P2R Meeting

Wavelet-based restoration methods: application to 3D microscopy images

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OVERVIEW

- Introduction
  - Encountered problems
- State of the art in microscopy
  - Methods acting in the space domain
  - Methods acting in a transformed domain
- Wavelet transforms
  - Classical discrete wavelet transform
  - Anisotropic version
- Proposed methods
  - Splitting denoising and deconvolution steps
  - Combining denoising and deconvolution steps
- Simulation results
- Future works

INTRODUCTION

➢ Image formation model :

$$i = \mathscr{P}(o * h)$$

where $\mathscr{P}$ is a Poisson noise, i the observed image, o the object and h the PSF.

➢ Our objective is to estimate the original image, *i.e.* obtaining $\hat{o}$ only having an observation i.

➢ In order to preserve thin structures, we want to use wavelet transforms to reach this goal.

➢ Pre-processings (Anscombe, Fisz) used to stabilize the noise variance in order to apply common methods of image restoration.

Background in confocal microscopy image restoration

Richardson-Lucy Algorithm

Under Poisson process assumption :

$$p(i \mid o) = \prod_{\mathbf{x}} \left(\frac{(h * o)(\mathbf{x})^{i(\mathbf{x})} e^{-(h*o)(\mathbf{x})}}{i(\mathbf{x})!} \right)$$

➢ Maximum likelihood estimation :

Minimize $$J_1(o) = \sum_{\mathbf{x}} \left(-i(\mathbf{x}) \log[(h * o)(\mathbf{x})] + (h * o)(\mathbf{x}) \right) + c^{st}$$

Iterative algorithm :

$$o_{n+1}(\mathbf{x}) = \left\{ \left[\frac{i(\mathbf{x})}{o_n(\mathbf{x}) * h(\mathbf{x})} \right] * h(-\mathbf{x}) \right\} . o_n(\mathbf{x})$$

➢ Regularisation by Total Variation [Dey *et al.*, 2004] :

$$J_1(o) + J_{reg}(o) = J_1(o) + \lambda \sum_{\mathbf{x}} |\nabla o(\mathbf{x})|$$

Associated iterative algorithm :

$$o_{n+1}(x) = \left\{ \left[\frac{i(x)}{o_n(x) * h(x)} \right] * h(-x) \right\} \frac{o_n(x)}{1 - \lambda\, div \left(\frac{\nabla o_n(x)}{|\Delta o_n(x)|} \right)}$$

- **Anscombe** transform :

$$\mathscr{A}(i(x,y,z)) = 2\sqrt{i(x,y,z)+\frac{3}{8}}.$$

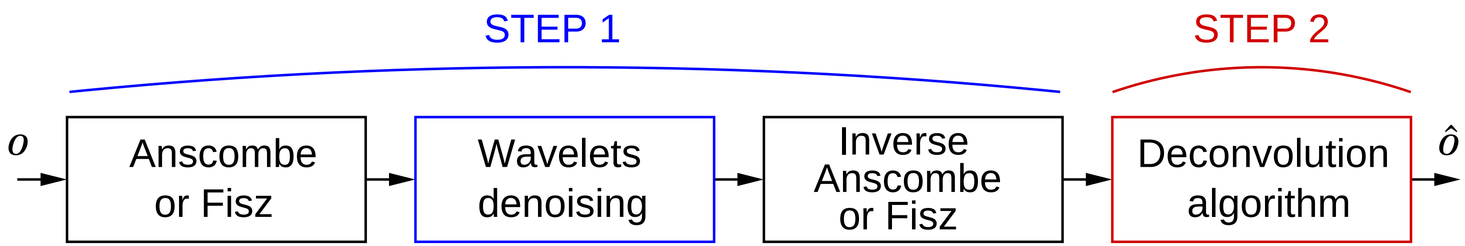
- **Fisz** transform :
  1. Process a non normalized Haar transform
  2. Transform the detail coefficients $d_{j,\mathbf{m}}(\mathbf{k})$ of level j, subband $\mathbf{m}$ and spatial position $\mathbf{k}$ as

$$d_{j,\mathbf{m}}(\mathbf{k}) = \begin{cases} d_{j,\mathbf{m}}(\mathbf{k})/\sqrt{a_{j,\mathbf{m}}}(\mathbf{k}) & \text{if } a_{j,\mathbf{m}}(\mathbf{k}) \neq 0 \\ 0 & \text{if } a_{j,\mathbf{m}}(\mathbf{k}) = 0 \end{cases}$$

     where $a_{j,\mathbf{m}}$ denotes the approximation coefficients.
  3. Reconstruct using the inverse non normalized Haar transform.

→ After such processings, the noise can be asymptotically considered as $\mathscr{N}(\mu,1)$.

