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Simplex Meshes: a General Representation for 3D Shape Reconstruction*

H. Delingette
Projet Epidaure
I.N.R.I.A.
06902 Sophia-Antipolis, BP 93, France

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Abstract

Simplex meshes are simply connected meshes that are topologically dual of triangulations. In a previous work, we have introduced the simplex mesh representation for performing recognition of partially occluded smooth objects[3]. In this paper, we present a physically-based approach for recovering three-dimensional objects, based on the geometry of simplex meshes. Elastic behavior is modelled by local stabilizing functionals, controlling the mean curvature through the simplex angle extracted at each vertex. Those functionals are viewpoint-invariant, intrinsic and scale-sensitive. Unlike deformable surfaces defined on regular grids, simplex meshes are highly adaptive structures, and we have developed a refinement process for increasing the mesh resolution at highly curved or inaccurate parts. Furthermore, operations for connecting simplex meshes are performed to recover complex models from parts with simpler shapes.

1 Introduction

The emergence of high resolution three-dimensional images either in the form of range data or voxel images, enforces the need for general shape reconstruction techniques. The difficulty stems from the necessary flexibility of object reconstruction systems to include a wide variety of man-made or natural shapes. Flexibility should be achieved both in terms of geometry and topology.

This paper presents a shape reconstruction algorithm that offers both geometric and topological adaptability. We use a simply connected mesh or *simplex mesh* as a surface representation in a deformable model fitting approach.

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2 Simplex Meshes

2.1 Topology

A simplex mesh has constant vertex connectivity. In order to represent three dimensional surfaces, we make use of 2-simplex meshes where each vertex is connected to three neighboring vertices. The structure of a simplex mesh is dual of the structure of a triangulation (see figure 1). However, this correspondance exists only in terms of topology but not in terms of geometry. In another words, we cannot associate an underlying triangulation to given simplex mesh and conversely. Therefore, the simplex mesh representation has different geometric properties than triangulations that make them better suited for surface reconstruction. However, both representations are general since they can represent all types of orientable surfaces.

We have coined the word *simplex mesh* in order to stress the existence of a 3-simplex, a tetrahedron, at each vertex. The structure of a simplex mesh is the one of a simply connected graph and does not in itself constitute a new surface representation. The main contribution of this paper, however, is to exhibit the topological and geometric properties inherent to those meshes and demonstrate their relevance for object reconstruction as well as object recognition.

We define a contour on a simplex mesh as a closed polygonal chain consisting of neighboring vertices on the simplex mesh. We restrict a contour to not intersect itself. Contours are deformable models as well, and they are handled independently of the simplex mesh where they are embedded. In terms of surface topology, contours on a 2SM can be classified in two

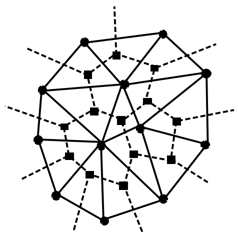

Figure 1: A 2-simplex mesh and its dual triangulation.

categories depending whether they are "dividing" or not. The combination of surface and contour deformation enables the recovery of objects with complex topology.

2.2 Mesh Transformation

Simplex Meshes as well as triangulations locally adaptive meshes. We define at set of four independent transformations $\{T_1^2, T_2^2, T_3^2, T_4^2\}$ for achieving the whole range of possible mesh tranformations. The first two transformations are Eulerian and therefore do not change the mesh topology. They consist in inserting or deleting edges in a face. The last two transformations correspond to either connecting two faces or cutting a mesh along a contour[2]. When the contour is dividing, the cutting operation results in splitting the mesh into two parts. Otherwise, it results in decreasing the genus of the mesh.

2.3 Geometry

We introduce the notion of *Simplex Angle* on a simplex mesh, that generalizes in many ways, the angle used in planar geometry. In the following sections, we will describe how this angle is linked to the mean curvature of a surface and how a shape description of a mesh may be uniquely determined.

2.3.1 Simplex Angle

Let $\mathcal{M}$ be an oriented simplex mesh of $\mathbb{R}^3$. Let $P_i \in \mathbb{R}^3$ be a vertex of a 2-simplex mesh, and $(P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)})$ its three neighbors. The three neighboring points define a plane $\mathcal{P}_i$ and its normal vector $\overrightarrow{N_i}$. If S_1 is the circumscribed circle at the three neighboring vertices $(P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)})$ vertices of radius r_i and S_2 be the circumscribed sphere at the four vertices $(P_i, P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)})$ of radius R_i, then the simplex angle at vertex P_i, $\varphi_i = \angle(P_i, P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)})$ is defined as:

$$\begin{aligned} \varphi_i \in [-\pi, \pi] : \\ \sin(\varphi_i) = \frac{r_i}{R_i} sign(\overrightarrow{P_i P_{N_1(i)}} \cdot \overrightarrow{N_i}) \\ \cos(\varphi_i) = \frac{\|O_i C_i\|}{R_i} * sign(\overrightarrow{O_i C_i} \cdot \overrightarrow{N_i}) \end{aligned} \quad (1)$$

The simplex angle φ_i is independent of the position of $P_{N_1(i)}, P_{N_2(i)}, P_{N_3(i)}$ on the circle S_1 and of P_i on a hemisphere of S_2. It is null when P_i and its three neighbors are coplanar. The simplex angle φ_i can be easily computed by considering an inversion of center P_i. More details on the simplex angle definition and properties can be found in [2].

