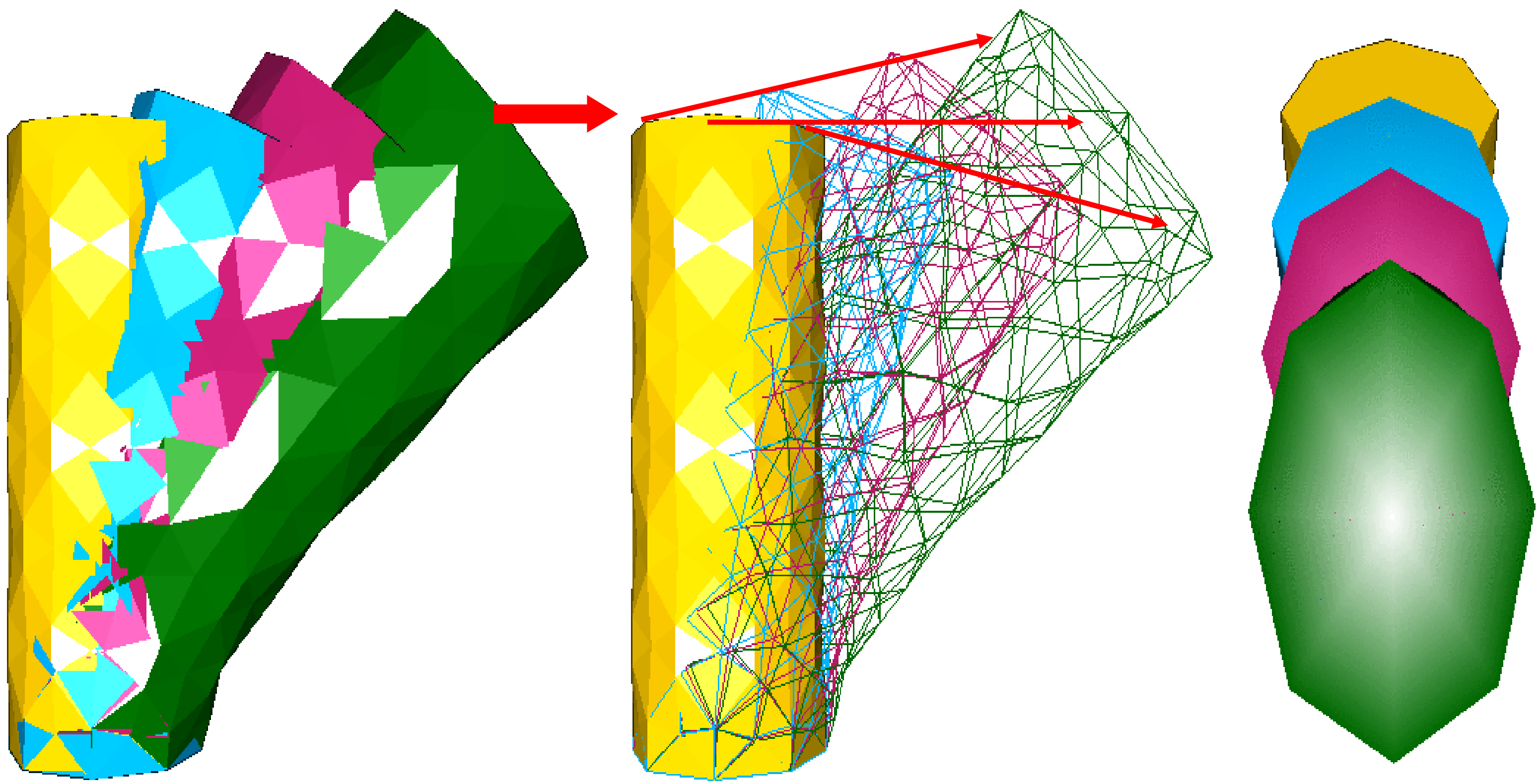
- Advantage :
  - Quadratic function of displacement

$$w = \frac{\lambda}{2}\left(div\, U\right)^2 + \mu\left\|\nabla U\right\|^2 - \frac{\mu}{2}\left\|rot\, U\right\|^2$$

- Drawback :
  - Not invariant with respect to global rotation
- Extension for anisotropic materials

Shortcomings of linear elasticity

- Non valid for « large rotations and displacements »