DENOISING + DECONVOLUTION ALGORITHM



➢ Step 1 : Denoising by coefficient thresholding using Steerable pyramids [Rooms *et al.*, 2005] or using wavelets [Boutet de Monvel *et al.*, 2001], [Colicchio *et al.*, 2007]

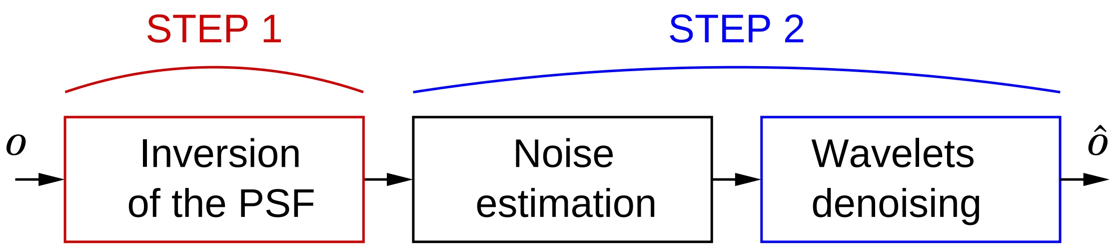
➢ Step 2 : Deconvolution using the RL algorithm or a MAP

➢ Pre-processing stage to stabilize the noise variance

➢ **DIFFICULTIES** :

- Is the PSF modified during Step 1 ? (In [Rooms *et al.*, 2005] they estimate it after denoising)
- What is the nature of the residual noise after Step 1 ?

DECONVOLUTION + DENOISING



➢ Step 1 : Inversion of the PSF [McNally *et al.*, 1999] taking into account it has zeros

➢ Step 2 : Denoising by coefficient thresholding using wavelet transforms

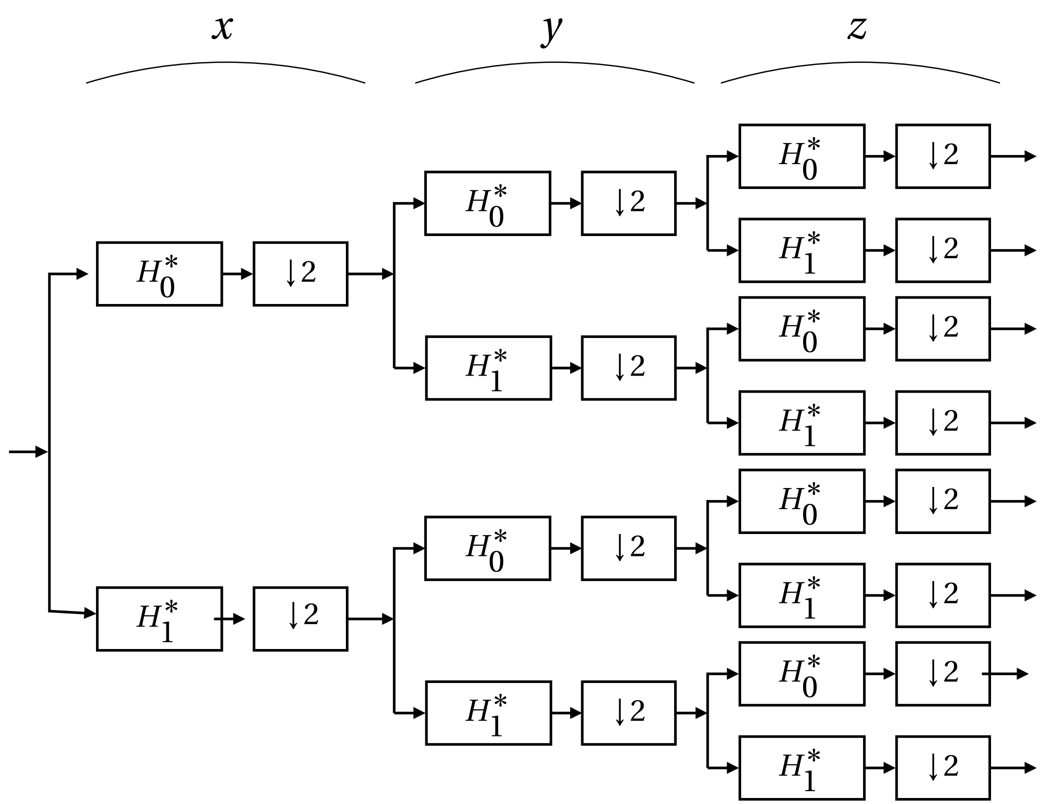
➢ Noise estimation stage : variance estimation ...

➢ **DIFFICULTIES** :

- The noise is amplified during Step 1
- What is the noise nature after Step 1 ?

