Satellite image deconvolution using complex wavelet packets

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- Problem statement
- Efficient representations and basis choice
- Complex wavelet transform
- Complex wavelet packet transform
- Transform thresholding
  - Different methods
  - Parameter estimation
  - Two algorithms : COWPATH 1 and 2
  - Results
- Conclusion and future work

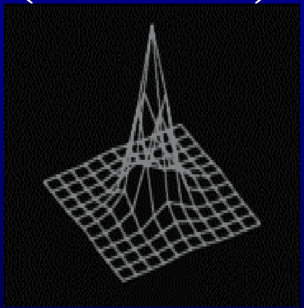
Observation equation

Observed images are **corrupted** :

$$Y = h * X_0 + N$$

Observed image

Convolution kernel
(*known PSF*)



Original image

Noise :
White and Gaussian
(*known variance*)

Problem statement

Ill-posed **inverse problem** [Hadamard 23]

- existence,
- unicity,
- stability of the solution ?

Inversion → **noise amplification**

Small errors of Y → high errors of X

Introduction

- **Monoscale methods** [Geman & McClure 85, Charbonnier 97, …]

  Regularization + edge preservation

  Find X by minimizing **U(X)** :

$$U(X) = \|Y - h * X\|^2 / 2\sigma^2 + \Phi(X)$$

Data term

Non-quadratic regularization term

- **Multiscale methods** [Mallat 89, Bijaoui 94, …]

  Multiresolution analysis → **wavelets**

  - Regularization of classical iterative methods (statistics)

    (shift invariant wavelet transform thresholding)

  - Multiresolution variational models

Introduction

- **Filtering after inversion** [Donoho, Mallat, Kalifa 99]
  - **Non-regularized inversion** (Fourier domain)
  - **Transform** (change the basis)
  - **Coefficient thresholding**
  - **Inverse transform** (return to image space)

Representations
for efficient filtering

Efficient **separation** of the signal and the deconvolved noise :
- compact reprensentation of the signal
- efficient compression of the noise in high frequencies

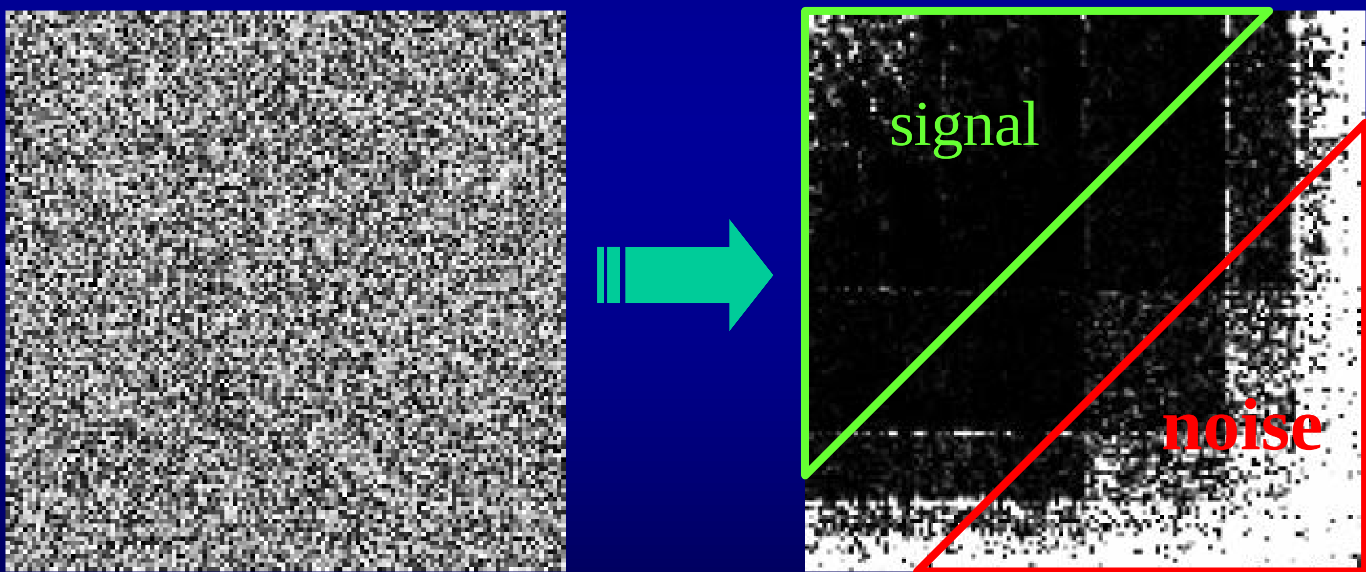


Image deconvolved without regularization

Transform

Filtering the deconvolved noise

- Cancel the coefficients corresponding only to the noise
- Thresholding the coefficients corrupted by noise

The deconvolved noise is colored !

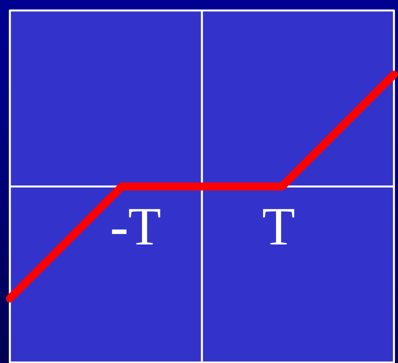
In the new basis, the coefficients of the noise transform must be **independent** → enable **separate** thresholding

→ **noise covariance « nearly diagonalized »** [Kalifa 99]

Choice of the basis

- Compact representation
- « nearly diagonal » noise covariance

***The thresholding estimator* is optimal** [Donoho, Johnstone 94]



$q_T(x)$ → Thresholding function

x → Coefficient in the new basis

Algorithm design

Rough deconvolution → Direct transform → thresholding → Inverse transform

Choice of the basis :

- compacity
- diagonalization
- reconstruction
- invariance properties

Choice of the thresholding function

Optimal threshold value ?

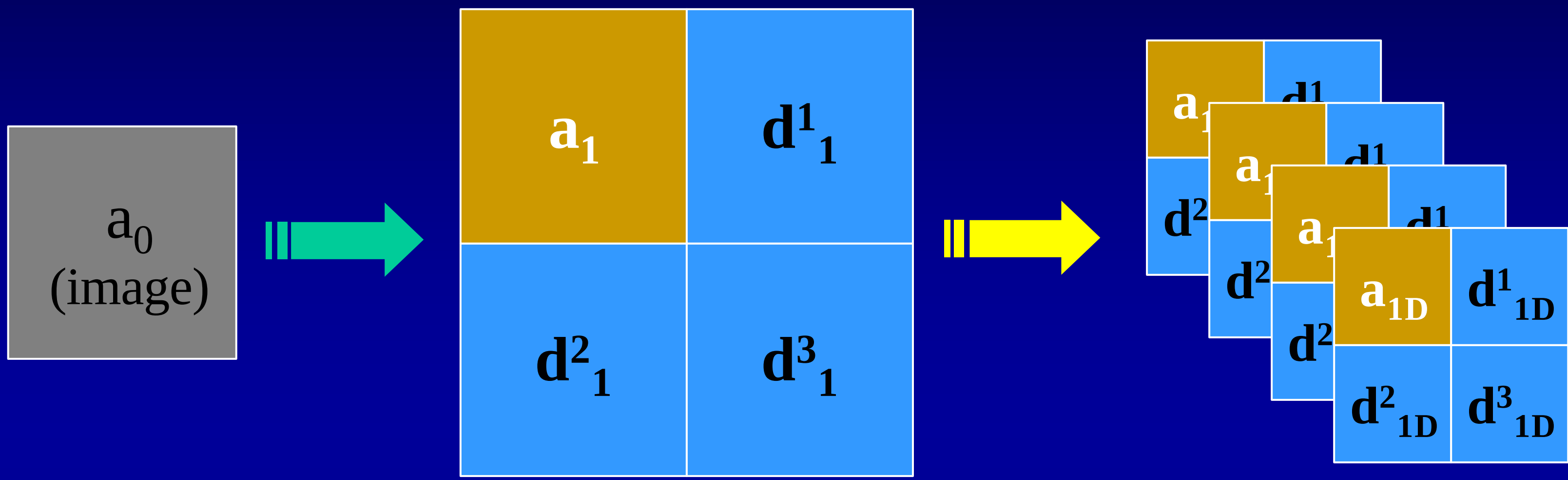
Complex wavelets

Properties :

- ☆ Shift invariance
- ☆ Directional selectivity
- ☆ Perfect reconstruction
- ☆ Fast algorithm O(N)

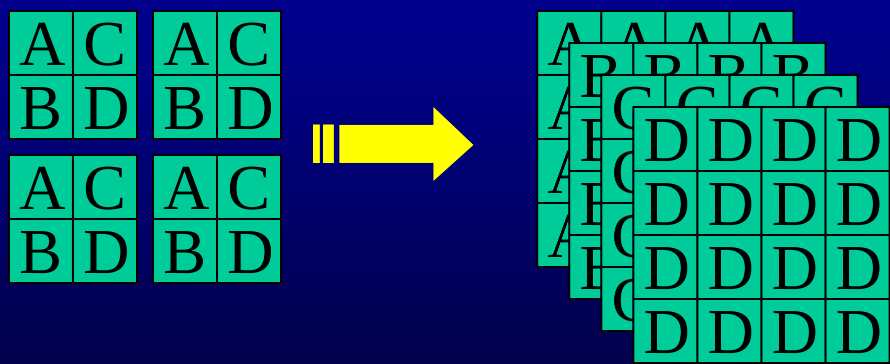
- **quad-tree** (4 parallel wavelet trees) [Kingsbury 98]
- filters **shifted** by ½ and ¼ pixel between trees
- combination of trees → **complex** coefficients
- **biorthogonal** wavelets
- **filter bank** implementation

