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Adaptive and Deformable Models based on Simplex Meshes*

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Abstract

Simplex meshes are simply connected meshes that are topologically dual of triangulations. In a previous work, we have introduced the simplex mesh representation for performing recognition of partially occluded smooth objects[5]. In this paper, we present a physically-based approach for recovering three-dimensional objects, based on the geometry of simplex meshes. Elastic behavior is modelled by local stabilizing functionals, controlling the mean curvature through the simplex angle extracted at each vertex. Those functionals are viewpoint-invariant, intrinsic and scale-sensitive. Unlike deformable surfaces defined on regular grids, simplex meshes are highly adaptive structures, and we have developed a refinement process for increasing the mesh resolution at highly curved or inaccurate parts. End contours are created in a semi-automatic way. Finally, operations for connecting simplex meshes are performed to recover complex models from parts of simpler shapes.

1 Introduction

The emergence of high resolution three-dimensional images either in the form of range data or voxel images, enforces the need for general shape reconstruction techniques. The difficulty stems from the necessary flexibility of object reconstruction systems to include a wide variety of man-made or natural shapes. Flexibility should be achieved both in terms of geometry and topology. Geometry relates to the local control of shape whereas topology relates to the global model structure.

Dynamically deformable models were first proposed by Terzopoulos et al. and have attracted significant interest for their intuitive and clay-like behavior. Several researchers have applied the dynamic model fitting scheme to range data or medical images[2][4][8][7]. Elastic models successfully address the problem of shape control. However, few researchers have proposed general adaptive reconstruction techniques for solving both geometric and topological aspects. [8][9][6]

This paper presents a shape reconstruction algorithm that offers both geometric and topological flexibility. As in Hoppe *et al.*[6], we chose to represent a surface with a discrete mesh without considering a continuous surface representation. We use a simply connected mesh or *simplex mesh* as a surface representation in a deformable model fitting approach. The simplex mesh representation has several desirable properties that makes them well suited for the recovery of geometric models from range data. The surface reconstruction system that we are presenting in this paper, has the following characteristics:

Generality The simplex mesh representation is general because it can represent all types of orientable surfaces regardless of their genus and end numbers.

Simplicity and Efficiency of Implementation The meshes and contours are considered as physically based models. The displacement between two iterations is derived from the computation of an internal and external force. Most models presented in this paper can be deformed in real-time.

Local Shape Functionals We have defined at set of shape functionals that are derived from the minimization of a local energy and have the properties of being viewpoint invariant and scale sensitive.

Adaptability Three different levels of adaptability was defined on a simplex mesh. A first algorithm consists in adapting the spacing of vertices in order to obtain a concentration of vertices at parts of high mean curvature. The second algorithm consists in refining the mesh by adding vertices

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at parts that do not correctly fit the data. Finally, we adapt the mesh topology by creating end contours where the data is incomplete.

Contour-Surface Interaction Contours or closed curves may be simply defined on a simplex mesh.

2 Simplex Meshes

2.1 Topology

A simplex mesh has constant vertex connectivity. In order to represent three dimensional surfaces, we make use of 2-simplex meshes where each vertex is connected to three neighboring vertices. The structure of a simplex mesh is dual of the structure of a triangulation (see figure 1). However, this correspondance exists only in terms of topology but not in terms of geometry. In another words, we cannot associate an underlying triangulation to given simplex mesh and conversely. Therefore, the simplex mesh representation has different geometric properties than triangulations that make them better suited for surface reconstruction. However, both representations are general since they can represent all types of orientable surfaces.

We have coined the word *simplex mesh* in order to stress the existence of a 3-simplex, a tetrahedron, at each vertex. The structure of a simplex mesh is the one of a simply connected graph and does not in itself constitute a new surface representation. The main contribution of this paper, however, is to exhibit the topological and geometric properties inherent to those meshes and demonstrate their relevance for object reconstruction as well as object recognition.

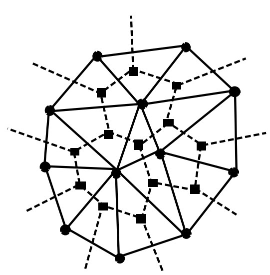

Figure 1: A 2-simplex mesh and its dual triangulation.

We define a contour on a simplex mesh as a closed polygonal chain consisting of neighboring vertices on the simplex mesh. We restrict a contour to not intersect itself. Contours are deformable models as well, and they are handled independently of the simplex mesh where they are embedded. In terms of surface topology, contours on a 2-simplex mesh can be classified in two categories depending whether they are "dividing" or not. The combination of surface and contour deformation enables the recovery of objects with complex topology.