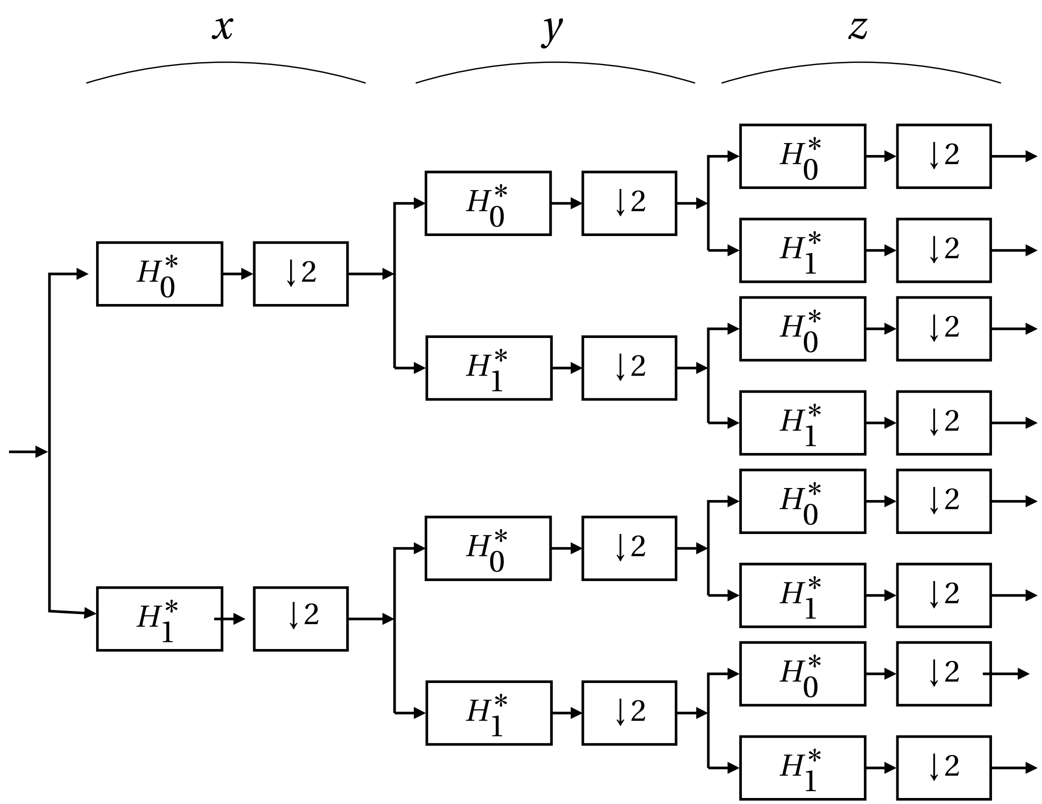
Wavelet Transforms

3D Dyadic Discrete Wavelet Transform



Dyadic analysis filter bank - 1 resolution level

3D Dyadic Discrete Wavelet Transform

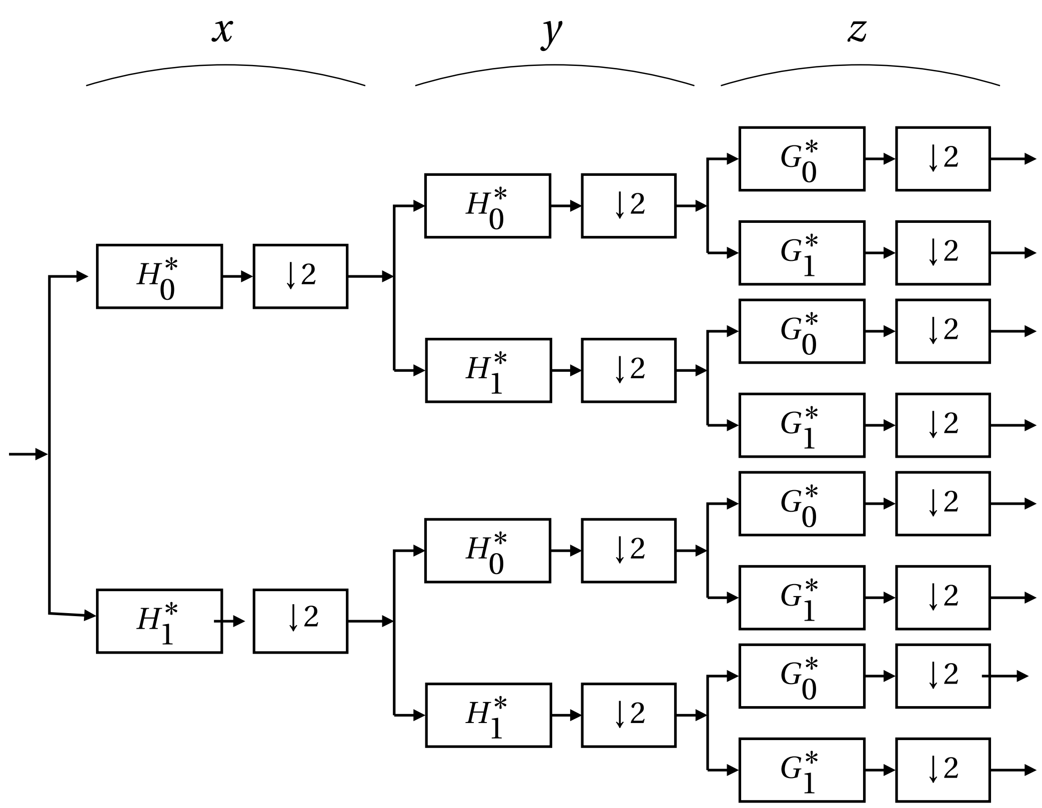


Dyadic analysis filter bank - 1 resolution level

➢ Fully decimated transform (no redundancy)

