Object representation and comparison inferred from its medial axis.

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Abstract

The skeleton and its associated medial axis give a very compact representation of objects, even in the case of complex shapes and topologies. They are powerful shape descriptors, bridging the gap between low-level and high-level object representations. Surprisingly, skeletons have been used in a relatively small number of applications.

This work deals with the question of using the potential strength of the skeleton and the medial axis. From the medial axis, we build adequate attributed relational graphs to organize in a structured way informations about object shape and topology contained in the medial axis. This representation then permits to compare in a meaningful way various objects using a graph matching algorithm. Synthetic results are presented.

Keywords*: euclidean skeleton; medial axis transform; topological characterisation; shape analysis; object representation; object comparison; ARG; graph matching.*

1. Introduction

Definitions for skeleton and medial axis of digitized objects have been proposed in the early sixties. The skeleton and its associated medial axis can be used as shape descriptors and are very well-suited for a large number of computer vision applications dealing with object representation. After their introduction, an impressive amount of work has been conducted to improve their computation. Therefore, developpement of applications has been put in abeyance. Nevertheless, it appears that after more than three decades, the skeleton and the medial axis have been used in a relatively small number of applications.

This work deals with the question of using the potential strength of the skeleton and the medial axis. Actually, all the authors agree in claiming that the skeleton and the medial axis are powerful tools, bridging the gap between low-level and high-level object representations, because they resume, synthetize and help the understanding of the object shape and topology.

In section 2, we first recall some definitions and then show how to extract object properties from its medial axis and then organize this knowledge into an Attributed Relational Graph (ARG) to provide a structural description of the object. In section 3, we see how to compare objects from their skeleton-based ARGs using an inexact consistent-labeling graph matching algorithm, and finally in section 4, we present experimental results about object comparison.

2. From the medial axis to a structured representation of the object

Let us first remind some basics. Intuitively, the skeleton of an object is the set of points which are equidistant from at least two points of the object boundary (we only consider the skeleton part which is inside the object). The skeleton of a 2D object is made of pieces of curves, called *skeleton parts*, linked together by junctions, and ended by frontier points. The medial axis is defined as a set containing the skeleton points and the distance vectors joining each skeletal point to its closest boundary point. Therefore, retrieving the object shape from its medial axis is straightforward. The medial axis is a more complete representation of the object than the skeleton. Indeed, objects with different shapes can yield the same skeleton, and only the medial axis will differ and allow to distinguish between these objects.

Some notations: let X be a digital object. Its medial axis is denoted: $\{SK(X), \rho_{SK}\}$, where $SK(X)$ is the skeleton of X and ρ is the distance map of X (inside X) defined by: $M \longmapsto \rho(M) = d(M, \overline{X}) = \inf_{P \in \overline{X}} d(M, P)$, being d a distance function, classicaly the euclidean distance. $\overline{X}$ denotes the complement of X, and finally ρ_{SK} is the restriction of ρ to $SK(X)$. As the skeleton of an object X is a thin set, it allows us to get a topological classification of $SK(X)$, denoted $SK_c(X)$: each point of the skeleton will be labelled as a Frontier point (**type F**), a Junction point (**type J**) or Skeleton part point (**type C**). In the following, we will use the n-D euclidean skeletonization twofold process presented in [4].

The medial axis of an object is a very compact and informative representation of the object, but the knowlege about the object shape and topology is not organized in a structured way. Thus, before to use this powerful shape descriptor for object representation, comparison, recognition or registration tasks, a further step is needed: organize this knowledge into some kind of structure.

Attributed Relational Graphs (ARGs) are relational structures which allow to describe objects using parametric and relational informations. An ARG consists of two sets: a *set of nodes* with various types of *attributes* eventually assigned to them and a *set of links* which represent various types of relations between the nodes and may take real values, their assigned weights (see [1] and [3] for a use of ARGs in image understanding and invariant matching).