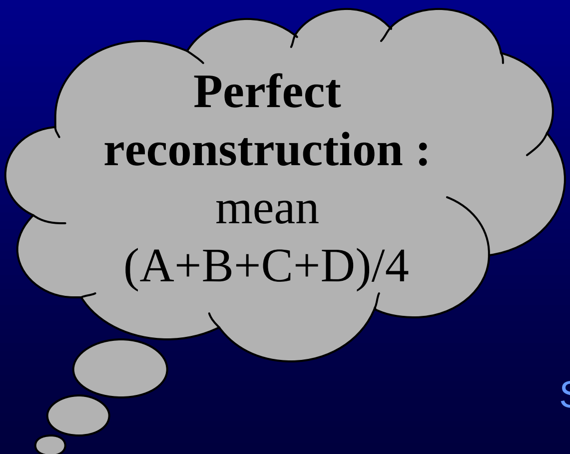
Quad-tree : 1st level



Non-decimated transform

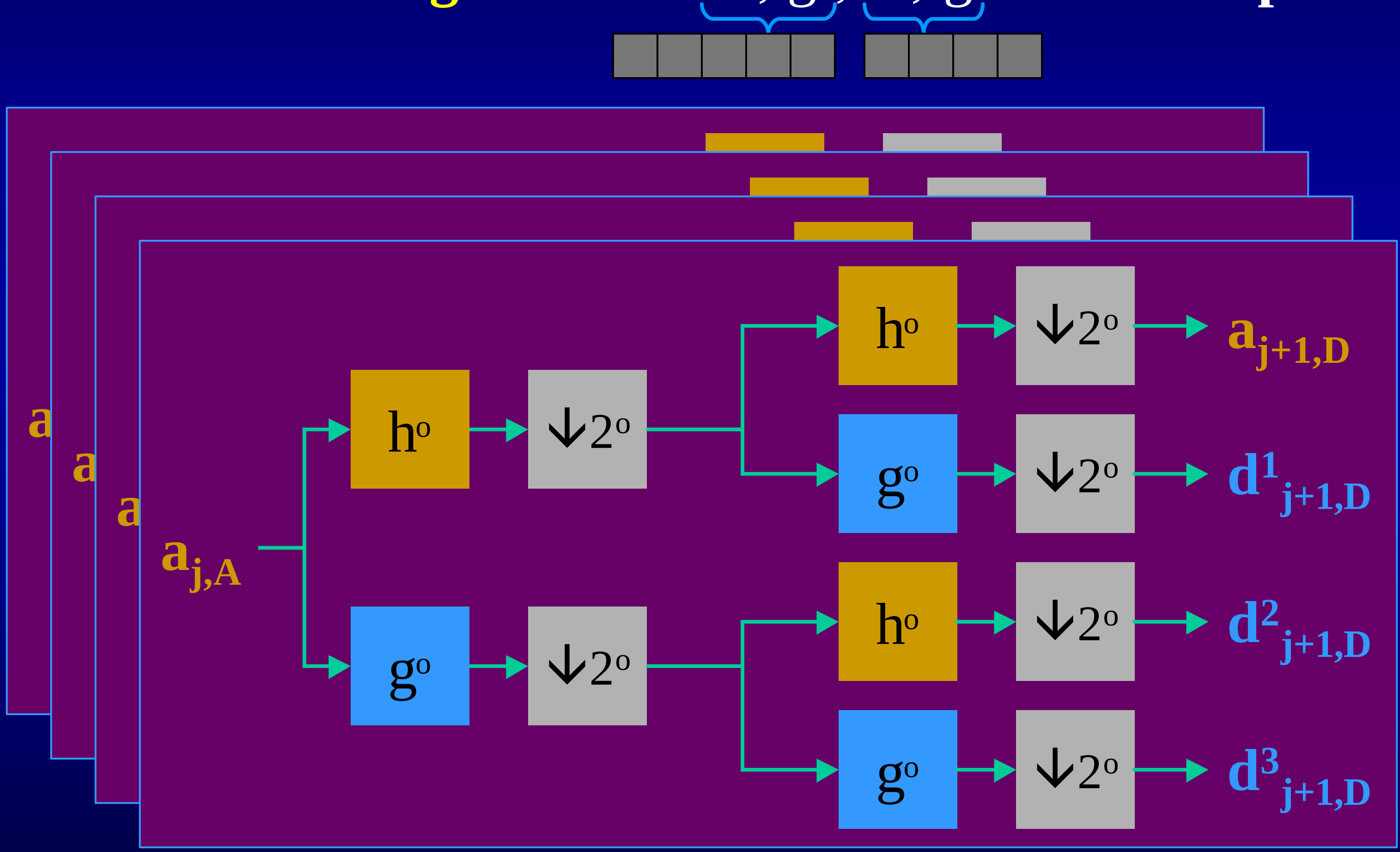
Parallel trees ABCD



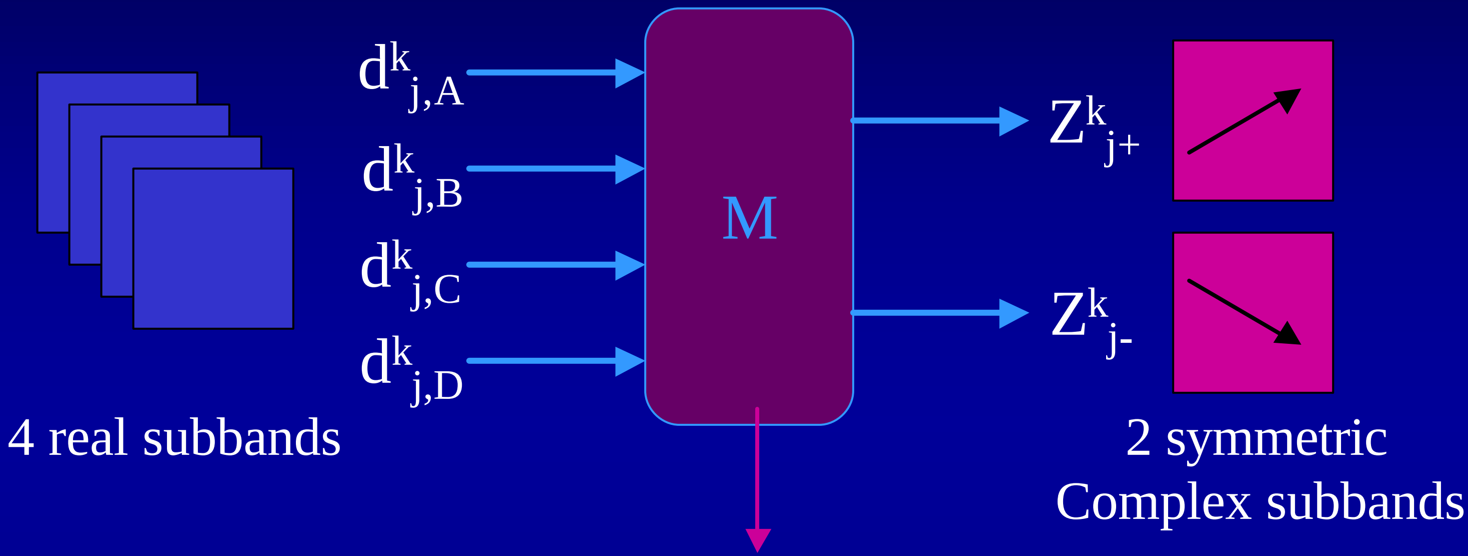


Quad-tree : level j

different length filters : h^o, g^o, h^e, g^e → **shift < pixel**



Complex coefficients



$$Z_+ = (A - D) + i\,(B + C)$$
$$Z_- = (A + D) + i\,(B - C)$$

The wavelet function is not a complex function.
Not exactly ‘complex’ wavelets !

Necessity of the packets

Complex wavelets :