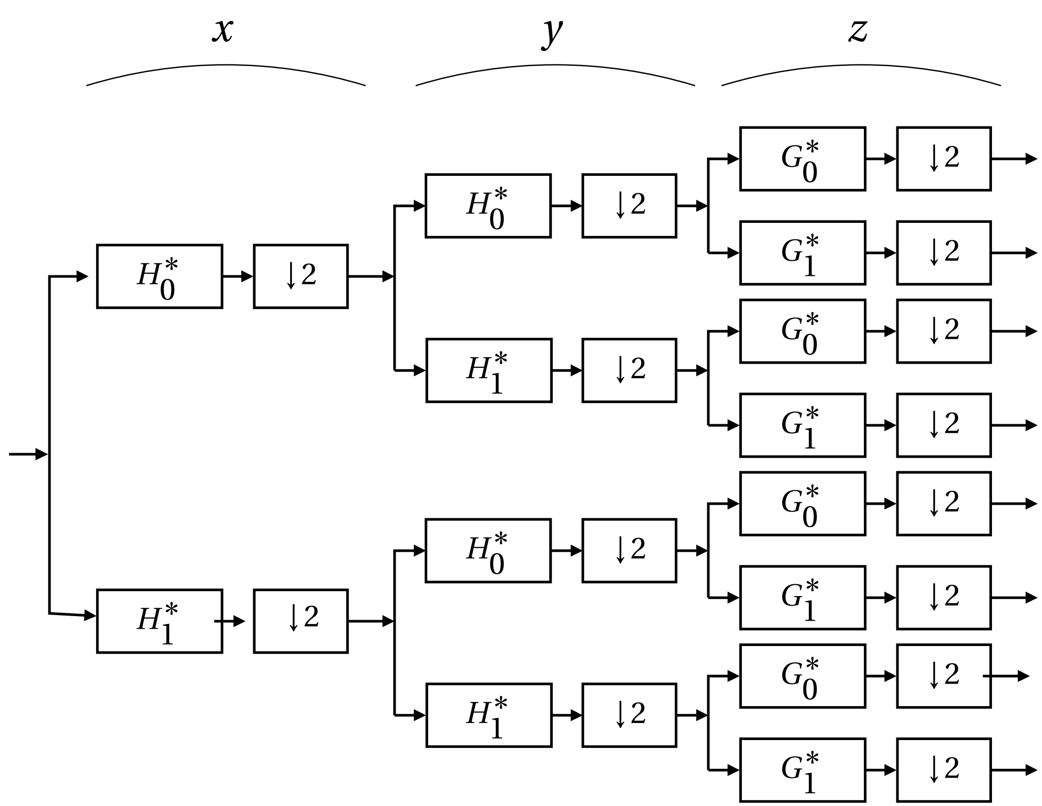
➢ Using 2-band filter banks

3D Dyadic anisotropic Discrete Wavelet Transform



Dyadic analysis filter bank, $\neq$ filters in z direction - 1 level

3D Dyadic anisotropic Discrete Wavelet Transform

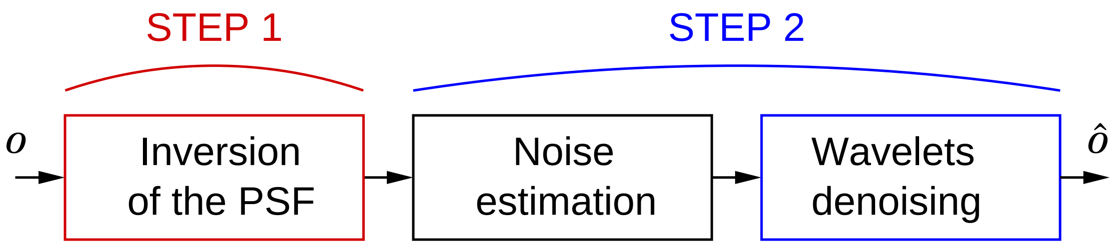


Dyadic analysis filter bank, ≠ filters in z direction - 1 level

- ➢ Fully decimated transform (no redundancy)
- ➢ Using 2-band filter banks
- ➢ The z direction is processed by an other wavelet