Obviously, an ARG is a well-suited structure to represent the knowledge extracted from an object medial axis. Indeed, the topologically classified euclidean skeleton is already split up into labelled parts, which will define different classes of primitives (namely *skeleton parts*, *junction components* and *frontier components*). These primitives will form the set of nodes of the ARG. To each class of primitives, we will attach attributes. To set up the set of links, we will define in a straightforward way different kinds of relations (topological or geometrical) between these primitives.

2.1. ARG nodes and associated attributes

Skeleton parts The skeleton parts are detected and labelled by a connected components extraction of **type C** pixels. From the skeleton parts labeling, we also infer a meaningful partition of the object into regions, each of these regions being associated to one of the skeleton parts. To each skeleton part $P^i_{SK(X)}$ and associated object region R^i_X we can attach the following attributes:
1. $P^i_{SK(X)}$ size compared to the skeleton size;
2. R^i_X size compared to the object size.
3. variation of the curvature sign along $P^i_{SK(X)}$;
4. variation of the distance map along $P^i_{SK(X)}$.

Junctions They can be detected and labelled by a connected components extraction of **type J** pixels of $SK_c(X)$. To each skeleton junction $J^i_{SK(X)}$ we can attach the following attributes:
1. its order as defined by the number of *skeleton parts* which meet at the junction $J^i_{SK(X)}$;
2. the value of ρ at the junction $J^i_{SK(X)}$ compared to the maximal value of ρ in $SK(X)$.

Frontiers They can be detected and labelled by a connected components extraction of **type F** pixels of $SK_c(X)$. To each skeleton frontier component we can attach the following attribute:
1. the relative value of ρ at the frontier. The larger the value of ρ is at the frontier, the smoother the curvature change at the boundary will be.

2.2. ARG links

Links or relations between the graph nodes are represented by adjacency matrices. For ARGs, there will be one adjacency matrix for each relation type.

Topological relations will express properties of the skeleton structure and will be expressed as logical flags, whereas geometrical relations will be related to distance informations available through the medial axis, and to informations about the shape of the skeleton parts, and they will be expressed as real variables, or weights.

Topological relations
1. Logical flag *"to be in contact with"* or *"to be neighbours"* (for all primitives);
2. Logical flag *"meet at the same jonction"* (for skeleton parts);
3. Logical flag *surrounding the same hole* (skeleton parts and junctions);
4. Logical flag *"to be in contact with the same skeleton part"* (for frontiers and junctions).

Geometrical relations
1. Real weight which measures the size difference between two skeleton parts;
2. Real weight which measures the relative distance between two junctions or frontiers;
3. Real weight which measures the difference in orientation between two skeleton parts.

3. Comparing objects from their ARGs

Consider the following problem: we have a series of objects and we would like to compare them and also get a ranking which would be relevant with respect to their shape and topology. For each object, we first compute its skeleton, its medial axis, characterize it topologically, get its labelled partition and build an ARG.

Then, to compare two objects of the series using their skeleton-based ARGs, we need to solve a graph matching problem. We use an error-correcting consistent-labeling graph matching algorithm which can handle ARGs and uses a nonlinear optimization method called graduated assignment (all details in [2]).

Given two ARGs G and g, with A and I nodes respectively, R link types and S attribute types, it finds the association matrix M such that the following objective function is minimized:

$$E=\frac{1}{2}\sum_{a,b}^{A}\sum_{i,j}^{I}M_{ai}M_{bj}\sum_{r}^{R}C^{(r)}_{aibj}+\sum_{a}^{A}\sum_{i}^{I}M_{ai}\sum_{s}^{S}C^{(s)}_{ai}$$

subject to: $\forall a, \forall i, \sum_{i=1}^{I} M_{ai} \leq 1, \sum_{a=1}^{A} M_{ai} \leq 1, M_{ai} \in \{0,1\}$

$\{C^{(r)}_{aibj}\}$ is the compatibility matrix for a r-link and is defined by: $C^r_{aibj} = cl^r(G^r_{ab}, g^r_{ij})$ (0 if either G^r_{ab} or g^r_{ij} is NULL); $\{C^{(s)}_{ai}\}$ is the similarity matrix for an attribute of type s and is defined by: $C^s_{ai} = cn^s(G^s_a, g^s_i)$;

$\{G_{ab}\}^r$ and $\{g_{ij}\}^r$ are the adjacency matrices for the r-link; $cl^r(.,.)$ is a measure of compatibility between a r-link in G and a r-link in g; $\{G_a^{(s)}\}$ and $\{g_i^{(s)}\}$ are vectors corresponding to the s-attribute of the nodes of G and g; $cn^s(.,.)$ is a measure of similarity between a node in G and a node in g, with respect to the same attribute s.