2.2 Mesh Transformation

Simplex Meshes as well as triangulations are locally adaptive meshes. We define at set of four independent transformations $\{T_1^2, T_2^2, T_3^2, T_4^2\}$ for achieving the whole range of possible mesh tranformations. The first two transformations are Eulerian and therefore do not change the mesh topology. They consist in inserting or deleting edges in a face. The last two transformations correspond to either connecting two faces or cutting a mesh along a contour[3]. When the contour is dividing, the cutting operation results in splitting the mesh into two parts. Otherwise, it results in decreasing the genus of the mesh.

2.3 Geometry

We introduce the notion of *Simplex Angle* on a simplex mesh, that generalizes in many ways, the angle used in planar geometry. In the following sections, we will describe how this angle is linked to the mean curvature of a surface and how a shape description of a mesh may be uniquely determined.

2.3.1 Simplex Angle

Let $\mathcal{M}$ be an oriented simplex mesh of $\mathbb{R}^3$. Let $P_i \in \mathbb{R}^3$ be a vertex of a 2-simplex mesh, and $(P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)})$ its three neighbors. The three neighboring vertices define a plane $\mathcal{P}_i$ and its normal vector $\overrightarrow{N_i}$. If S_1 is the circumscribed circle at the three neighboring vertices $(P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)})$ of radius r_i and S_2 be the circumscribed sphere at the four vertices $(P_i, P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)})$ of radius R_i, then the simplex angle at vertex P_i, $\varphi_i = \angle(P_i, P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)})$ is defined as:

$$\begin{aligned} \varphi_i &\in [-\pi, \pi] : \\ \sin(\varphi_i) &= r_i R_i sign(\overrightarrow{P_i P_{N_1(i)}} \cdot \overrightarrow{N_i}) \\ \cos(\varphi_i) &= \|O_i C_i\| R_i * sign(\overrightarrow{O_i C_i} \cdot \overrightarrow{N_i}) \end{aligned} \qquad (1)$$

The simplex angle φ_i is independent of the position of $P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)}$ on the circle S_1 and of P_i on a hemisphere of S_2. It is null when P_i and its three neighbors are coplanar. The simplex angle φ_i can be easily computed by considering an inversion of center P_i. More details on the simplex angle definition and properties can be found in [3].

We then define the notion of *discrete mean curvature* H_i on a simplex mesh as the inverse of the radius of the circumscribed sphere:

$$H_i = sign(\overrightarrow{P_i P_{N_1(i)}} \cdot \overrightarrow{N_i}) R_i = \sin(\varphi_i) r_i \quad (2)$$

We justify this definition by demonstrating that the mean curvature at a point on a three dimensional surface is the inverse radius of the sphere that best approximate the surface at that point [3].

2.3.2 Metric Parameters

In addition to the simplex angle, we introduce two additional parameters called *metric parameters* $\{\epsilon_{1i}, \epsilon_{2i}\}$ that describe how a vertex is located with respect to his 3 neighbors. If we consider the orthogonal projection F_i of P_i on the plane defined by its 3 neighbors, then F_i may be written as a weighted average of the 3 neighboring vertices:

$$F_i = \epsilon_{1i} P_{N_1(i)} + \epsilon_{2i} P_{N_2(i)} + \epsilon_{3i} P_{N_3(i)} \quad (3)$$
$$\epsilon_{1i} + \epsilon_{2i} + \epsilon_{3i} = 1 \quad (4)$$

The position of a vertex P_i is completely determined by the position of its 3 neighbors and the knowledge of $(\epsilon_{1i}, \epsilon_{2i}, \varphi)$:

$$P_i = \epsilon_{1i} P_{N_1(i)} + \epsilon_{2i} P_{N_2(i)} + \epsilon_{3i} P_{N_3(i)} + L(r_i, d_i, \varphi_i)\vec{N_i} \quad (5)$$

where d_i is the distance between F_i and the center of the circumscribed circle C_i and where $L(r_i, d_i, \varphi_i)$ is a function described in [3].

The $3n$ values $\{(\epsilon_{1i}, \epsilon_{2i}, \varphi_i)\}$ of a simplex model $\mathcal{M}$ consisting of n vertices completely describe the model shape, up to an isometry and scale.

3 Smoothness and Shape Control

3.1 Equations of Motion

We propose a modeling scheme based on deformable and adaptive mesh. The dynamics of each vertex is given by a Newtonian law of motion:

$$m \frac{d^2 P_i}{dt^2} = -\gamma \frac{dP_i}{dt} + \overrightarrow{F_{int}} + \overrightarrow{F_{ext}} \quad (6)$$

where m is the mass unit of a vertex and γ is the damping factor. $\overrightarrow{F_{int}}$ contrains either the shape of a mesh to be smooth whereas $\overrightarrow{F_{ext}}$ constrains the mesh to be close from some three-dimensional data.