- Compact representation
- Poor representation of the deconvolved noise

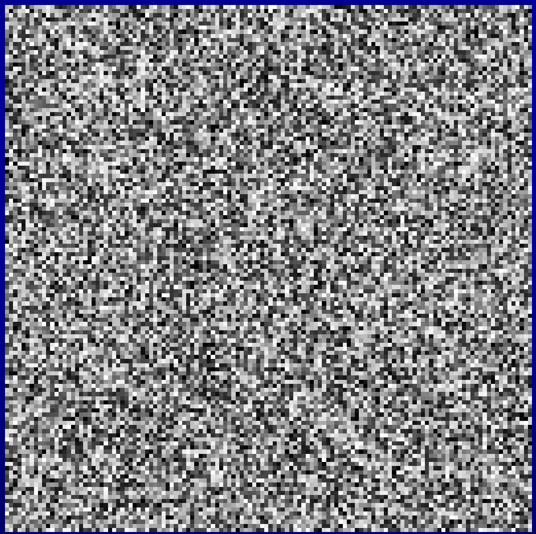
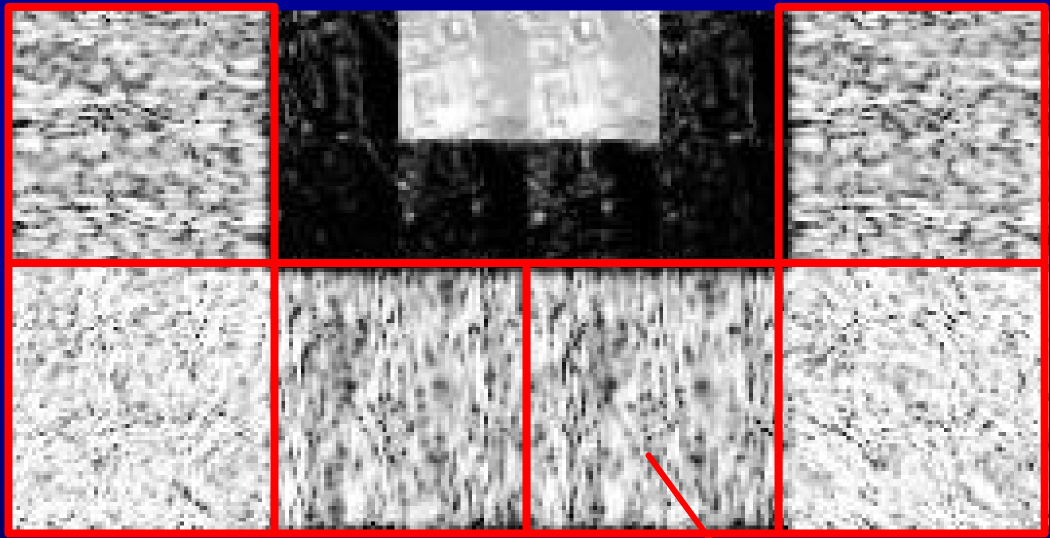


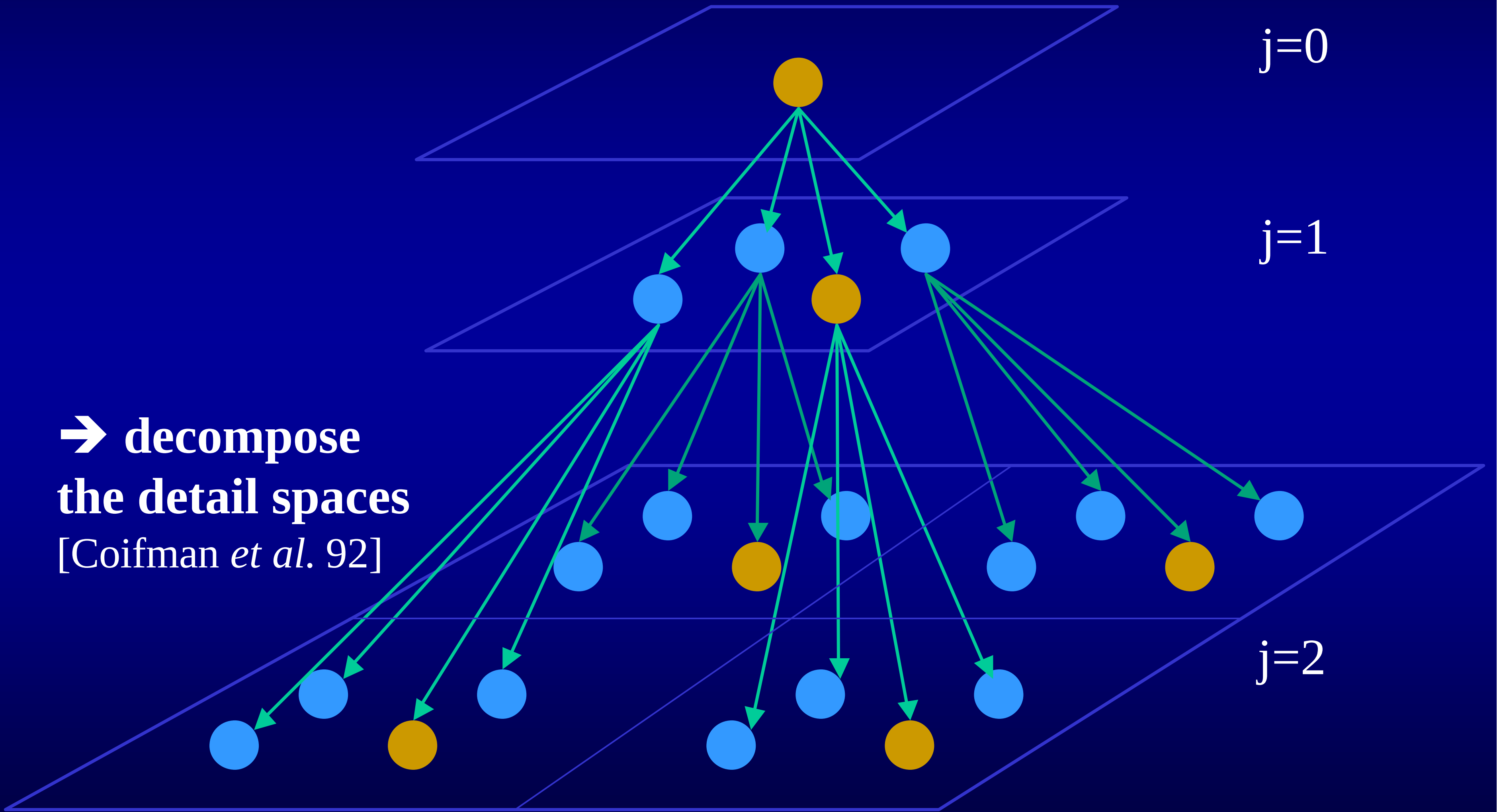
Image deconvolved without regularization



Transform

High frequencies not recoverable

Wavelet packets

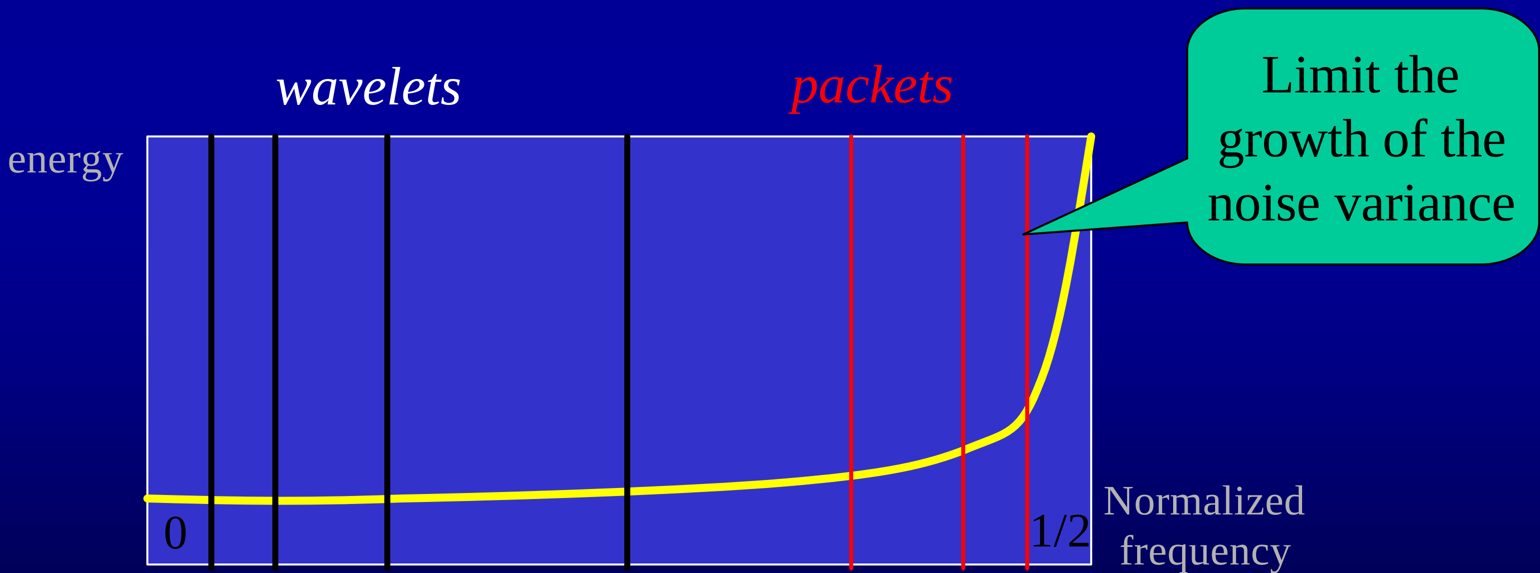


➔ decompose the detail spaces

[Coifman *et al.* 92]

Choice of the tree

- **no unicity** of the decomposition tree
- application **dependent**
- deconvolution : **must adapt to the deconvolved noise**



Deconvolved noise power spectrum

Complex wavelet packets (CWP)

➔ **decompose the detail spaces of the complex wavelet transform**

for each tree A,B,C,D

Original image

Transform

Complex wavelet packets (CWP)

Complex Wavelet Packets :