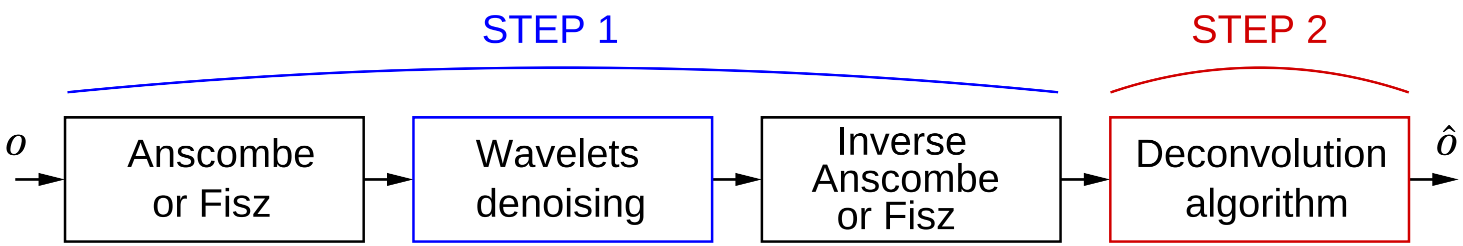
Deconvolution algorithms

➢ Considering deconvolution then denoising



- Step 1 : Deconvolution by PSF inversion
- Step 2 : Denoising using 3D DWT (thresholding).

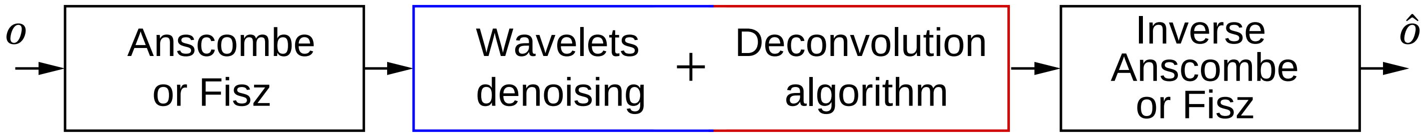
➢ Considering denoising then deconvolution



- Step 1 : Denoising by coefficient thresholding using 3D DWT
- Step 2 : Deconvolution using the RL algorithm or a standard algorithm if the noise is no longer Poisson.

SECOND APPROACH

➢ Consider a combined approach operating in the transform domain



- Using an algorithm as in [Daubechies *et al.*, 2004] or [Combettes *et al.*, 2005]
- First considering wavelet bases (3D DWT)
- Then extending this approach to wavelet frames (redundant decomposition like the 3D CWT transform) which implies more complicated algorithms (future work).

PROPOSED ALGORITHM

Considering an additive noise n leading to the observation $i = h * o + n$.

Objective :

$$\underset{o \in \mathrm{H}}{\text{minimize}} \ \frac{1}{2}\|h * o - i\|^2 + \sum_{k \in K} \phi_k(\langle o \mid e_k \rangle).$$

Algorithm :

$$o_{n+1} = o_n + \lambda_n (\text{prox}_{\gamma_n \phi_k} < o_n + \gamma_n (h^* (i - h * o_n)) | e_k > - o_n)$$

- e_k stands for an orthonormal basis.
- We can choose

$$\phi = \omega |.|$$

$$\phi = \omega |.|^2$$

- ω is a a fixed parameter and ϕ a fixed regularization function.
- γ : step size and $\lambda = 1$: relaxation parameter.

Simulation results

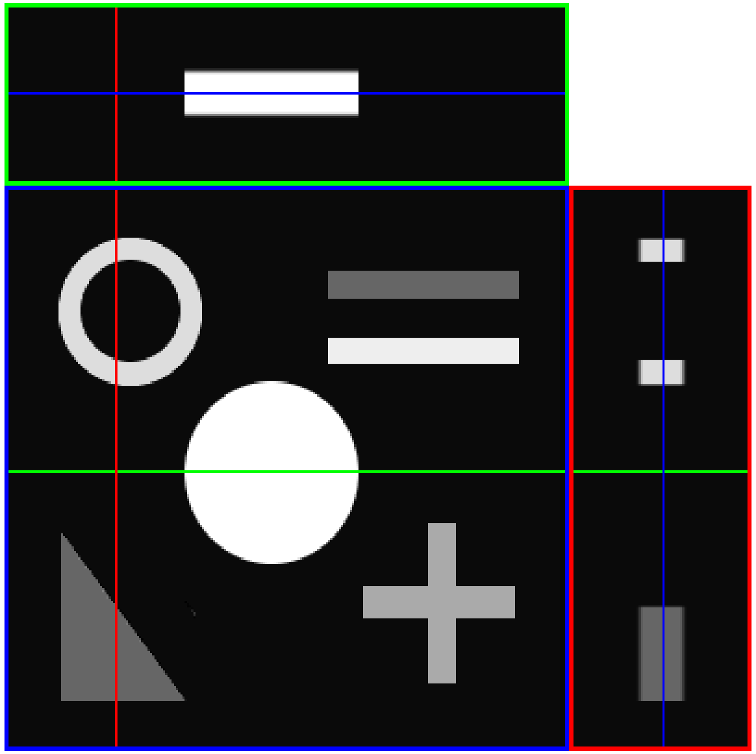
Tested methods :