M is an $(A \times I)$ association matrix which at the end of the minimization process gives the correspondences between one set of primitives and the other: $M_{ai} = 1$ if node *a* in G corresponds to node *i* in g, 0 otherwise. Note that the algorithm does not always converge to an exact permutation matrix, thus a clean-up heuristic has to be defined. For the experiments presented in this paper, we use a very simple heuristic: we set in each column of the association matrix M the maximum element to 1 and others to 0.

Note that the value of the objective function E at the end of the process directly gives a qualitative measure of the matching. It can therefore be used to set up a ranking of a series of objects.

4. Experimental results

To evaluate the capabilities of the skeleton-based ARG matching algorithm, we conduct 2 experiments on object comparison: from an original object, we get 3 distorded copies; we then look for correspondences between object parts in the series; finally we set up a ranking of the objects in the series based on the ARG matching result.

The preprocessing consists in the following steps: 1) compute skeleton and medial axis of each object of the series; 2) topologically characterize the skeleton; 3) infer labelled partition of all skeleton parts and object parts of each object; 4) build the ARG of each object. We also select some of the possible nodes, attributes and links to build the ARG: **nodes** - skeleton parts; **attributes** - skeleton part size compared to skeleton size, object part size compared to object size, variation of the distance map along skeleton part; **links** - logical flag *"meet at the same jonction"*, logical flag *"surrounding the same hole"*, real weight which measures the size difference between two skeleton parts, where the size is computed as the number of pixels which compose the parts. The compatibility measure $cl^r(G^r_{ab}, g^r_{ij})$ and the similarity measure $cn^s(G^s_a, g^s_i)$ are taken as the absolute differences of their arguments.

In the first experiment (Figure 1), the 3 distorded copies are obtained through the following successive deformations: 1. similitude; 2. similitude + global shape change; 3. similitude + flip + local shape change. In the second experiment (Figure 2), the deformations are: 1. similitude; 2. similitude + local shape change; 3. similitude + topology change applied to 2. The four objects with their skeletons superimposed as well as their labelled partition can be seen in the first lines of each figure. We then infer the corresponding ARGs.

Then we perform the skeleton-based ARG matching from 1 to 2, 1 to 3 and 1 to 4, resulting in a set 4 association matrices $M_{1\mapsto 2}$, $M_{1\mapsto 3}$ and $M_{1\mapsto 4}$ which give the correspondences between skeleton parts in each object of the series. Using these correspondences, we can propagate the labels of the original object regions to the distorded objects (bottom lines of each figure; each region is numbered with its label).

For the first experiment, each similar region in each of the objects has been assigned the same label, in other words the correspondences between all parts of the 4 objects have been perfectly retrieved. For the second experiment, the correspondences have been almost perfectly retrieved. In particular, it is important to note that for the last object, the labeling is kept consistent in the object even if it has been modified around the missing hole (labels $7, 10, 14$ have disappeared because of the topology change).

Concerning ranking, values of the objective function of the ARG matching (corresponding to $M_{1\mapsto 2}$, $M_{1\mapsto 3}$ and $M_{1\mapsto 4}$) are for the first experiment $(1.02, 1.70, 4.70)$ and for the second experiment $(12.19, 15.10, 26.69)$. This ranking is what is visually expected, as the dissimilarity between objects in the 2 series increases from left to right.

5. Conclusion

We have presented a method which allows to exploit the informations about object shape and topology contained in an object medial axis. First we have seen how to organize this information into an Attributed Relational Graph, and then how to compare series of objects using their skeleton-based ARGs through a graph matching algorithm.