3.2 Internal Forces

Internal forces determine the response of a physically-based model to external constraints. In this paper, we do not derive the internal force expression from an almost quadratic smoothness energy as in [8][2]. Instead, we chose to minimize a local energy $\mathcal{S}_i = \frac{\alpha_i}{2} P_i P_i^{\star 2}$. $P_i^\star$ is computed from equation 5 with a value of $\varphi_i = \varphi_i^\star$. The choice of $\varphi_i^\star$ determines the types of constraints enforced on the mesh:

Normal Discontinuity We set $\varphi_i^\star = \varphi_i$. The surface can freely bend around vertex P_i.

Normal Continuity constraint We have simply $\varphi_i^\star = 0$. Hence, the internal force is: $\vec{F_{int}} = \alpha_i(\epsilon_{1i} P_{N_1(i)} + \epsilon_{2i} P_{N_2(i)} + \epsilon_{3i} P_{N_3(i)} - P_i)$.

Mean Curvature Continuity Constraint $\varphi_i^\star$ is chosen such that the mean curvature in P_i, $H_i^\star$, is the weighted average of the mean curvature in a neighborhood $N^r(P_i)$: $H_i^\star = \sum_{j \in N^r(P_i)} e_{ij} * H_j$. r is the size of the neighborhood on which the smoothing is performed. This parameter is called the *rigidity* and it effects the dynamics of the surface model.

Shape Constraint Given the constant φ_i^0 by setting $\varphi_i^\star = \varphi_i^0$ we constrain the simplex angle at P_i to φ_i^0. In this case, since we are using constant metric parameters, this amounts to constraining the shape of the mesh up to an isometry and scale.

The expression of those internal forces has the advantages of being intrinsic, viewpoint invariant and scale dependant. Similar type of constraints with similar formulation hold for contours.

3.3 External Forces

In a surface reconstruction scheme based on deformable models, external forces constrain the closeness of fit to some three dimensional data. At each vertex P_i , the closest point $M_{Cl(i)}$ on the data is computed and the force is computed as:

$$\overrightarrow{F_{ext}} = \beta_i G \left(\frac{\| \overrightarrow{P_i M_{Cl(i)}} \|}{D} \right) (\overrightarrow{P_i M_{Cl(i)}} \cdot \vec{N_i}) \vec{N_i} \quad (7)$$

$G(x)$ is the stiffness function that is constant between 0 and 1 and rapidly decreases at values greater than

1. D corresponds to the maximum distance at which a data point strongly attracts a vertex, and it is computed as a fraction of the overall data diameter. Indeed, in order to avoid the effect of outliers, the mesh model is only attracted toward data points that are relatively close.

The computation of the closest point depends on the data type. For structured range data, or volumetric images, it is computed in a $O(1)$ complexity by projecting the normal line on the image. For scattered data points, we use a kd-tree structure to get the data points located inside a sphere of radius D and centered on P_i.

4 Topology Control

4.1 Mesh Adaptivity

The metric parameters control the relative distance of P_i from its three neighboring vertices. While we have previously used the simplex angle to control the variation of mean curvature, we will define in this section a procedure to adapt the spacing of vertices to the mean curvature of the mesh. The notion of adaptive mesh has been studied by several researchers[8][9][6][1]. In all cases, their aim is to provide an optimal shape description from a mesh with a fixed number of vertices, by concentrating vertices at highly curved parts. Our algorithm uses that approach but is characterized by the following properties:

- The concentration of vertices is governed by the local minimization of an energy $\mathcal{E}_i$. This energy expresses the link between the metric parameters and the variation of mean curvature.
- Vertices of low mean curvature migrate toward neighboring vertices of relatively larger mean curvature. Therefore, the concentration of vertices is governed by the relative variation of curvature.
- Vertices of high mean curvature have metric parameters close to $\frac{1}{3}$ in order to obtain a uniform concentration at highly curved parts.