St-Venant Kirchoff Elasticity

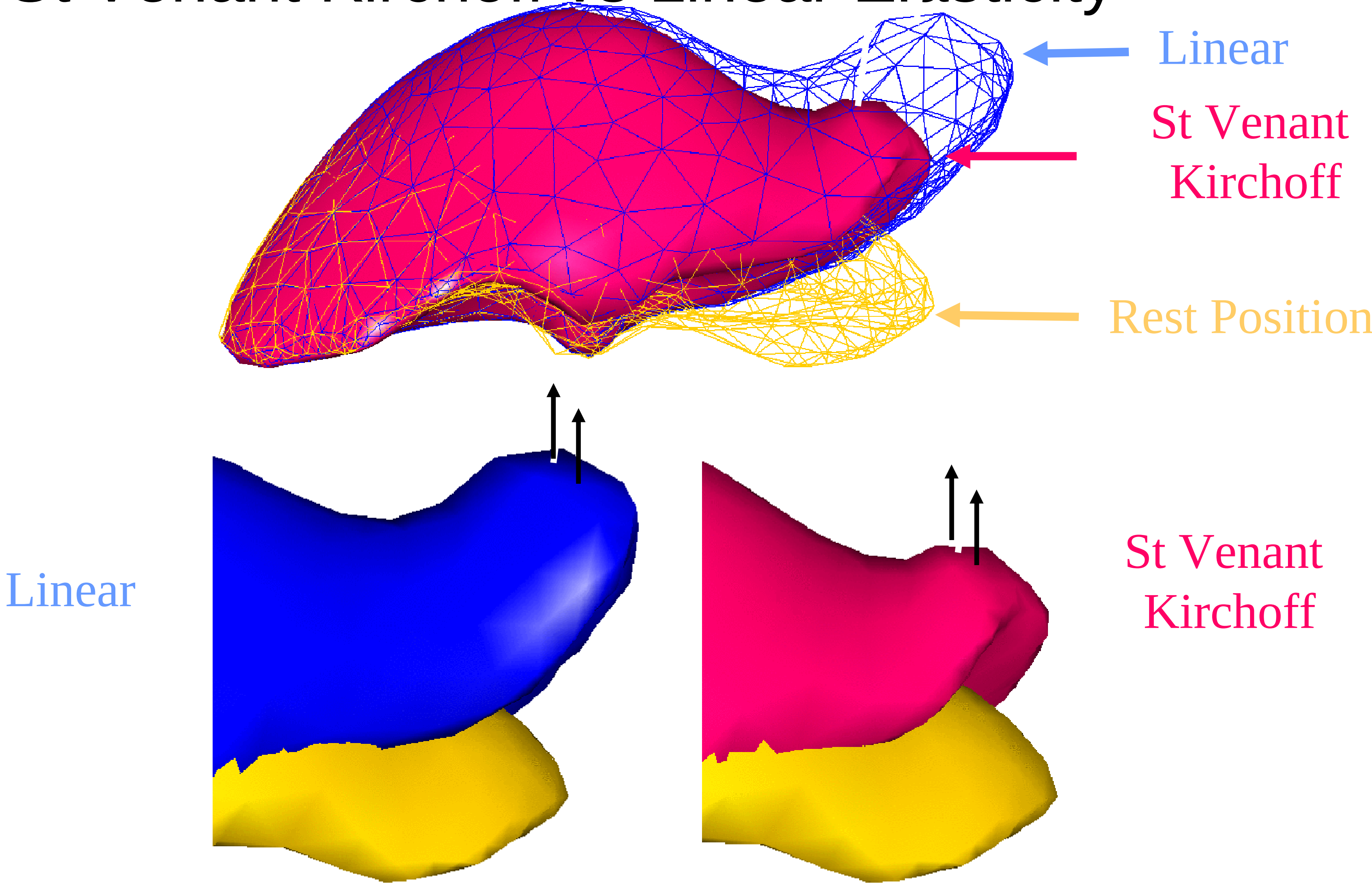
- Isotropic Energy

$$w(X) = \frac{\lambda}{2}\left(tr\,E\right)^2 + \mu\, tr\,E^2$$

(λ, μ) **: Lamé coefficients**

- Advantage :
  - Generalize linear elasticity
  - Invariant to global rotations
- Drawback :
  - Poor behavior in compression
  - Quartic function of displacement
- Extension for anisotropic materials

St Venant Kirchoff vs Linear Elasticity



Other Hyperelastic Material

$$w(X) = \frac{\mu}{2} trE + f\left(I_3\right)$$

- Neo-Hookean Model

$$w(X) = \frac{\mu}{2} e^{trE} + f\left(I_3\right)$$

- Fung Isotropic Model

$$w(X) = \frac{\mu}{2} e^{trE} + \frac{k_1}{k_2}\left(e^{k_2 (I_4 - 1)} - 1\right) + f\left(I_3\right)$$

- Fung Anisotropic Model

$$w(X) = c_1\left(e^{\gamma trE}\right) + c_2\, trE^2 + f\left(I_3\right)$$

- Veronda-Westman

$$w(X) = c_{10} trE + c_{01} trE^2 + f\left(I_3\right)$$

- Mooney-Rivlin :

Estimating material parameters

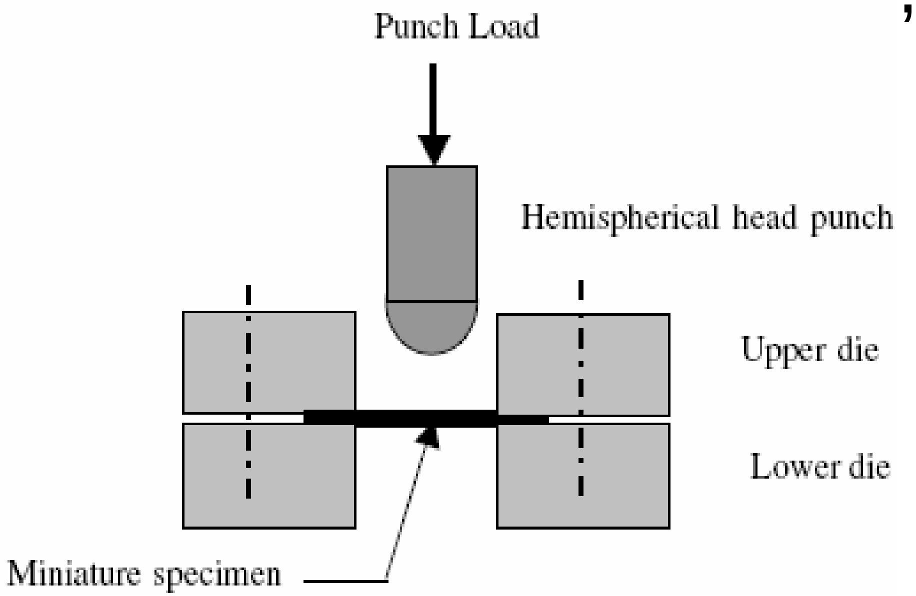
- Complex for biological tissue :
  - Heterogeneous and anisotropic materials
  - Tissue behavior changes between in-vivo and in-vitro
  - Ethics clearance for performing experimental studies
  - Effect of preconditioning
  - Potential large variability across population