- Compact representation
- Nice representation of the deconvolved noise

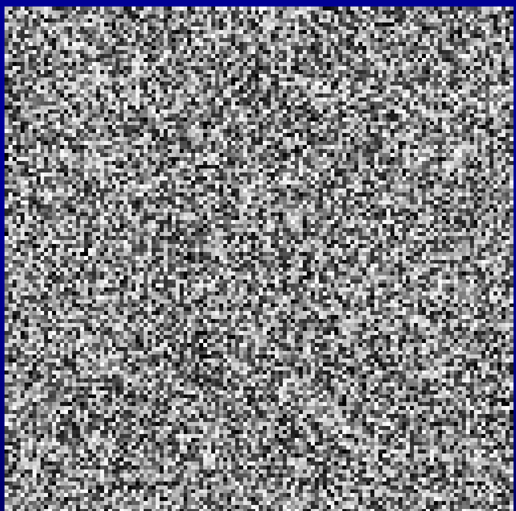
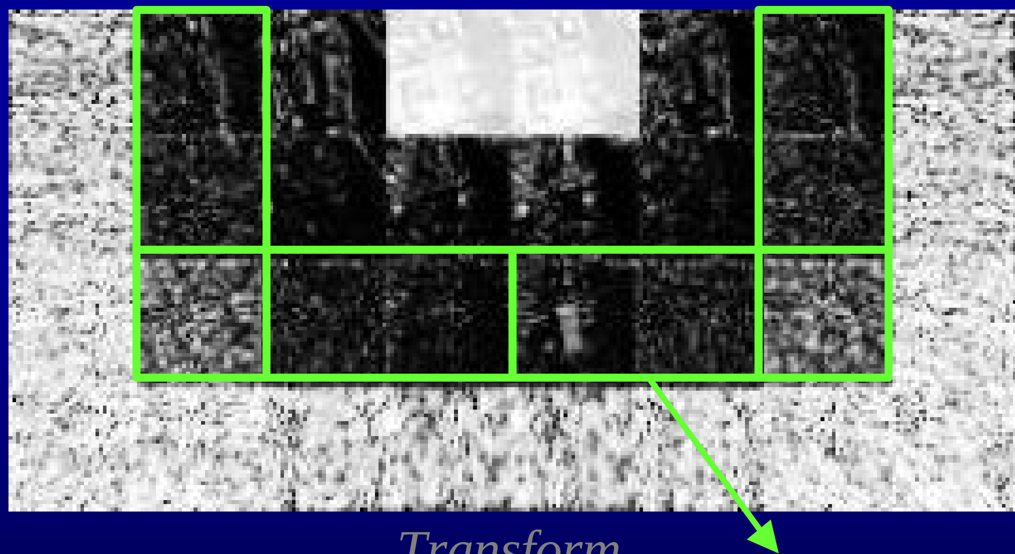


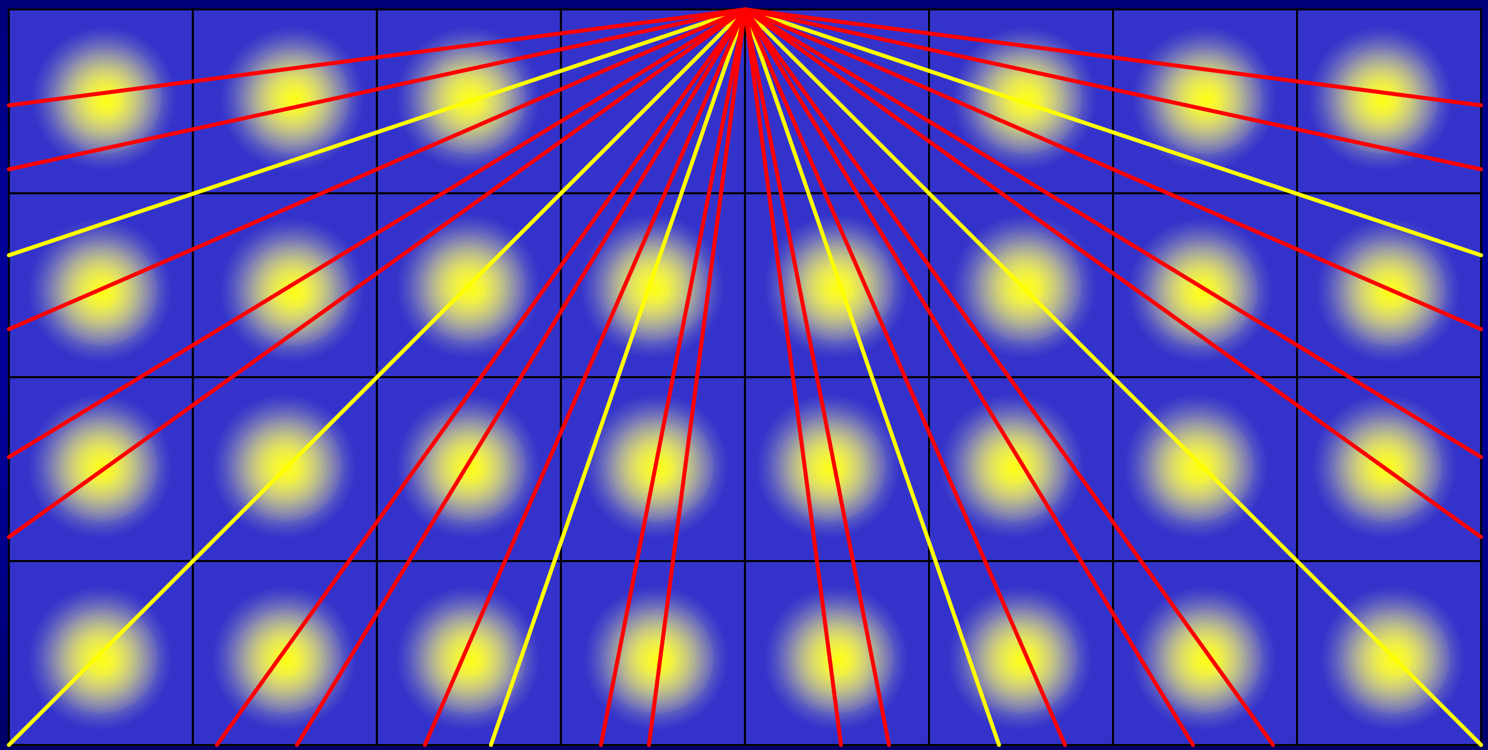
Image deconvolved without regularization