We proceed by periodically adapting the metric parameters to the mean curvature of a mesh. if ϵ_i^t is the value of the metric parameters at iteration t, then we compute the metric parameter at iteration $t+p$ as:

$$\epsilon_i^{t+p} = \epsilon_i^t + \frac{1}{2}\vec{\bigtriangledown}\mathcal{E}_i$$
$$\mathcal{E}_i = \frac{1}{2}(\epsilon_i^\star - \epsilon_i)^2$$

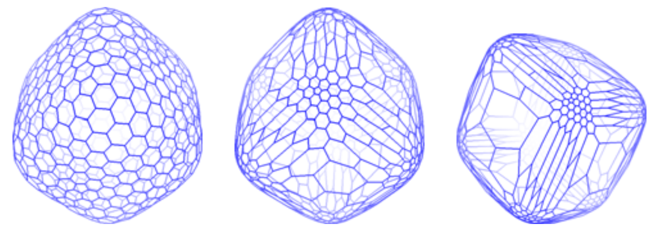

Figure 2: **Left** the mesh fit on the isosurface. The metric parameters are everywhere equal to $\frac{1}{3}$; **Center** The adaptive mesh with a value of $\gamma_i = 0.15$; **Right** The adaptive mesh with a value of $\gamma_i = 0.20$.

where $\epsilon_i^\star$ is computed as a function of the variation of the absolute value of the mean curvature. We derive the expression of $\epsilon_i^\star$ by first considering the mean value of the absolute mean curvature $|\bar{H}_i| = (|H_{N_1(i)}| + |H_{N_2(i)}| + |H_{N_3(i)}|)/3$. We then compute the relative mean curvature deviation vector $\delta|H|_i$:

$$\delta|H|_i = \begin{pmatrix} \frac{|H_{N_1(i)}| - |\bar{H}_i|}{|H_i|} \\ \frac{|H_{N_2(i)}| - |\bar{H}_i|}{|H_i|} \\ \frac{|H_{N_3(i)}| - |\bar{H}_i|}{|H_i|} \end{pmatrix}$$

We link the value of the reference metric parameter $\epsilon_i^\star$ with the relative mean curvature deviation vector:

$$\epsilon_i^\star = \frac{1}{3} + \gamma_i \delta|H|_i \quad (8)$$

γ_i is a constant that controls the extent of the adaptivity of the simplex mesh and is usually chosen between 0.03 and 0.25. However, since we guaranty all metric parameter ϵ_{ki} to be greater than 0.05 and lower than 0.833, we may have a γ_i substantially lower than 0.05. Finally we have:

$$\epsilon_i^{t+p} = \epsilon_i^t + \frac{1}{2}(\epsilon_i^\star - \epsilon_i^{t+p}) \quad (9)$$

The coefficient $\frac{1}{2}$ enables a smoother variation of the metric parameters over time. In practice, we choose to update the metric parameters every $p = 10$ iteration in order to stabilize the mesh before re-evaluating the mean curvature over the mesh.

Figure 2 shows the effect of the mesh adaptation on an isosurface shaped as a cube.

4.2 Refinement

We introduce an algorithm for automatically increasing the mesh resolution at parts of high curvature or at parts whose distance to the range data is higher than a threshold. The refinement process is completed in an iterative way. First, we evaluate, for every face model, a criterion measuring the need for refinement. Then, faces whose criterion exceeds a given threshold, are refined and the mesh is deformed during a constant number of iterations. The refinement process is repeated until all faces criteria are below the threshold. This approach has the advantage of recovering models satisfying both geometric constraints (regularity and closeness of fit) and topological constraints (optimal vertex spacing).

The criterion is computed as the face area multiplied by three dimensionless coefficients. The first one is the measure of Gaussian curvature on the face. It can be computed as the area of a spherical polygon on the Gauss sphere [3]. The second coefficient is the ratio of the distance to the closest data point and the reference distance D. The third coefficient measures the elongation of a face in order to refine in priority large elongated faces. The threshold has the dimension of a surface area and therefore the refinement process is guaranteed to stop since the faces area decrease at every iteration. In order to keep the number of vertices per face as close as possible to six, we either use a T_2^2 operation or a T_5^2 operation [3] depending whether the face has more or less than five vertices.

4.3 End Contour Creation

We have implemented an algorithm that adapts the simplex model topology to the three dimensional data, by creating holes or ends at parts where the mesh is distant from the data. We proceed by first computing at each vertex the distance to the closest data point. We then create zones, i.e. set of faces whose vertices are located at a distance greater than a given threshold from the data. A set of contours surrounding those zones are created and operation T_3^2 is performed to remove the artificial part. Contours are systematically created around the newly created faces. Those deformable contours will deform to fit the shape of the hole existing in the data set.

4.4 Building Models from Parts

Recovering a complex model by deforming and refining a simple primitive is a difficult task since the mesh would have to automatically change its genus and avoid numerous local minima. Moreover, current range techniques cannot acquire a complex shaped object within a single image. A more natural approach for recovering complex objects, consists in connecting separately built models, corresponding to approximately convex subsets. In this framework, different meshes are fit on different subsets of a same objects. Then two meshes are connected with a single transformation operation and the zone around the connection is smoothed to remove the normal and curvature discontinuities.