Soft Tissue Characterization

- Different possible methods
  - In vitro rheology
  - In vivo rheology
  - Elastometry
  - Solving Inverse problems

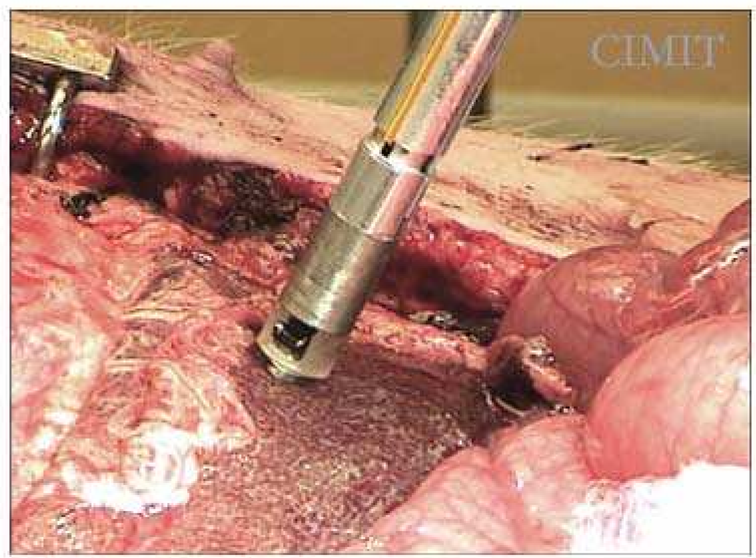
Soft Tissue Characterization

- In vitro rheology
  - can be performed in a laboratory. Technique is mature
  - Not realis ... , …)



Soft Tissue Characterization

- In vivo rheology
  - can provide stress/strain relationships at several locations
  - Influence of boundary conditions not well unders



Source : Cimit. Boston USA

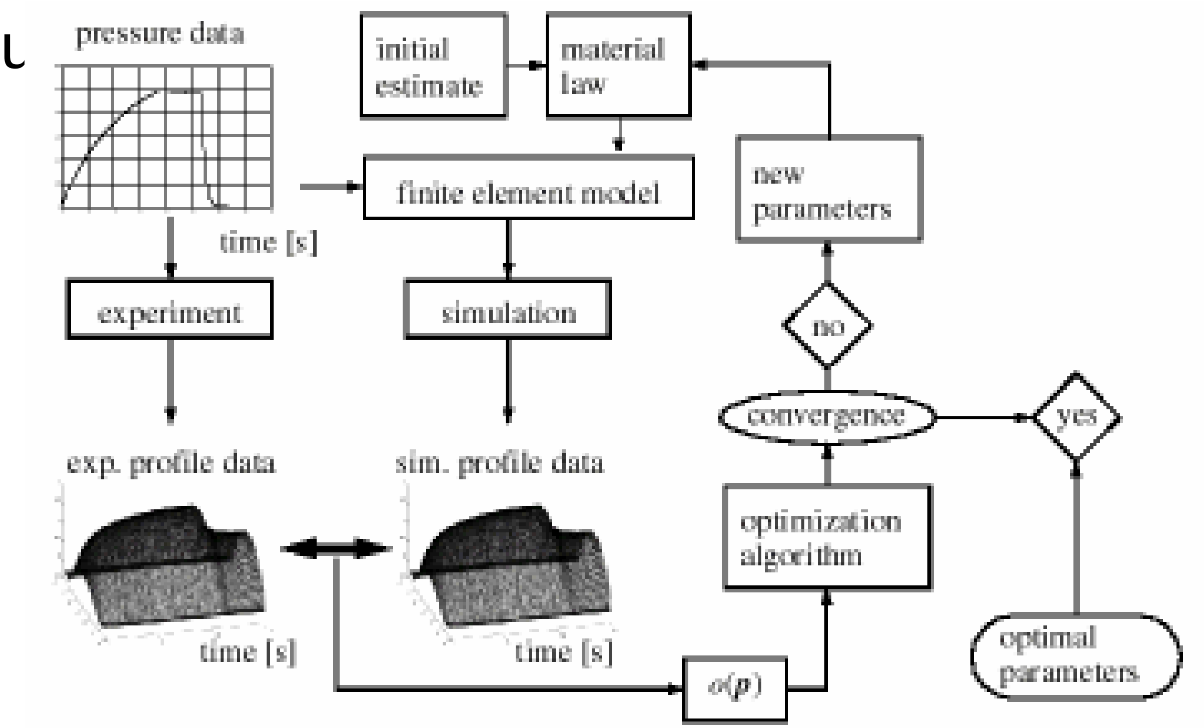
Soft Tissue Characterization

- Elastometry (MR, Ultrasound)
  - mesure property inside any organ non invasively
  - validati … als

Source Echosens, Paris

Soft Tissue Characterization

- Inverse Problems
  - well-suited for surgery simulation (computational approach)
  - requ … ation



Soft Tissue Characterization

- Still difficult to find “reliable” soft tissue material parameters
- Example : Liver soft tissue characterization

First Author	Experimental Technique	Liver Origin	Young Modulus (kPa)
Yamashita [111]	Image-Based	Human	Not Available
Brown [15]	*in-vivo*	Porcine Liver	≈ 80
Carter [17]	*in-vivo*	Human Liver	≈ 170
Dan [27]	*ex-vivo*	Porcine Liver	≈ 10
Liu [62, 61]	*ex-vivo*	Bovine Liver	Not Available
Nava [76]	*in-vivo*	Porcine Liver	≈ 90
Miller [74]	*in-vivo*	Porcine Liver	Not Available
Sakuma [92]	*ex-vivo*	Bovine Liver	Not Available

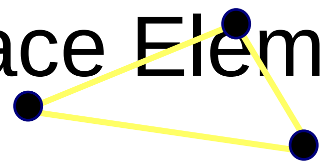
Table 2: List of published articles providing some quantitative data about the biomechanical properties of the liver.