The preliminary results that we have presented are fairly promising as they indicate that using a skeleton-based ARG matching allows to get consistent correspondences between object parts and to classify objects based on shape and topology similarities. We will now look at applications of this method for object recognition and object matching tasks.

6. Acknowledgements

This research has been supported in part by the UE, DG XII (TMR program), contract Nr. ERBFMBICT-977647 and by the Spanish Ministry of Education, contracts Nr. TIC-1134-C02-01 and TIC-97-1150 (DGICYT).

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Figure 1. Top line: Original object (left one) + 3 distorded copies with superimposed skeletons and independantly computed labelled partitions. Bottom line: 4 objects labelled by propagating labels of the original object (left one) through the skeleton-based ARG matching (see text for details).

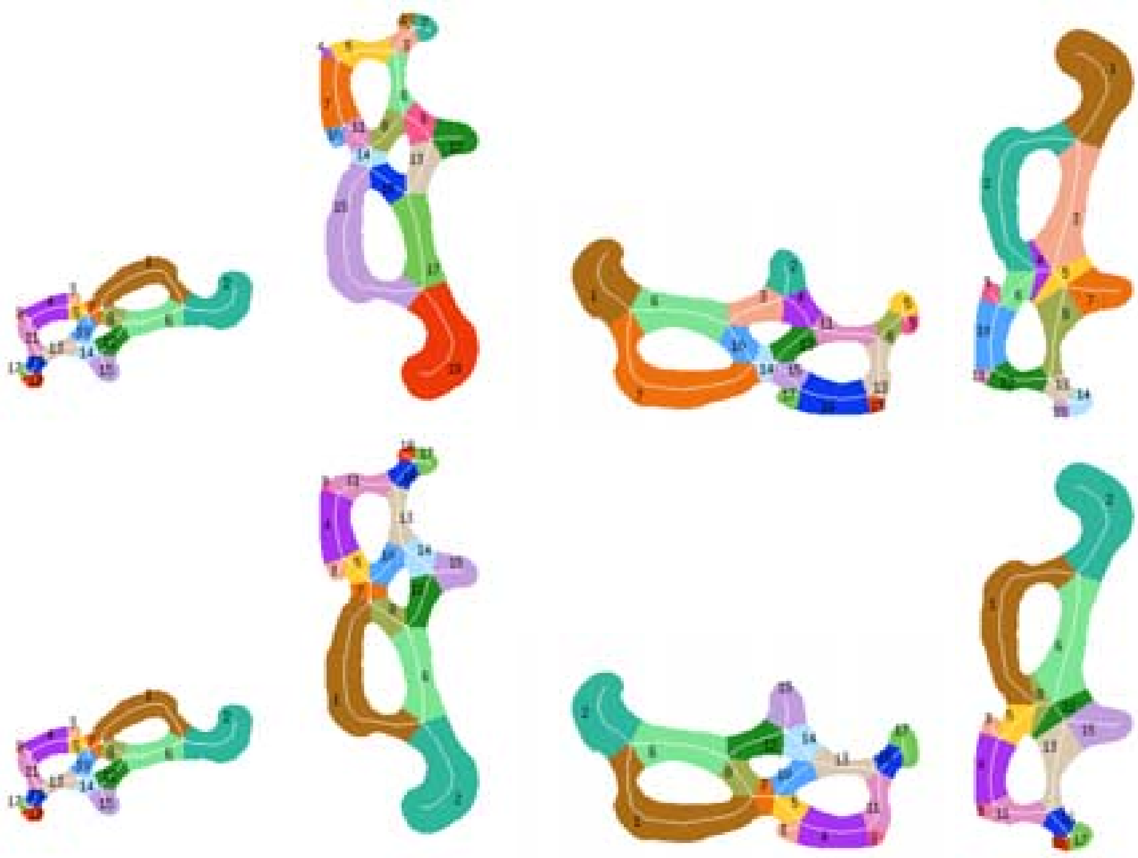

Figure 2. Top line: Original object (left one) + 3 distorded copies with superimposed skeletons and independantly computed labelled partitions. Bottom line: 4 objects labelled by propagating labels of the original object (left one) through the skeleton-based ARG matching (see text for details).