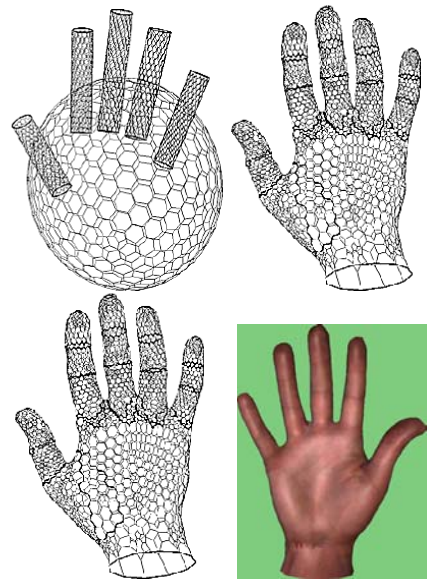

Figure 3: **Upper left** Six initial meshes; **Upper right** Resulting hand model after connecting the finger models to the palm model; **Lower left** Face model after refinement;**Lower right** Rendered face model.

5 Experimental Results

We have applied the simplex mesh modeling on a variety of structured and unstructured three-dimensional datasets. The recovery of three-dimensional objects from range data proceeds in two stages. In a first stage, the model is initialized as one of four primitives with a limited number of vertices. We use a high value of the rigidity parameter to obtain a smooth and stable deformation toward the image data. However, the resulting model tends to be inaccurate at locations of high curvature. In a second stage, we apply the iterative adaptation and refinement process over the whole mesh or over selected parts while decreasing the rigidity parameter to its minimum. A new equilibrium is reached when highly curved parts are sufficiently refined and adapted and when vertices closely approximate the raw data.

Human Face Figure 4 shows the different stages of the fitting process on a human face digitized with a Cyberware Inc. range finder. The first fit does not

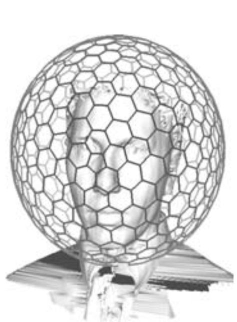

Figure 4: **Left** The initial mesh is a sphere with 720 nodes; **Center** the mesh is fit on the surface. The mesh grossly approximates the surface because it is smoothed over a large extent and because the mesh is not adapted; **Right** The mesh is now iteratively adapted and refined. We chose a small value of $\gamma_i = 0.06$ and a medium threshold for controlling the refinement;

correctly fit the nose and the chin, but nicely interpolates the missing data corresponding to the hair (figure b).' Figures c displayd the result of the combined adaptation and refinement algorithms. The final mesh has 1700 vertices. The mesh has been cut at the level of the neck and the bounding contour is deformed to fit the data.

Human Skull We use a $60 \times 60 \times 60$ CT Scan image of a human skull. The isosurface is a closed surface of complex topology. Fitting a mesh on a isosurface greatly differs from previous cases because the data is already segmented and complete. The aim here is to recover a simplified model of the skull in order to perform interactive deformation on a surgery simulator. In this example, we first deform a 2000 vertices spherical simplex mesh toward the skull data. The model then grossly oversmooth the skull especially around the jaw. The adaptation of the mesh greatly decreases its overall distance to the data. No refinement was performed in this example.

6 Conclusion

We have presented a general algorithm for recovering three dimensional objects from range data. Simplex meshes modeling differs from previous reconstruction techniques by their highly malleable and their essentially local nature.

Future work will focus on the extraction of geometric representation of anatonomical parts from medical images by elastically deforming some generic simplex models.

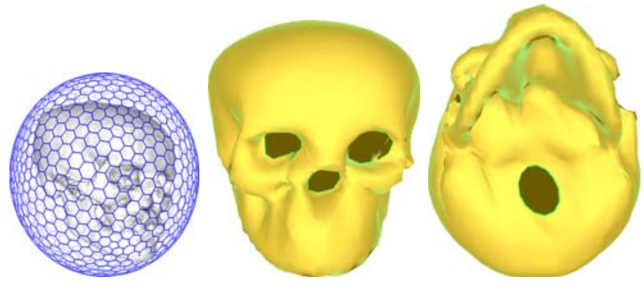

Figure 5: **Left** A sphere with 2000 vertices is initialized around the skull data extracted from a CT Scan Image; **Center** Rendered view of the adapeted mesh. End contours located at the eyeballs, the nose and the foramen are automatically created; **Right** Bottom view of the skull

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