Discretisation techniques

- Four main approaches :
  - Volumetric Mesh Based
  - Surface Mesh Based
  - Meshless
  - Particles

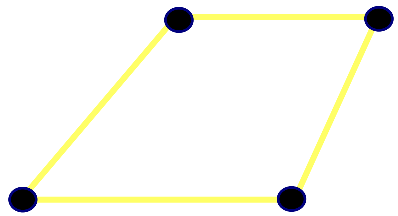
Different types of meshes

- Surface Elements :



Triangle

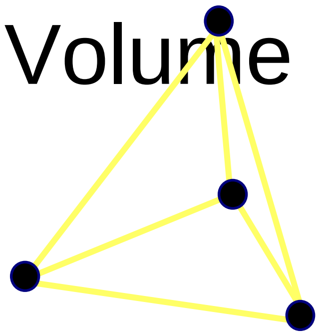
3, 12 nodes and more



Quad

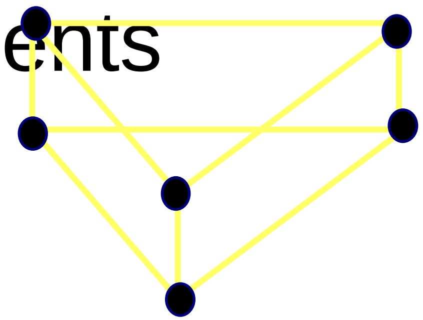
4, 8 nodes and more

- Volume Elements



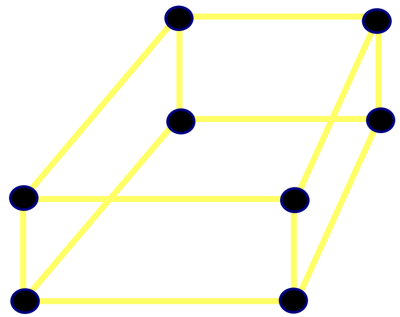
Tetrahedron

4, 10 nodes



Prismatic

6, 15 nodes and more



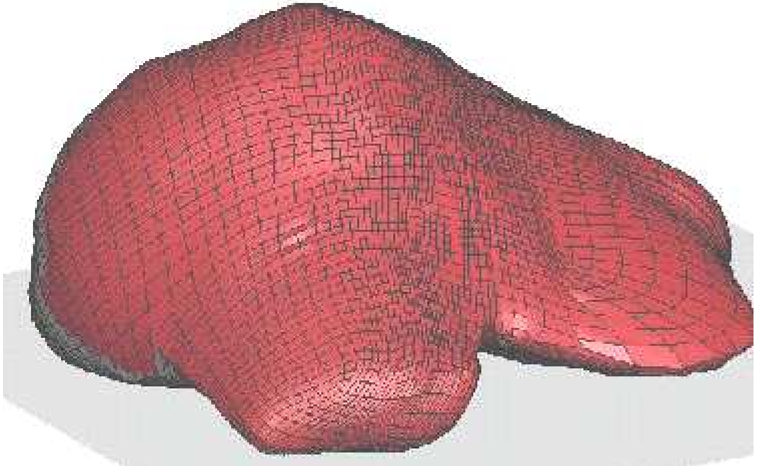
Hexahedron

8, 20 nodes and more

Structured vs Unstructured meshes

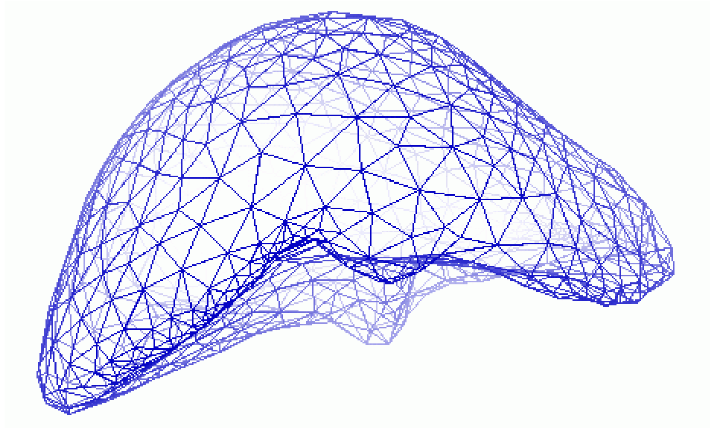
- Example 1 : Liver meshed with hexahedra

3 months work
(courtesy of ESI)



- Example 2: Liver meshed with tetrahedra

Automatically
generated (1s)