We then define the notion of *discrete mean curvature* H_i on a simplex mesh as the inverse of the radius of the circumscribed sphere:

$$H_i = \frac{sign(\overrightarrow{P_i P_{N_1(i)}} \cdot \overrightarrow{N_i})}{R_i} = \frac{\sin(\varphi_i)}{r_i} \quad (2)$$

We justify this definition by demonstrating that the mean curvature at a point on a three dimensional surface is the inverse radius of the sphere that best approximate the surface at that point [2].

2.3.2 Metric Parameters

In addition to the simplex angle, we introduce two additional parameters called *metric parameters* $\{\epsilon_{1i}, \epsilon_{2i}\}$ that describe how a vertex is located with respect to his 3 neighbors. If we consider the orthogonal projection F_i of P_i on the plane defined by its 3 neighbors, then F_i may be written as a weighted average of the 3 neighboring points:

$$F_i = \epsilon_{1i} P_{N_1(i)} + \epsilon_{2i} P_{N_2(i)} + \epsilon_{3i} P_{N_3(i)} \quad (3)$$

$$\epsilon_{1i} + \epsilon_{2i} + \epsilon_{3i} = 1 \quad (4)$$

The position of a vertex P_i is completely determined by the position of its 3 neighbors and the knowledge of $(\epsilon_{1i}, \epsilon_{2i}, \varphi)$:

$$P_i = \epsilon_{1i} P_{N_1(i)} + \epsilon_{2i} P_{N_2(i)} + \epsilon_{3i} P_{N_3(i)} + L(r_i, d_i, \varphi_i)\vec{N_i} \quad (5)$$

where d_i is the distance between F_i and the center of the circumscribed circle C_i and where $L(r_i, d_i, \varphi_i)$ is a function described in [2].

The $3n$ values $\{(\epsilon_{1i}, \epsilon_{2i}, \varphi_i)\}$ of a simplex model $\mathcal{M}$ consisting of n vertices completely describe the model shape, up to an isometry and scale.

3 Smoothness and Shape Control

3.1 Equations of Motion

We propose a modeling scheme based on deformable and adaptive mesh. The dynamics of each vertex is given by a Newtonian law of motion:

$$m\frac{d^2P_i}{dt^2} = -\gamma\frac{dP_i}{dt} + \vec{F_{int}} + \vec{F_{ext}} \qquad (6)$$

where m is the mass unit of a vertex and γ is the damping factor. $\vec{F_{int}}$ contrains either the shape of a mesh to be smooth whereas $\vec{F_{ext}}$ constrains the mesh to be close from some three-dimensional data.

3.2 Internal Forces

Internal forces determine the response of a physically-based model to external constraints. In this paper, we do not derive the internal force expression from an almost quadratic smoothness energy as in [4][1]. Instead, we chose to minimize a local energy $\mathcal{S}_i = \frac{\alpha_i}{2}P_iP_i^{\star 2}$. $P_i^\star$ is computed from equation 5 with a value of $\varphi = \varphi^\star$. The choice of $\varphi^\star$ determines the types of constraints enforced on the mesh:

Normal Discontinuity We set $\varphi_i^\star = \varphi_i$. The surface can freely bend around vertex P_i.

Normal Continuity constraint We have simply $\varphi_i^\star = 0$. Hence, the internal force is: $\vec{F_{int}} = \alpha_i(\epsilon_{1i}P_{N_1(i)} + \epsilon_{2i}P_{N_2(i)} + \epsilon_{3i}P_{N_3(i)} - P_i)$.

Mean Curvature Continuity Constraint $\varphi_i^\star$ is chosen such that the mean curvature in P_i, $H_i^\star$, is the weighted average of the mean curvature in a neighborhood $N^r(P_i)$: $H_i^\star = \sum_{j\in N^r(P_i)} e_{ij} * H_j$. r is the size of the neighborhood on which the smoothing is performed. This parameter is called the *rigidity* and it effects the dynamics of the surface model.

Shape Constraint Given the constant φ_i^0 by setting $\varphi_i^\star = \varphi_i^0$ we constrain the simplex angle at P_i to φ_i^0. In this case, since we are using constant metric parameters, this amounts to constraining the shape of the mesh up to an isometry and scale.