Transform

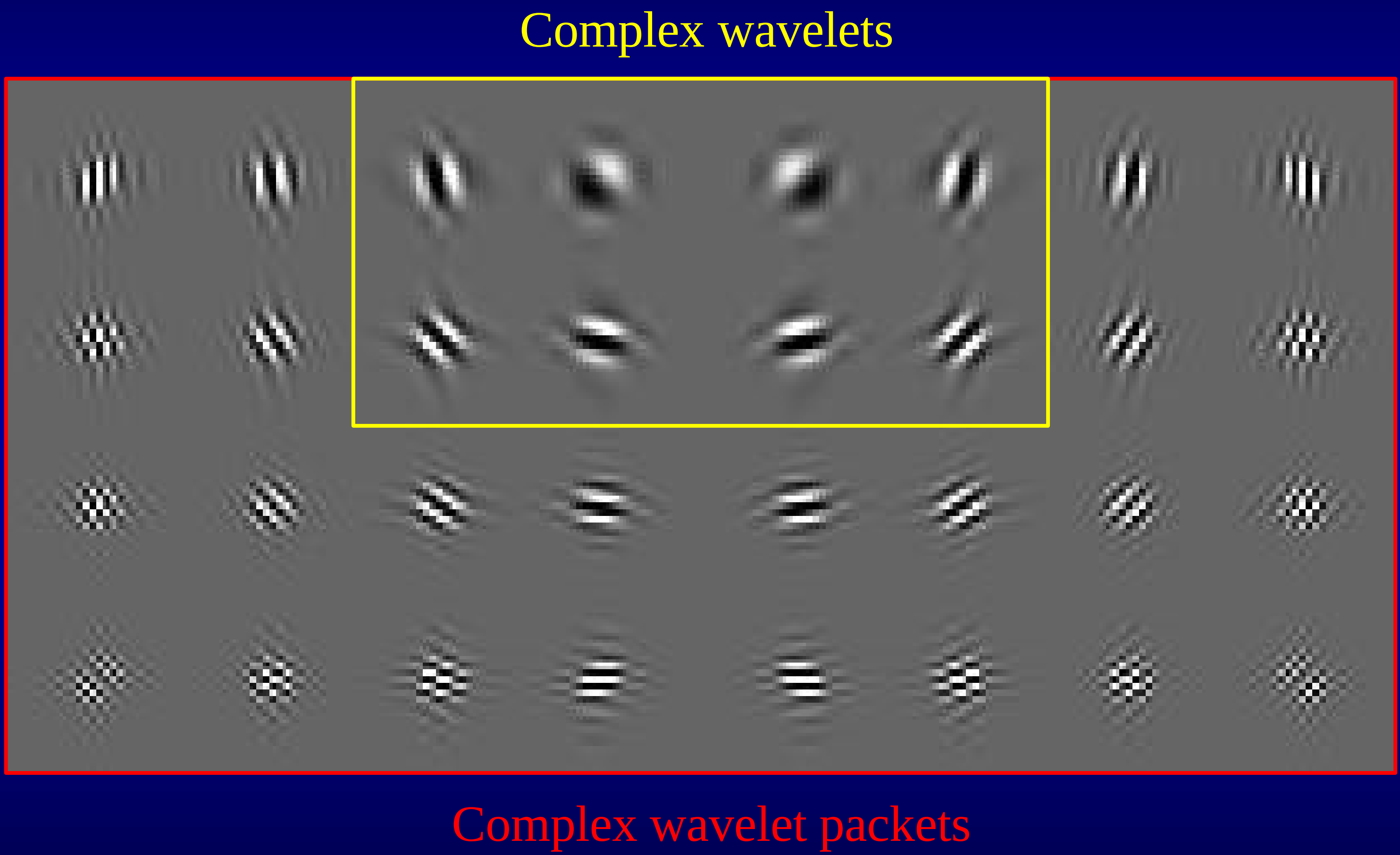
High frequencies recoverable

Frequency plane partition



Directional selectivity

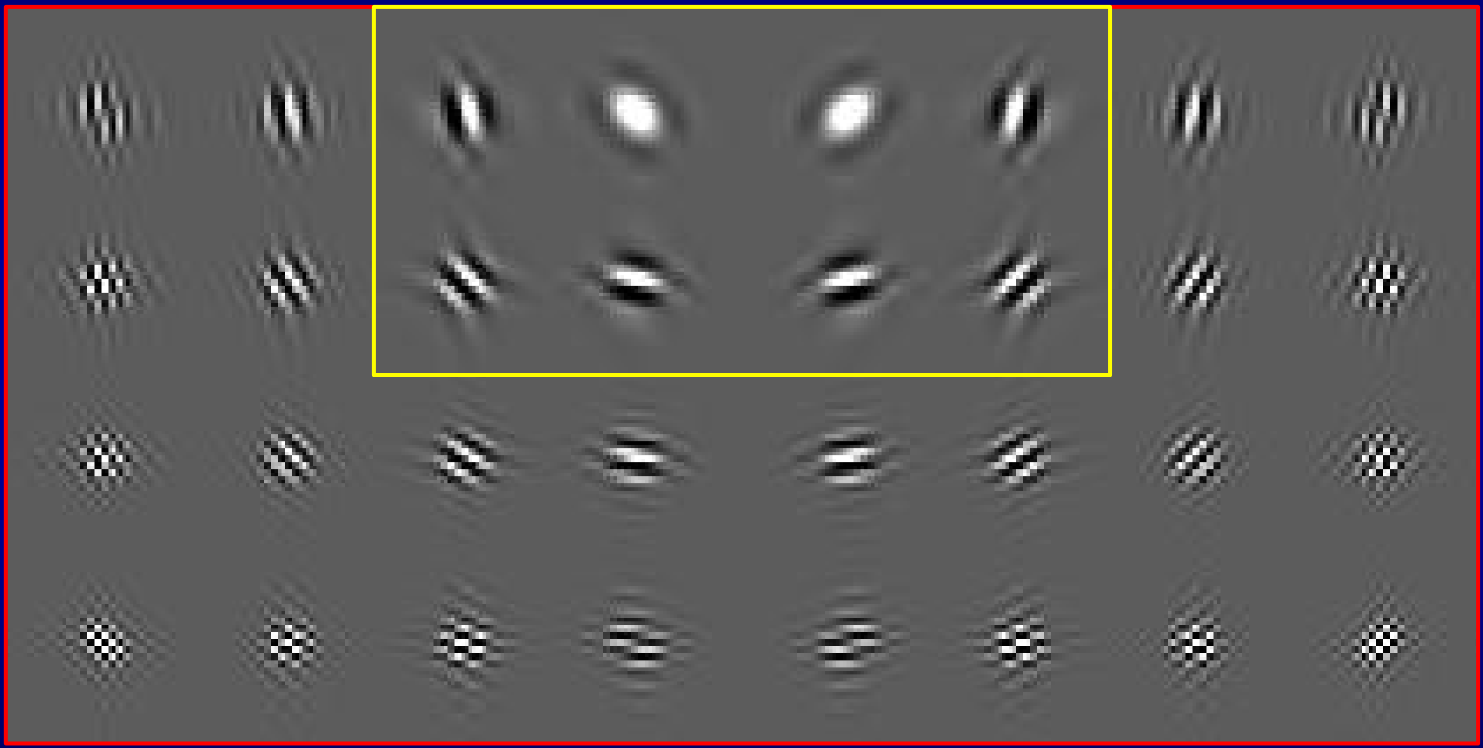
impulse responses – real part



Directional selectivity

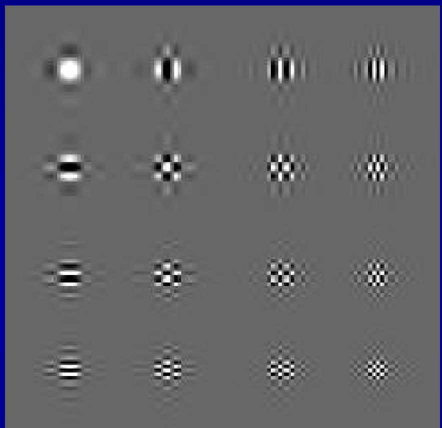
impulse responses – imaginary part

Complex wavelets



Complex wavelet packets

Comparison with real wavelet packets



Impulse responses

No shift invariance

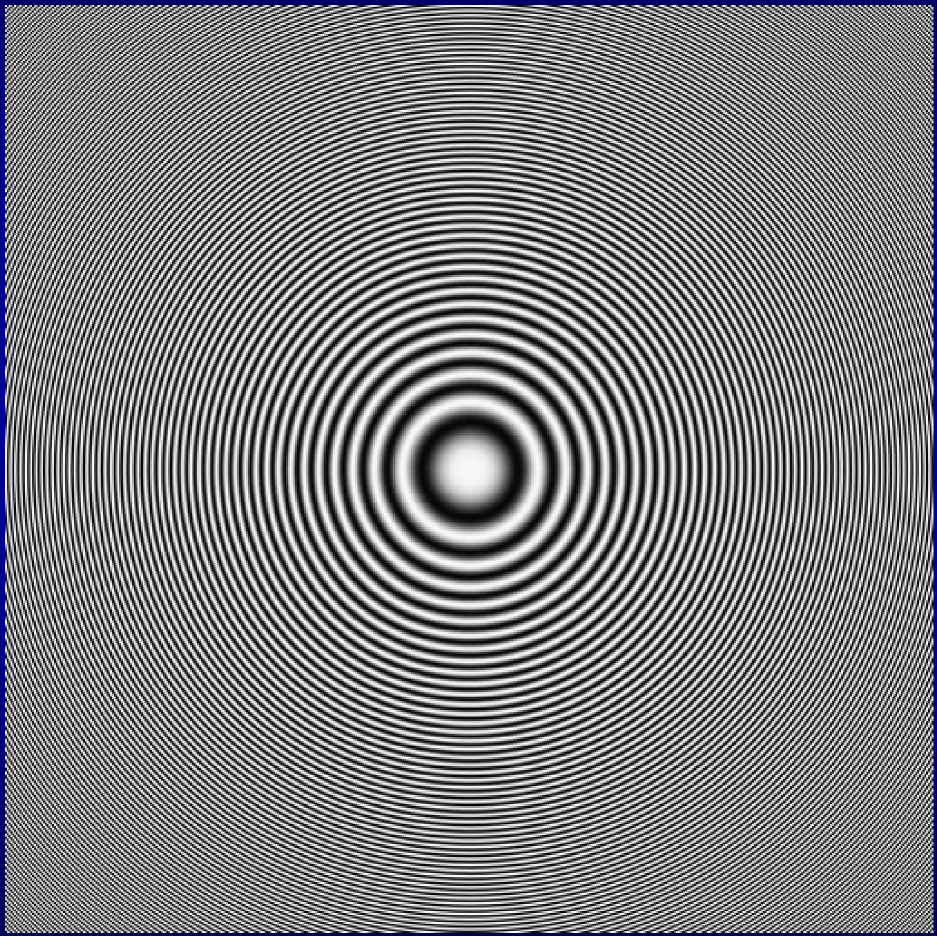
➔ artefacts (mean over translations)

No rotation invariance

Privileged directions : horizontal / vertical

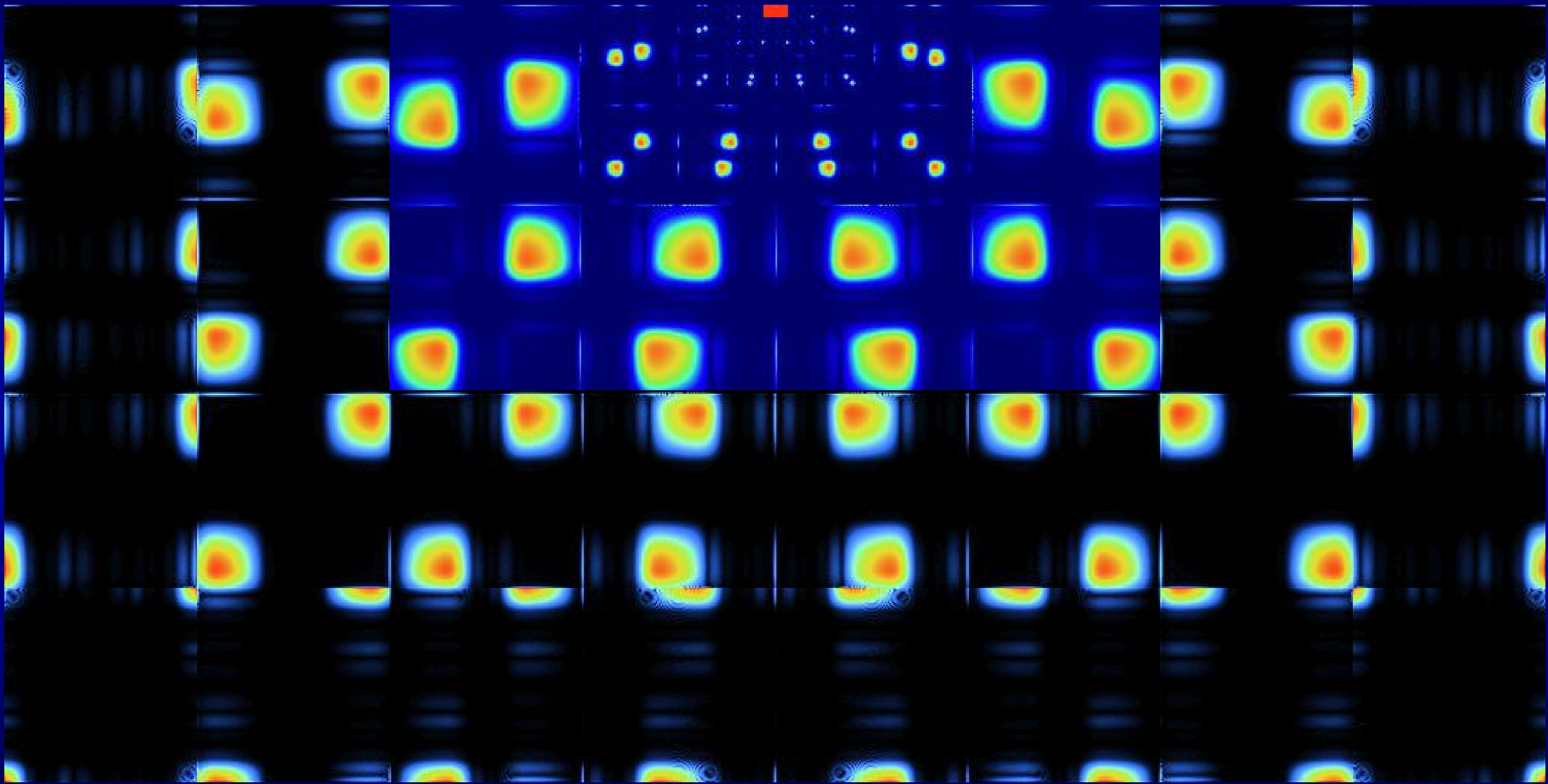
➔ poor texture representation
variously oriented (diagonals)