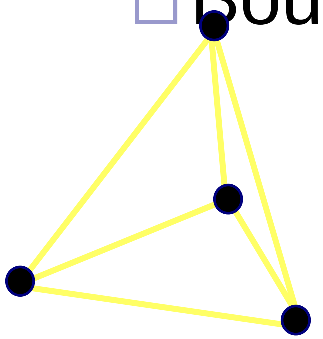


Volumetric Mesh Discretization

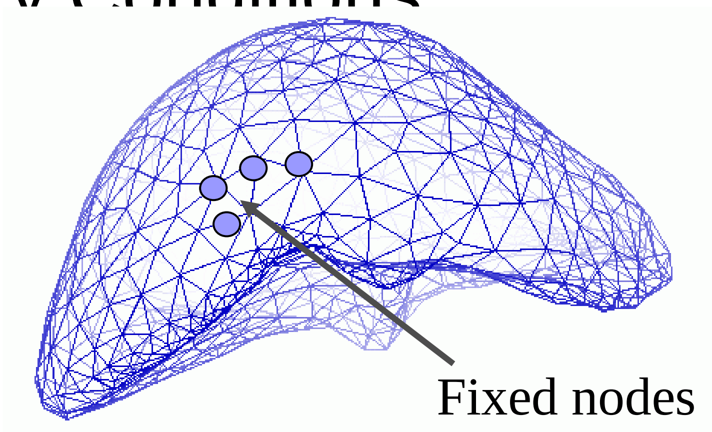
- Classical Approaches :
  - Finite Element Method (weak form)
  - Rayleigh Ritz Method (variational form)
  - Finite Volume Method (conservation eq.)
  - Finite Differences Method (strong form)
- FEM, RRM, FVM are equivalent when using linear elements

Rayleigh-Ritz Method

- Step1 : Choose
  - Finite Element (e.g. linear tetrahedron)
  - Mesh discrediting the domain of computation
  - Hyperelastic Material with its parameters
  - Boundary Conditions



Tetrahedron
4 nodes



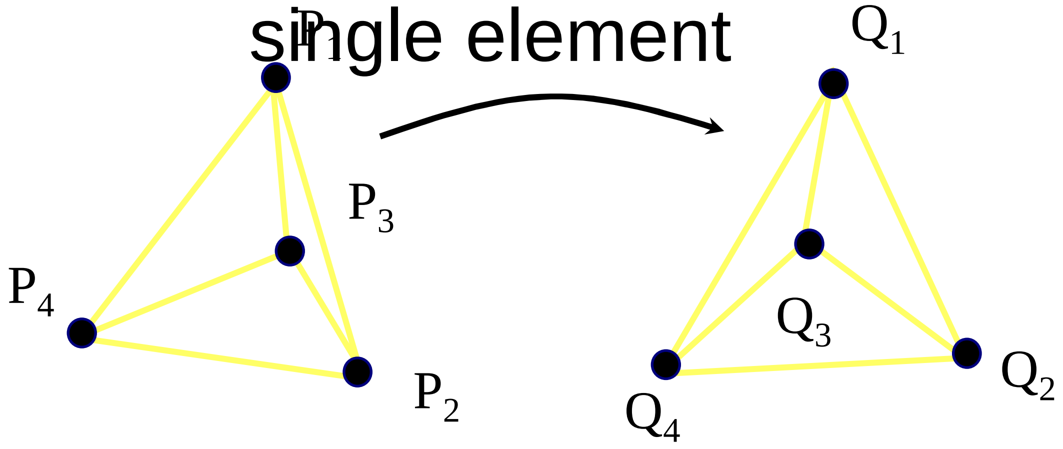
$$w(X) = \frac{\lambda}{2}\left(tr\,E\right)^2 + \mu\, tr\, E^2$$

Young Modulus

Poisson Coefficient

Rayleigh-Ritz Method

- Step2
  - Write the elastic energy required to deform a single element



$$u(P_i) = Q_i - P_i = U_i$$

$$u(X) = \sum_{i=1}^{4} \lambda_i(X)\, u(P_i)$$

$$\nabla \lambda_i(X) = -\frac{M_i}{6V(T)}$$

$$W_{T_i} = \sum_{jk} U_j^t \left[\mathsf{K}_{jk}^{\mathsf{T}_i}\right] U_k$$

$$[\mathbf{K}_{jk}^{T_i}] = \frac{1}{36.\mathrm{V}(T_i)} \left(\lambda_i \mathbf{M}_k \mathbf{M}_j^{T} + \mu_i \mathbf{M}_j \mathbf{M}_k^{T} + \mu_i \left(\mathbf{M}_j \cdot \mathbf{M}_k\right)[\mathbf{Id}_{3x3}]\right)$$

$$trE = -\sum_i \frac{M_i \cdot U_i}{6V(T)}$$

Rayleigh-Ritz Method

- Step3
  - Sum to get the total elastic energy

$$W(U) = \int_{\Omega_h} w(T_1, T_2, T_3)\, dX = \sum_{T_i} W_{T_i} = U^T K U$$

  - Write the conservation of energy

$$\underbrace{W(U)}_{\text{Internal Energy}} = \underbrace{F^T U}_{\text{Nodal Forces}} + \underbrace{\int_{\Omega} \rho(X)\left(X \cdot g\right) dX}_{\text{Gravity Potential Energy}}$$

Rayleigh-Ritz Method

- Step3
  - Write first variation of the energy :

Linear Elasticity

$$KU = R$$

Static case

$$M\ddot{U} + C\dot{U} + KU = R(t)$$

Dynamic case

HyperElasticity=NonLinear Elasticity

$$K(U) = R$$

Static case

$$M\ddot{U} + C\dot{U} + K(U) = R(t)$$

Dynamic case

Surface-Based Methods

- Only consider the mesh surface under some hypothesis :
  - Linear Elastic Material (sometimes homogeneous)
  - Only interact with organ surface
- Pros :
  - No need to produce volumetric meshes
  - Much faster than volumetric computation
- Cons :
  - Only linear material
  - No cutting

Surface-Based Methods

- Possible approaches :
  - Boundary Element Models (BEM)
    - Based on the Green Function of the linear elastic operator
    - Requires homogeneous material
  - Matrix Condensation
    - Full Matrix inversion
  - Iterative Precomputed Generation
    - Solve 3*Ns equations F=KU

Other Methods

- Meshless Methods
  - Use only points inside and specific shape functions
  - Can better optimize location of DOFs
  - Can cope with large deformations
  - Deformation accuracy unknown
- Particles
  - Smooth Particles Hydrodynamics that interact based on a state equation