The expression of those internal forces has the advantages of being intrinsic , viewpoint invariant and scale dependant. Similar type of constraints with similar formulation hold for contours.

3.3 External Forces

In a surface reconstruction scheme based on deformable models, external forces constrain the closeness of fit to some three dimensional data. At each vertex P_i , the closest point $M_{Cl(i)}$ on the data is computed and the force is computed as:

$$\vec{F}_{ext} = \beta_i G\left(\frac{\| \overrightarrow{P_iM}_{Cl(i)} \|}{D}\right)(\overrightarrow{P_iM}_{Cl(i)} \cdot \vec{N}_i)\vec{N}_i \qquad (7)$$

$G(x)$ is the stiffness function that is constant between 0 and 1 and rapidly decreases at values greater than 1. D corresponds to the maximum distance at which a data point strongly attracts a vertex, and it is computed as a fraction of the overall data diameter. Indeed, in order to avoid the effect of outliers, the mesh model is only attracted toward data points that are relatively close.

The computation of the closest point depends on the data type. For structured range data, or volumetric images, it is computed in a $O(1)$ complexity by projecting the normal line on the image. For scattered data points, we use a kd-tree structure to get the data points located inside a sphere of radius D and centered on P_i.

4 Topology Control

4.1 Refinement

We introduce an algorithm for automatically increasing the mesh resolution at parts of high curvature or at parts whose distance to the range data is higher than a threshold. The refinement process is completed in an iterative way. First, we evaluate, for every face model, a criterion measuring the need for refinement. Then, faces whose criterion exceeds a given threshold, are refined and the mesh is deformed during a constant number of iterations. The refinement process is repeated until all faces criteria are below the threshold. This approach has the advantage of recovering models satisfying both geometric constraints (regularity and closeness of fit) and topological constraints (optimal vertex spacing).

The criterion is computed as the face area multiplied by three dimensionless coefficients. The first one is the measure of Gaussian curvature on the face. It can be computed as the area of a spherical polygon on the Gauss sphere [2]. The second coefficient is the ratio of the distance to the closest data point and the reference distance D. The third coefficient measures the elongation of a face in order to refine in priority large elongated faces. The threshold has the dimension of a surface area and therefore the refinement process is guaranteed to stop since the faces area decrease at every iteration. In order to keep the number of vertices per face as close as possible to six, we either use a T_2^2 operation or a T_5^2 operation [2] depending whether the face has more than five vertices or not.

4.2 Building Models from Parts

Recovering a complex model by deforming and refining a simple primitive is a difficult task since the mesh would have to automatically change its genus and avoid numerous local minima. Moreover, current range techniques cannot acquire a complex shaped object within a single image. A more natural approach for recovering complex objects, consists in connecting separately built models, corresponding to approximately convex subsets. In this framework, different meshes are fit on different subsets of a same objects. Then two meshes are connected with a single transformation operation and the zone around the connection is smoothed to remove the normal and curvature discontinuities.

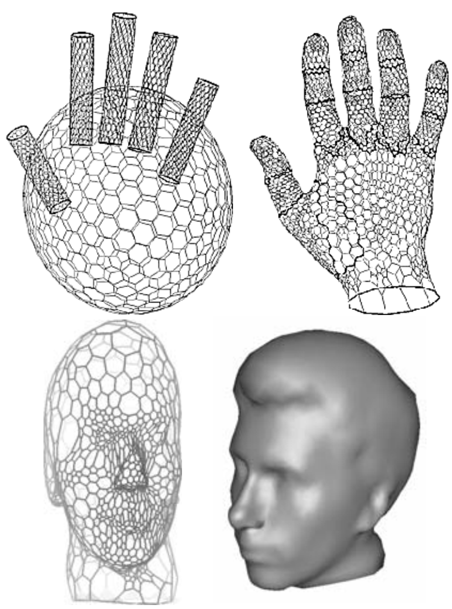

Figure 2: **upperleft** Six initial meshes. **upper right** Resulting hand model after connecting the finger models to the palm model. **lower left** Face model after refinement. **lower right** Rendered face model.

Figure 3: A mechanical part with a genus of one was created from range data

5 Results

We have applied the simplex mesh modeling on a variety of structured and unstructured three-dimensional datasets. The recovery of three-dimensional objects from range data proceeds in two stages. In a first stage, the model is initialized as one of four primitives with a limited number of vertices. We use a high value of the rigidity parameter to obtain a smooth and stable deformation toward the image data. However, the resulting model tends to be inaccurate at locations of high curvature. In a second stage, we apply the iterative refinement process over the whole mesh or over selected parts while decreasing the rigidity parameter to its minimum. A new equilibrium is reached when highly curved parts are sufficiently refined and when vertices closely approximate the raw data.

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