Example



Test image, 512x512

Example



Complex wavelet packet transform, level 6

Transform thresholding

Filter **only the magnitude** → enable **shift invariance**

$$\hat{x} = \theta_T(x) = x\, a_T(|x|)$$

recall : observed images are corrupted :

$$\mathbf{Y = h * X_0 + N}$$

each coefficient of the subband **k**
of the CWP transform is corrupted :

$$\mathbf{x} = x + \mathbf{n}$$

Observed coefficient
(CWP transform of
the deconvolved image)

Original coefficient
unknown
(original image
CWP transform)

Deconvolved noise
standard dev. σ_k

Thresholding functions

Data : image deconvolved without regularization

↓

Fix a thresholding function q_T

Optimal threshold computation : minimize the risk

- Minimax risk [Donoho 94]
- subband modeling

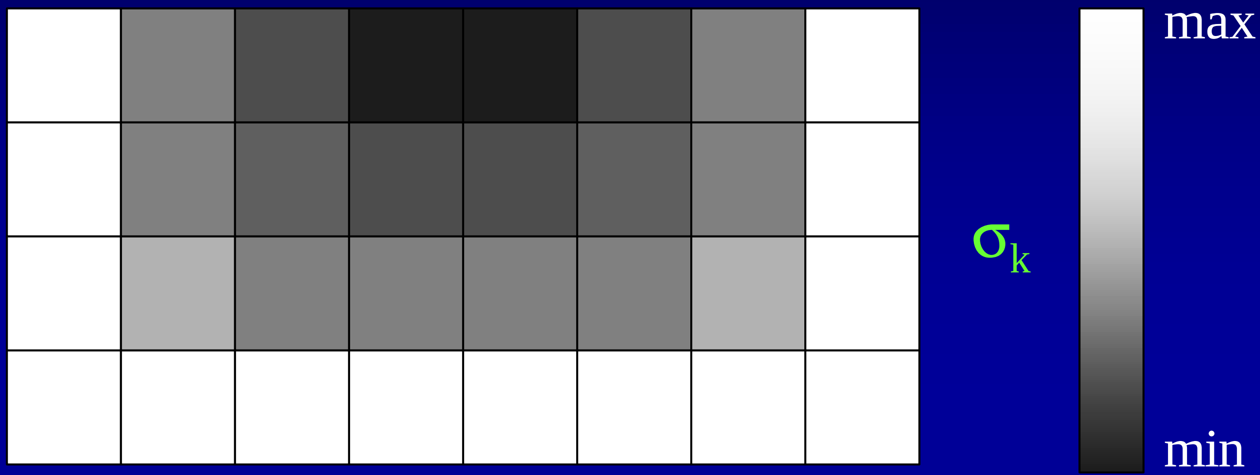
Bayesian methods :

Coefficient estimation by MAP → function q_T

Models for the subbands :

- Homogeneous generalized Gaussian
- Inhomogeneous Gaussian

Deconvolved noise variance



Estimation of σ_k :

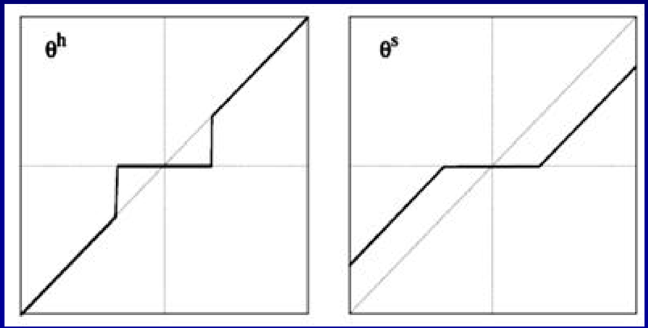
- simulation (CWP transform of a white Gaussian noise)
- direct computation, with known h and σ

$$\sigma_k^2 = \sigma^2 \sum_{i,j} \left| \frac{\mathrm{FFT}\left[R^k\right]_{ij}}{\mathrm{FFT}\left[h\right]_{ij}} \right|^2$$

Impulse response for subband k

Optimal risk

- Impose a thresholding function q_T



- **Minimize the risk** of the thresholding estimator

$$r(\hat{X}, X_0) = E\left[\left\|\hat{X} - X_0\right\|^2\right] = E\left[\sum_m \left|\theta_T(x) - \xi\right|^2\right]$$

- Theoretical results [Donoho, Johnstone 94]

  not useful in practice (too large threshold) [Kalifa 99]

- Subband modeling (Generalized Gaussian [Mallat 89])

  → model parameters estimation

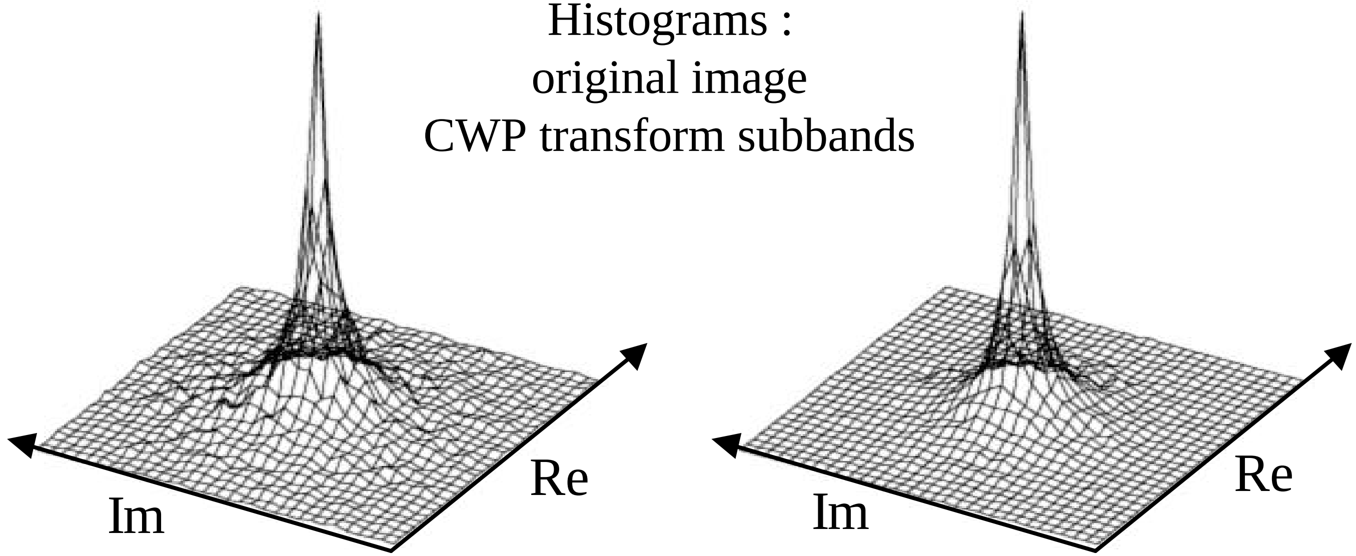
Subband modeling

Generalized Gaussian : $P(\xi) = \frac{1}{Z_{\alpha,p}} e^{-|\xi/\alpha|^p}$

α,p model parameters

Experimental study :

Histograms :
original image
CWP transform subbands



Bayesian methods

- Subband modeling
  → parameter estimation

- No arbitrary choice of thresholding function

- estimate **x** by **Maximum A Posteriori** (MAP)