Dynamic evolution and numerical integration

- Dynamic evolution
  - Discrete models = lumped mass particles submitted to forces
  - Newtonian evolution (1st order differential system):

$$\begin{cases} \delta \boldsymbol{P} = \boldsymbol{V}.dt \\ \delta \boldsymbol{V} = \boldsymbol{M}^{-1}\boldsymbol{F}(\boldsymbol{P},\boldsymbol{V}).dt \end{cases}$$

  - Explicit schemes:
    - Euler: $\begin{cases} \delta \boldsymbol{P} = \boldsymbol{V}_t.dt \\ \delta \boldsymbol{V} = \boldsymbol{M}^{-1}\boldsymbol{F}(\boldsymbol{P}_t, \boldsymbol{V}_t).dt \end{cases}$
    - Runge-Kutta: several evaluations to better extrapolate the new state [press92]

$\rightarrow$ Unstable for large time-step !!

  - Semi-Implicit schemes:
    - Euler: $\begin{cases} \delta \boldsymbol{P} = \boldsymbol{V}_{t+dt}.dt \\ \delta \boldsymbol{V} = \boldsymbol{M}^{-1}\boldsymbol{F}(\boldsymbol{P}_t, \boldsymbol{V}_t).dt \end{cases} \rightarrow \begin{cases} \boldsymbol{P}_{t+dt} = 2\boldsymbol{P}_t - \boldsymbol{P}_{t-dt} + \boldsymbol{M}^{-1}\boldsymbol{F}(\boldsymbol{P}_t, \boldsymbol{V}_t).dt^2 \\ \boldsymbol{V}_{t+dt} = (\boldsymbol{P}_{t+dt} - \boldsymbol{P}_t)dt^{-1} \end{cases}$
    - Verlet [teschner04]

Evolution

- Implicit schemes [terzopoulos87], [baraff98], [desbrun99], [volino01], [hauth01]
  - First-order expansion of the force:

$$F(P_{t+dt}\,, V_{t+dt}) \approx F(P_t\,, V_t) + \partial F/\partial P\ \delta P + \partial F/\partial V\ \delta V$$

  - Euler implicit

$$\rightarrow \begin{cases} \delta P = V_{t+dt} . dt \\ \delta V = H^{-1} Y \end{cases} \quad \text{with} \quad \begin{aligned} H &= I - M^{-1}\, \partial F/\partial V\ dt - M^{-1}\, \partial F/\partial P\ dt^2 \\ Y &= M^{-1}\, F(P_t\,, V_t) + M^{-1}\, \partial F/\partial P\ V_t\, dt^2 \end{aligned}$$

  - Backward differential formulas (BDF) : Use of previous states

→ Unconditionally stable for any time-step

… But requires the inversion of a large sparse system

- Choleski decomposition + relaxation
- Conjugate gradient
- Speed and accuracy can be improve through preconditioning (alteration of **H**)

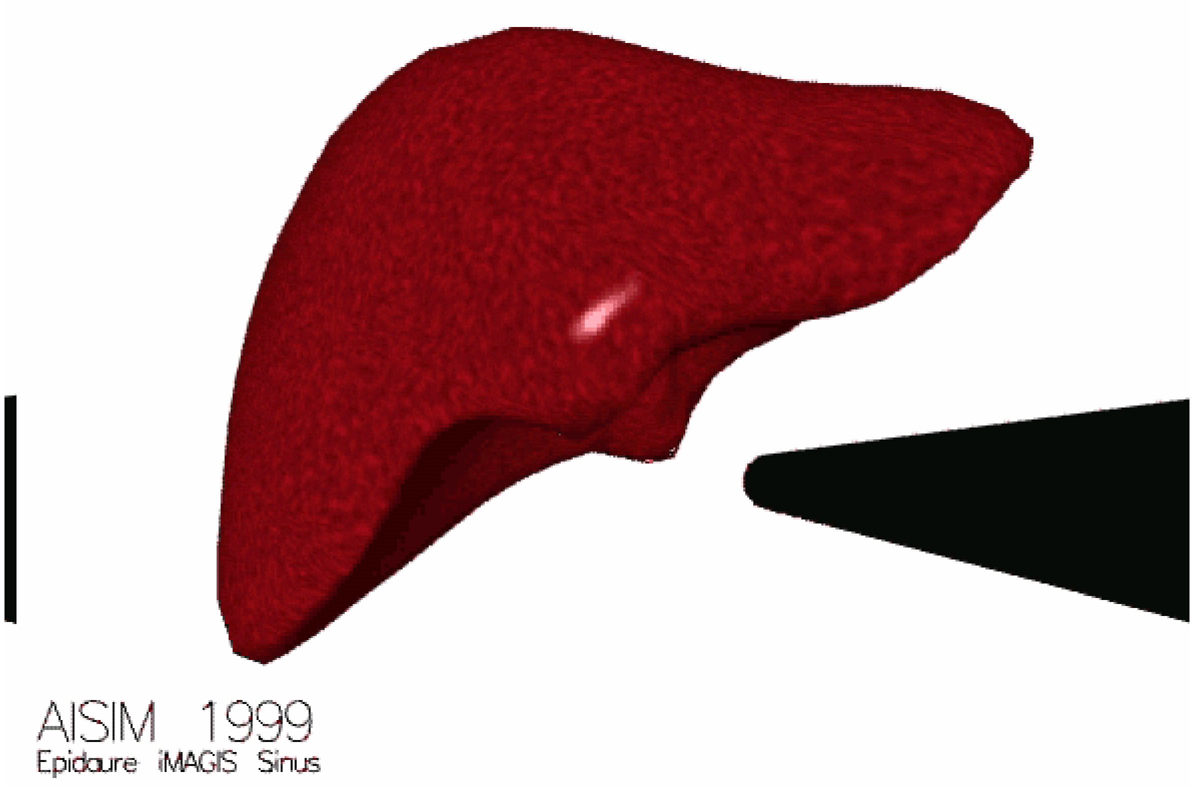
Towards Realistic Interactive Simulation