Method 1 :
1. Preprocessing
2. Wavelet denoising
3. Inverse Preprocessing
4. RL algorithm

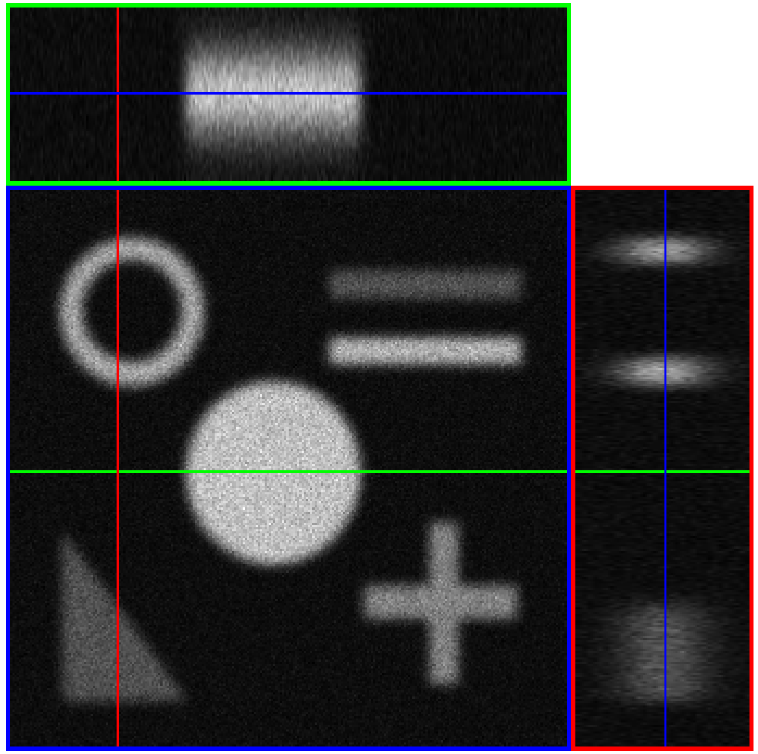
Method 2 :
1. PSF inversion
2. Noise estimation
3. Wavelet denoising

Method 3 :
1. Preprocessing
2. Deconvolution + denoising
3. Inverse Preprocessing

SYNTHETIC IMAGE



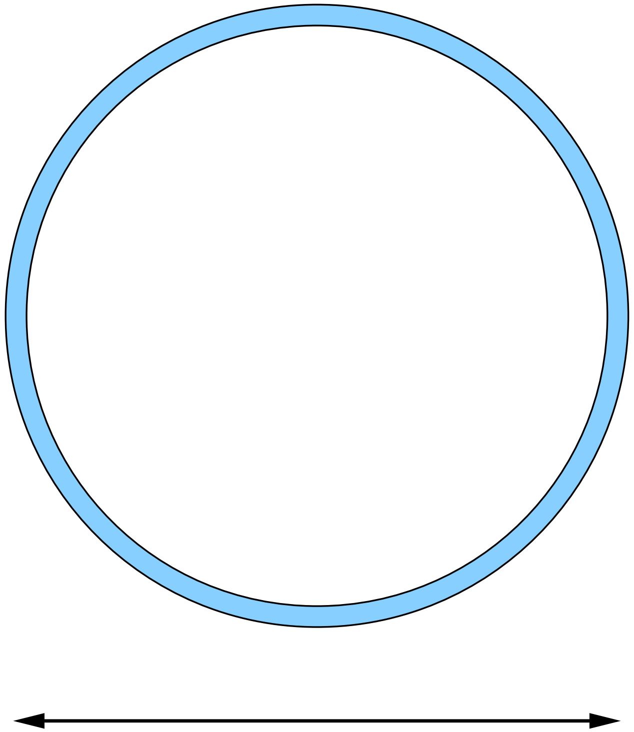
Original



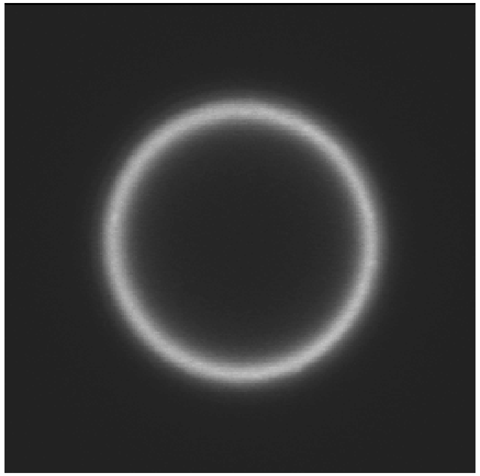
Degraded

I-div Init.	**Meth. 1**	**Meth. 2**	**Meth. 3**	**Meth. 3 anis**
6.98	1.17	2.11	1.09	0.90