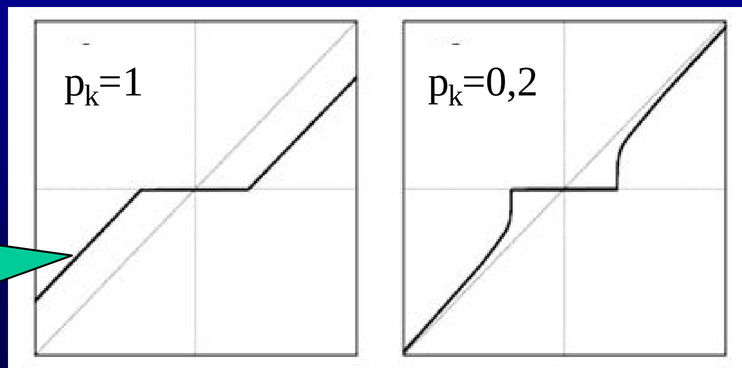
$$\text{Max } P(\xi|x) = \text{Max } P(x|\xi)\,P(\xi)$$

$$\left.\begin{array}{l} P(\xi) \propto e^{-|\xi/\alpha|^p} \\ P(x|\xi) \propto e^{-|x-\xi|^2/2\sigma^2} \end{array}\right\} \longrightarrow \hat{x} = \text{Min}_\xi \; |x-\xi|^2/2\sigma^2 + |\xi/\alpha|^p$$

Estmation of the model parameters α,p : Maximum Likelihood, …

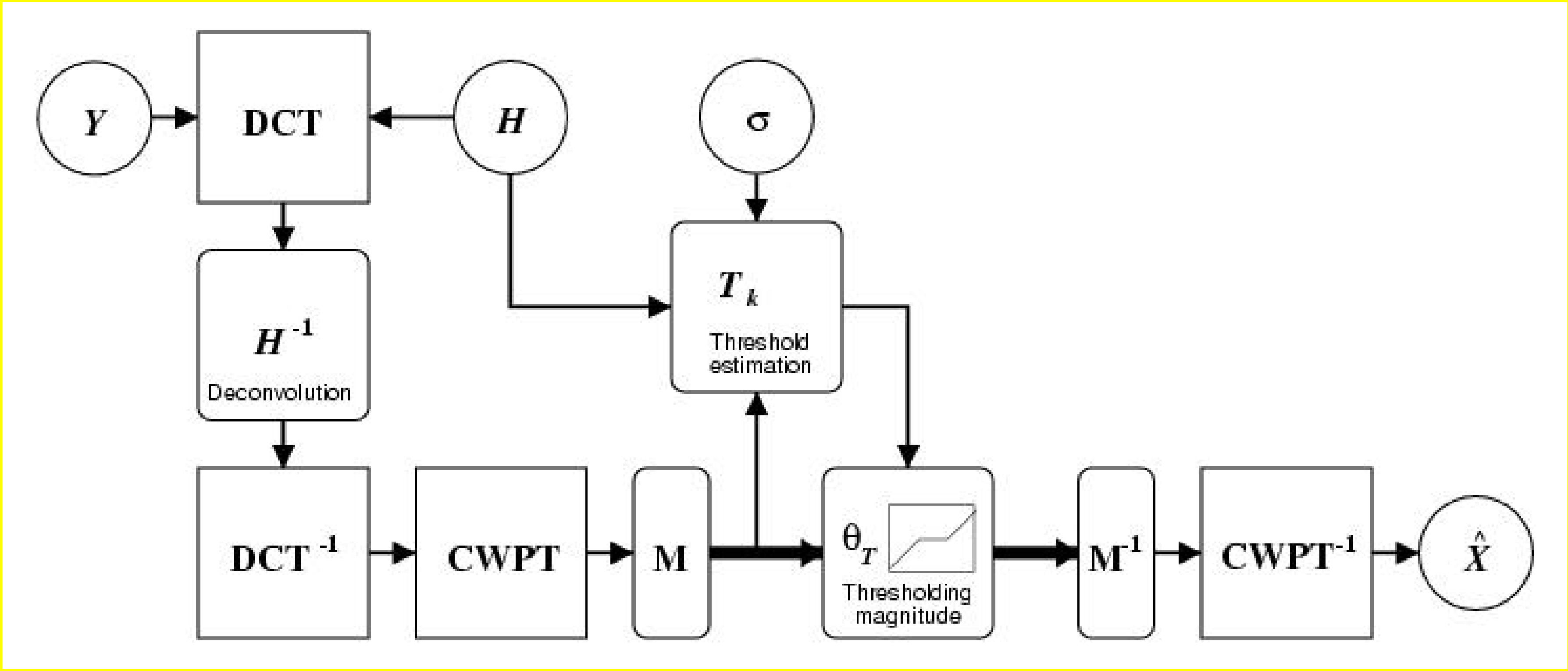
$$\boxed{\hat{x} = \theta(x)}$$

Classical thresholding functions for particular values of p_k



COWPATH 1.0

« **CO**mplex **W**avelet **P**ackets **A**utomatic **Th**resholding »



Inhomogeneous Gaussian Model

Insufficiency of homogeneous models
(constant areas / edges / textures)

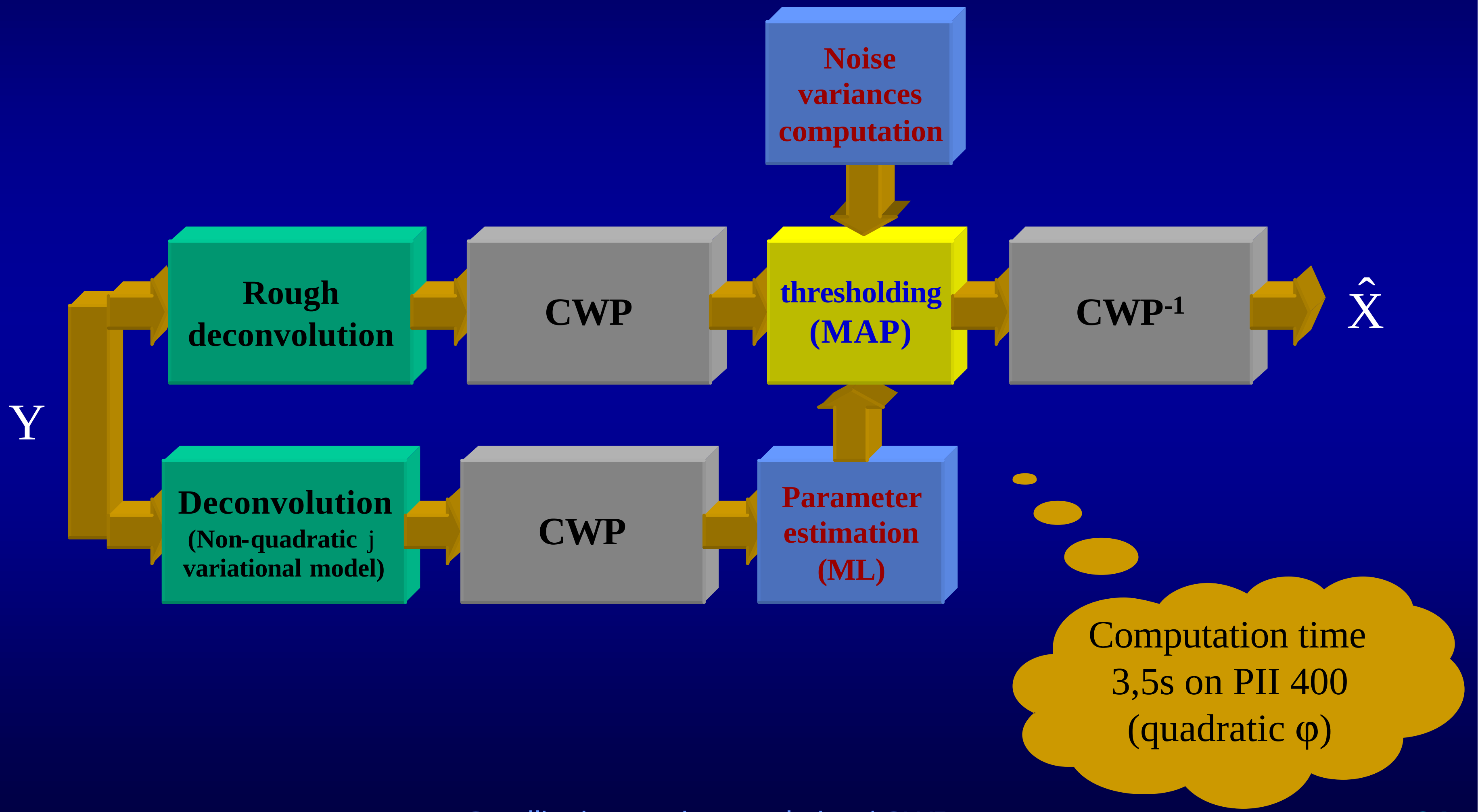
Parameter s_{ij} : depends on the location of the coefficient ξ_{ij}

$$P(\xi_{ij}) = \frac{1}{2\pi s_{ij}^2} e^{-|\xi_{ij}|^2 / 2 s_{ij}^2}$$

Estimation problems for the model parameters s_{ij}
(not enough data / number of unknown parameters !)