- Surgery Simulation must cope with several difficult technical issues :
  - Soft Tissue Deformation
  - Collision Detection
  - Collision Response
  - Haptics Rendering
- Real-time Constraints :
  - 25Hz for visual rendering
  - 300-1000 Hz for haptic rendering

Example of Soft Tissue Models

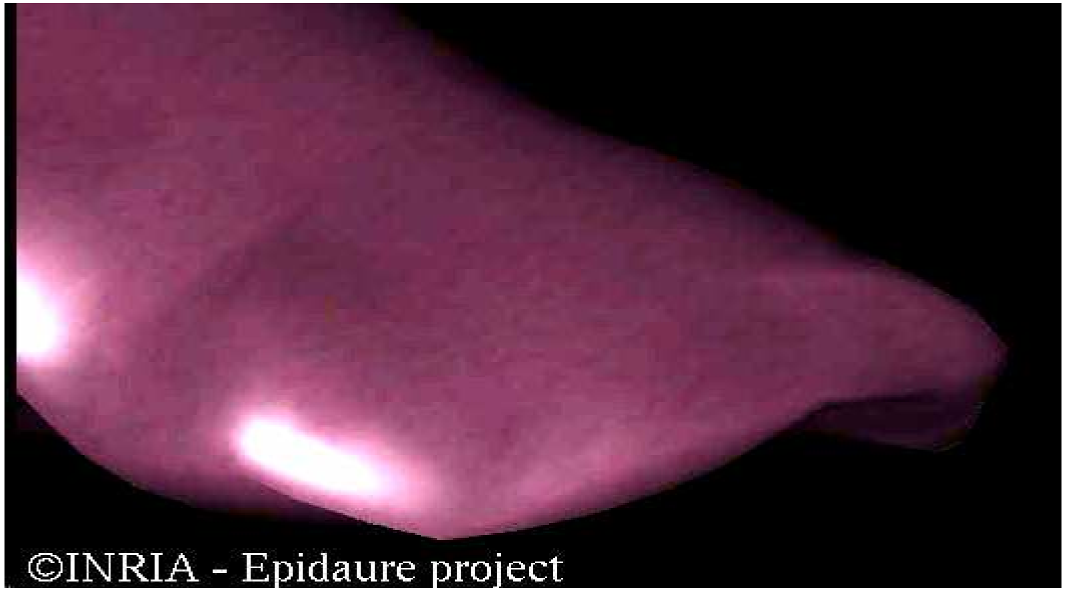
	Pre-computed Elastic Model	Tensor-Mass and Relaxation-based Model	Non-Linear Tensor-Mass Model
Computational Efficiency	+ + +	+	-
Cutting Simulation	-	++	++
Large Displacements	-	-	+