PHANTOM IMAGE



PHANTOM IMAGE

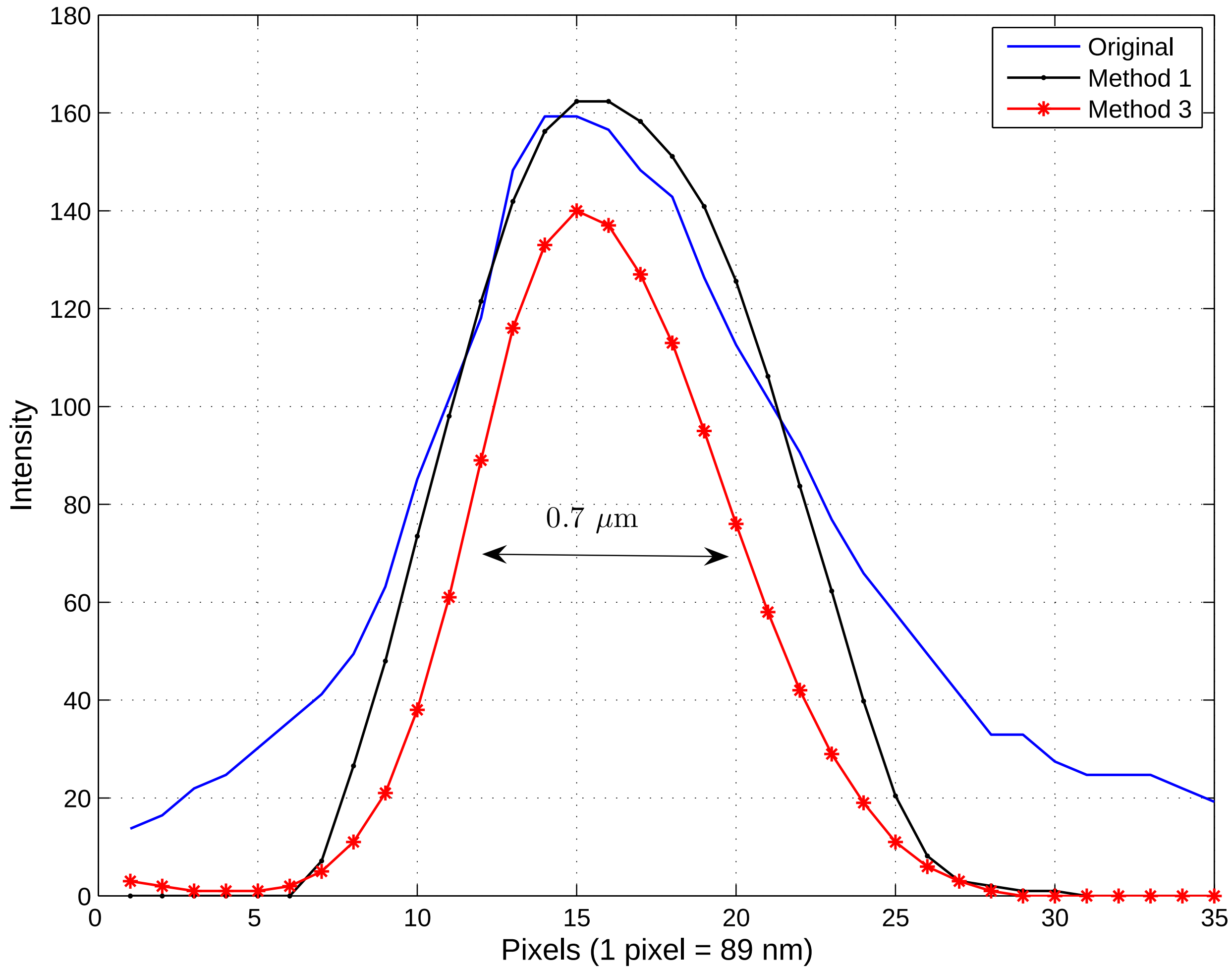


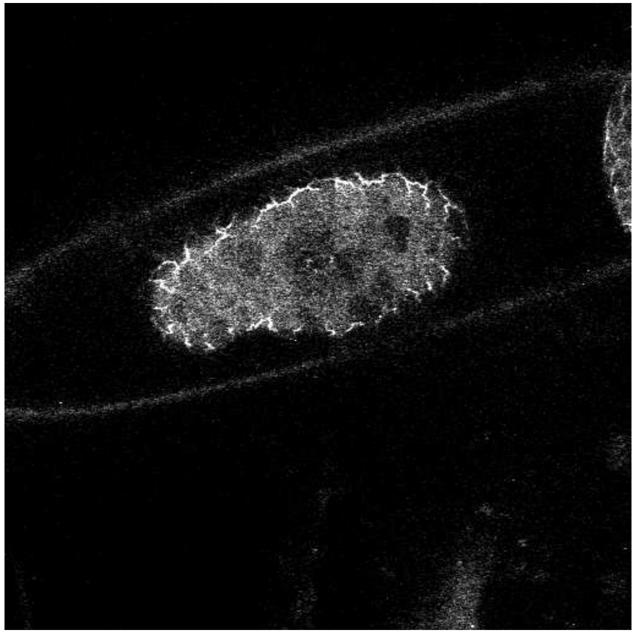
Observed

Restored meth. 1

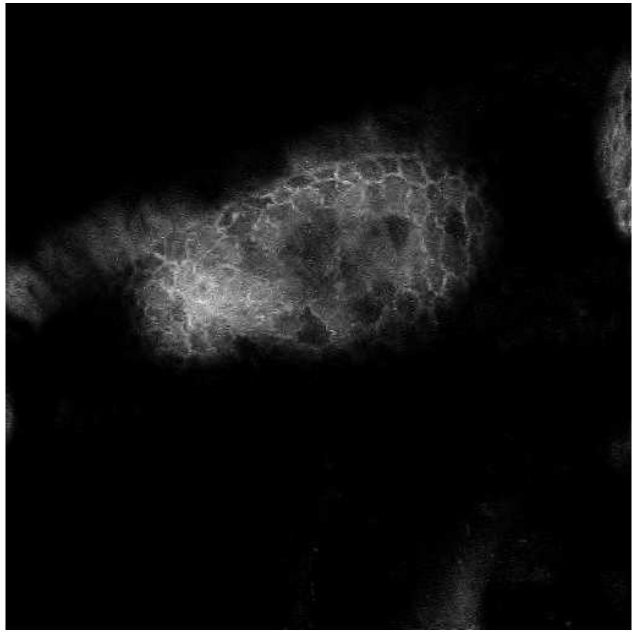
Restored meth. 3

PHANTOM IMAGE

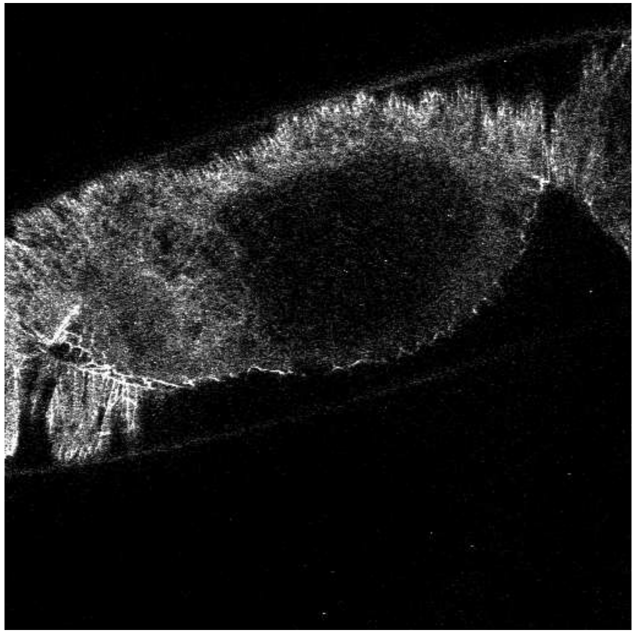




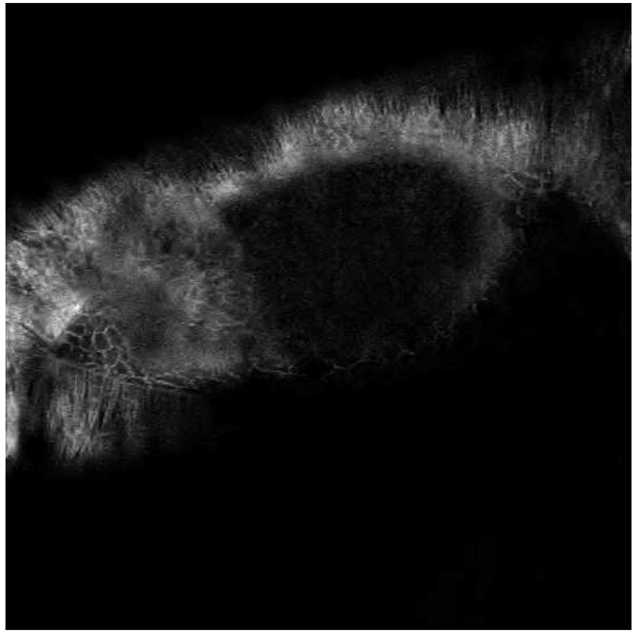
Original



Restored



Original



Restored

Perspectives

FUTURE WORKS

- Considering an M-band wavelet transform instead of a dyadic wavelet transform
- Using the dual-tree wavelet transform which is directional and which has a limited redundancy
- Directly considering Poisson noise instead of pre-processing data
- Consider a mixture of Poisson and Gaussian noise (MPG) [Zhang *et al.*, 2007]
- Using a "local" method partitioning the image into smaller areas in order to make assumptions on "local" noise properties
- ...