Precomputed linear elastic model



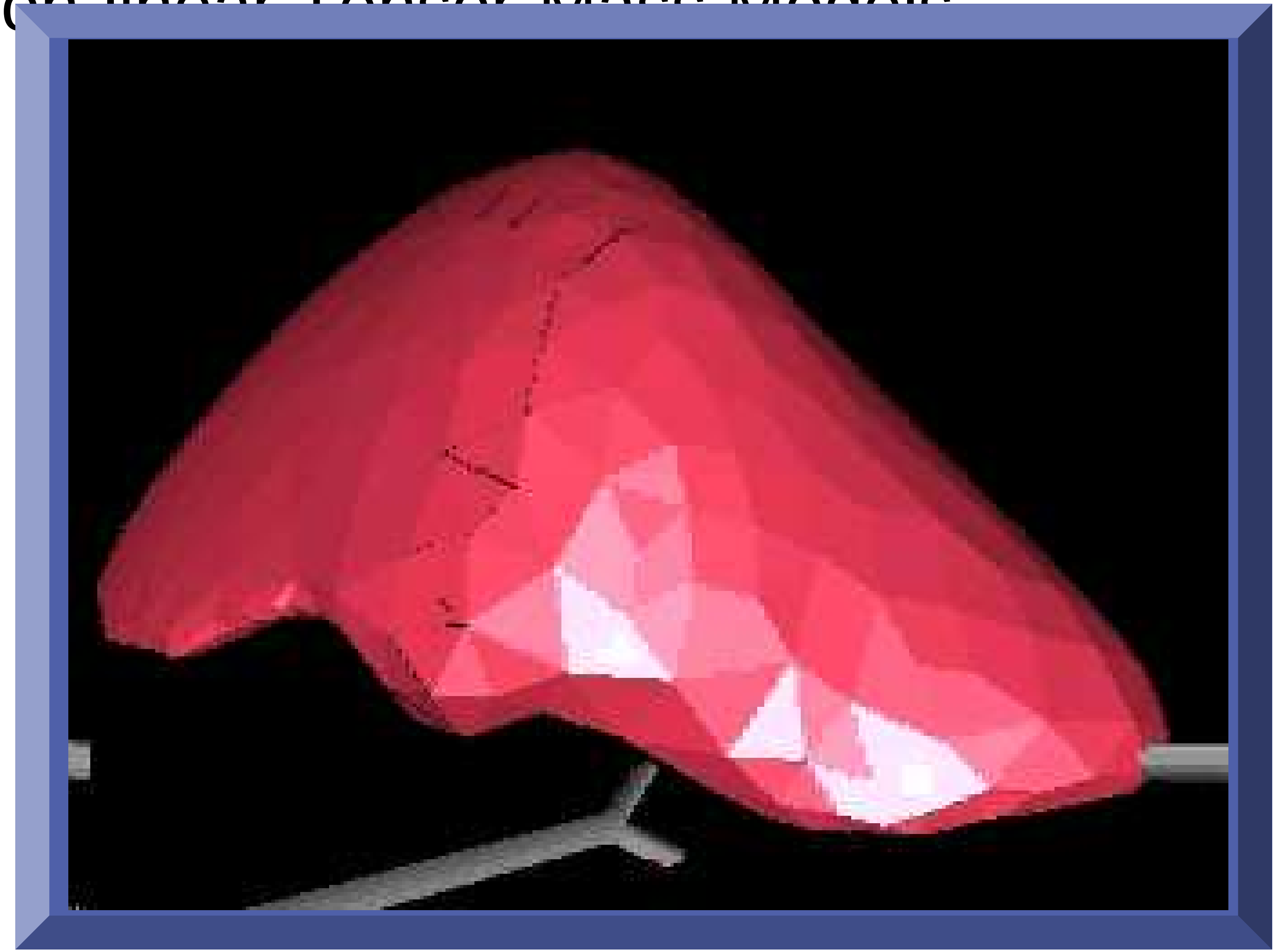
9517 Tetrahedra

Tensor-Mass Models (low resolution)

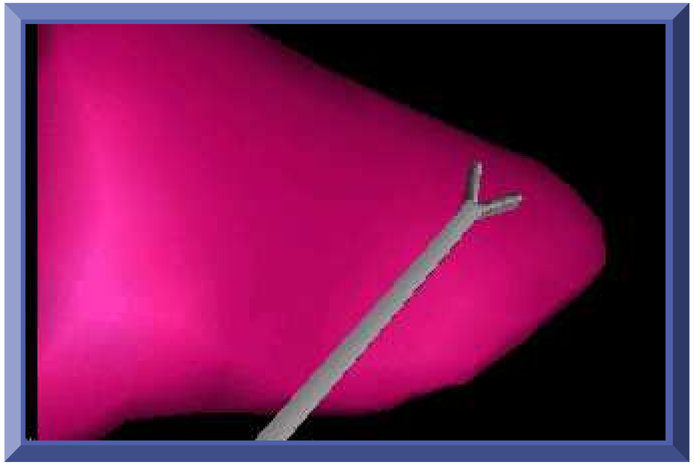


N = 1394   (6342 Tétraèdres)

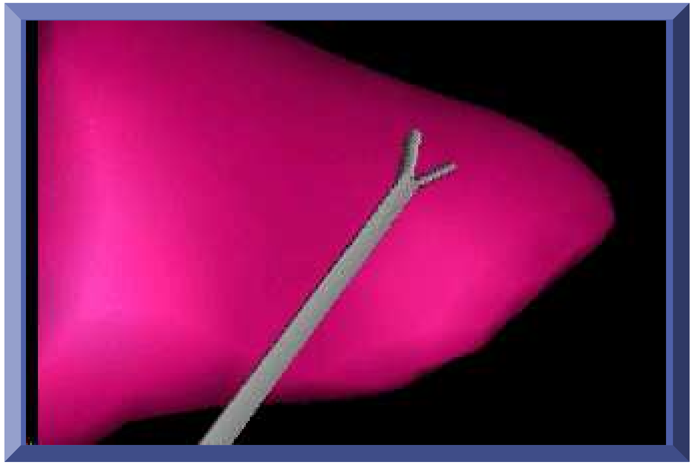
Non-linear Tensor Mass Models



Simulation of surgical gestures

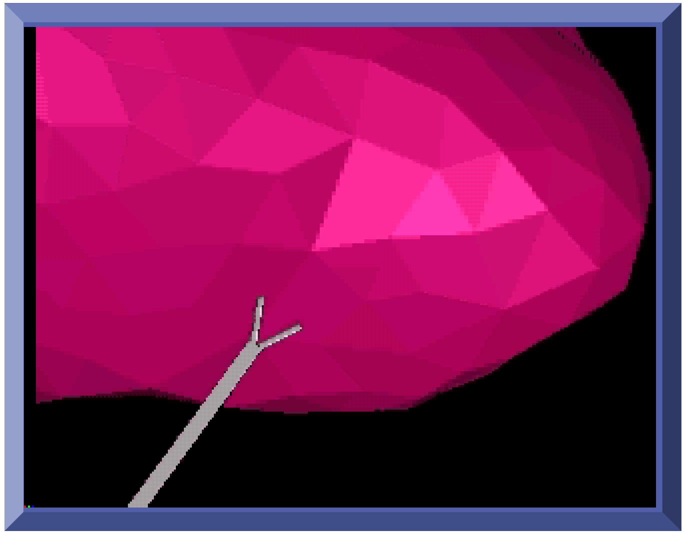


Gliding



Gripping

Cutting (pliers)



Cutting (US)

H[redacted]

Cardiac Simulation

- 4 Cardiac Phases:
  - Filling
  - Isovolumetric Contraction
  - Ejection
  - Is

- 2 Volumetric Conditions:
  - Pressure Field in the endocardium
  - Isovolumetric Constraint of myocardium

Slowed

6 times

More Information in …

- Asclepios : 3D Segmentation, Simulation Platform, Soft Tissue Modeling

  http://www.inria.fr/asclepios

- CardioSense3D : Cardiac Simulation,

  http://www.inria.fr/CardioSense3D

- SOFA : Open Software Plateform for Medical Simulation

  http://www.sofa-framework.org/