Hybrid method : parameter estimation
from a ‘good’ approximation of the original image
Complete Data Maximum Likelihood

COWPATH 2.0


Noise
variances
computation
Y
Rough
deconvolution
CWP
thresholding
(MAP)
CWP-1
X̂
Deconvolution
(Non-quadratic j
variational model)
CWP
Parameter
estimation
(ML)
Computation time
3,5s on PII 400
(quadratic φ)

Nîmes, original image 512 x 512

Nîmes, blurred and noisy image (σ~1.4)

Nîmes, COWPATH 1 result

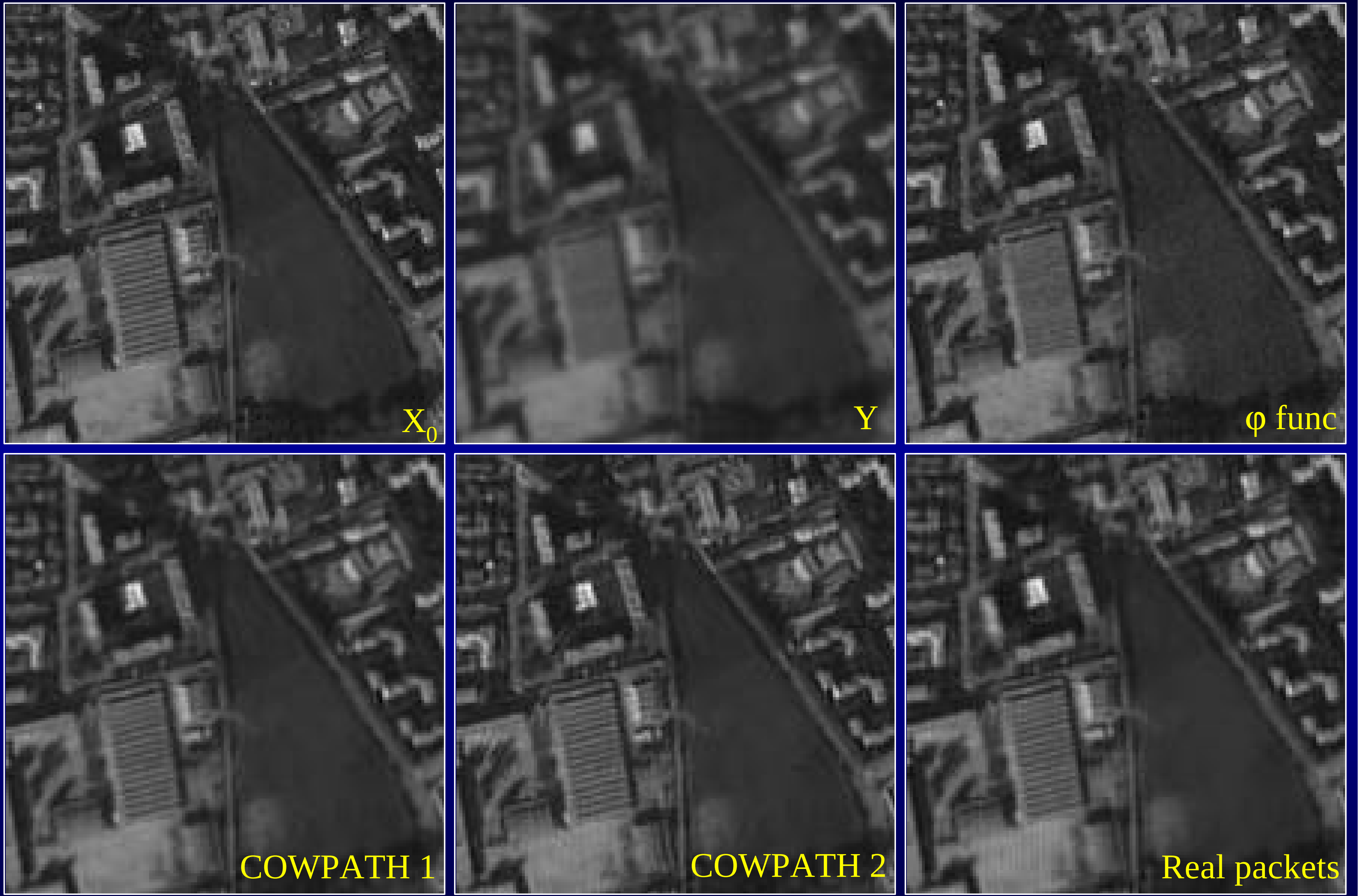
Nîmes, COWPATH 2 result

Nîmes, COWPATH 2 result - enlarged

Nîmes, deconvolution using real wavelet packets [Kalifa, Mallat 99]

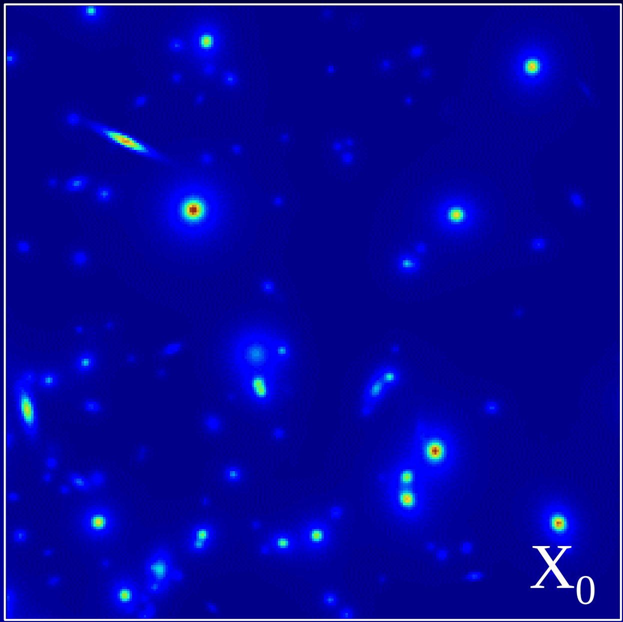
Nîmes, deconvolution using RHEA (non-quadratic φ function regularization [Jalobeanu 98])

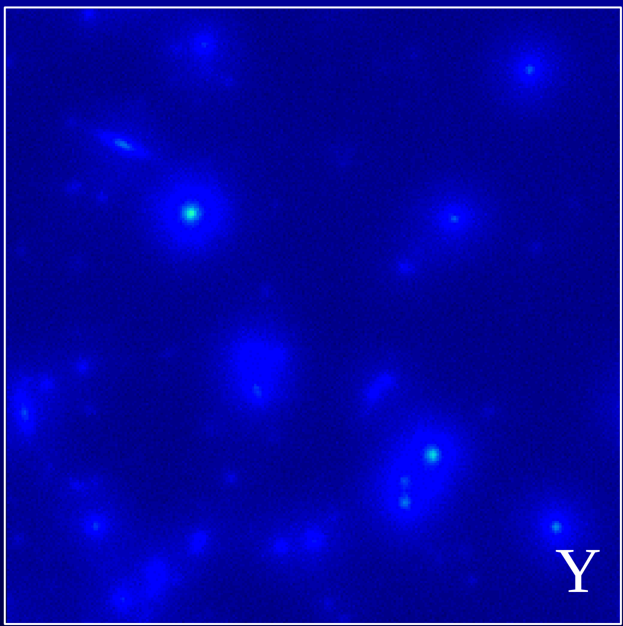
Nîmes, deconvolution using a quadratic regularization (~Wiener)

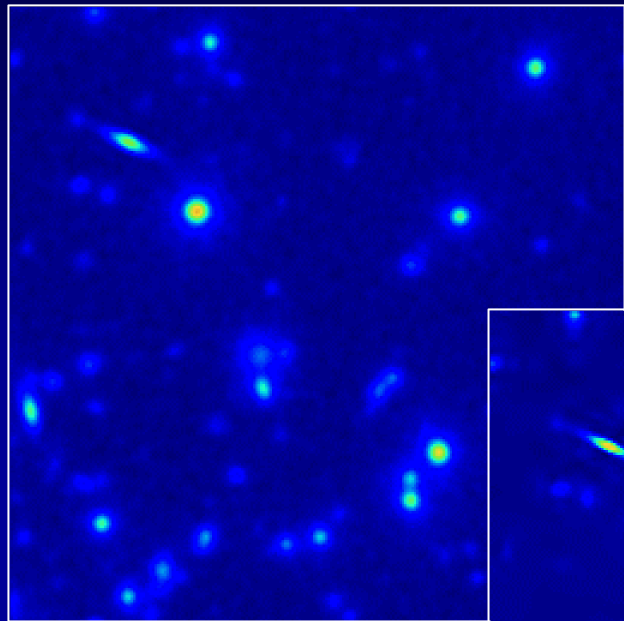


Result comparison

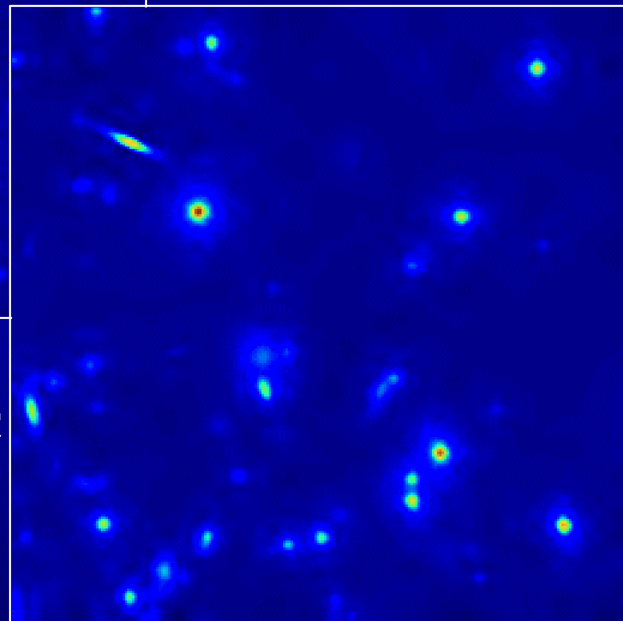
Results / Astronomy (Hubble PSF)



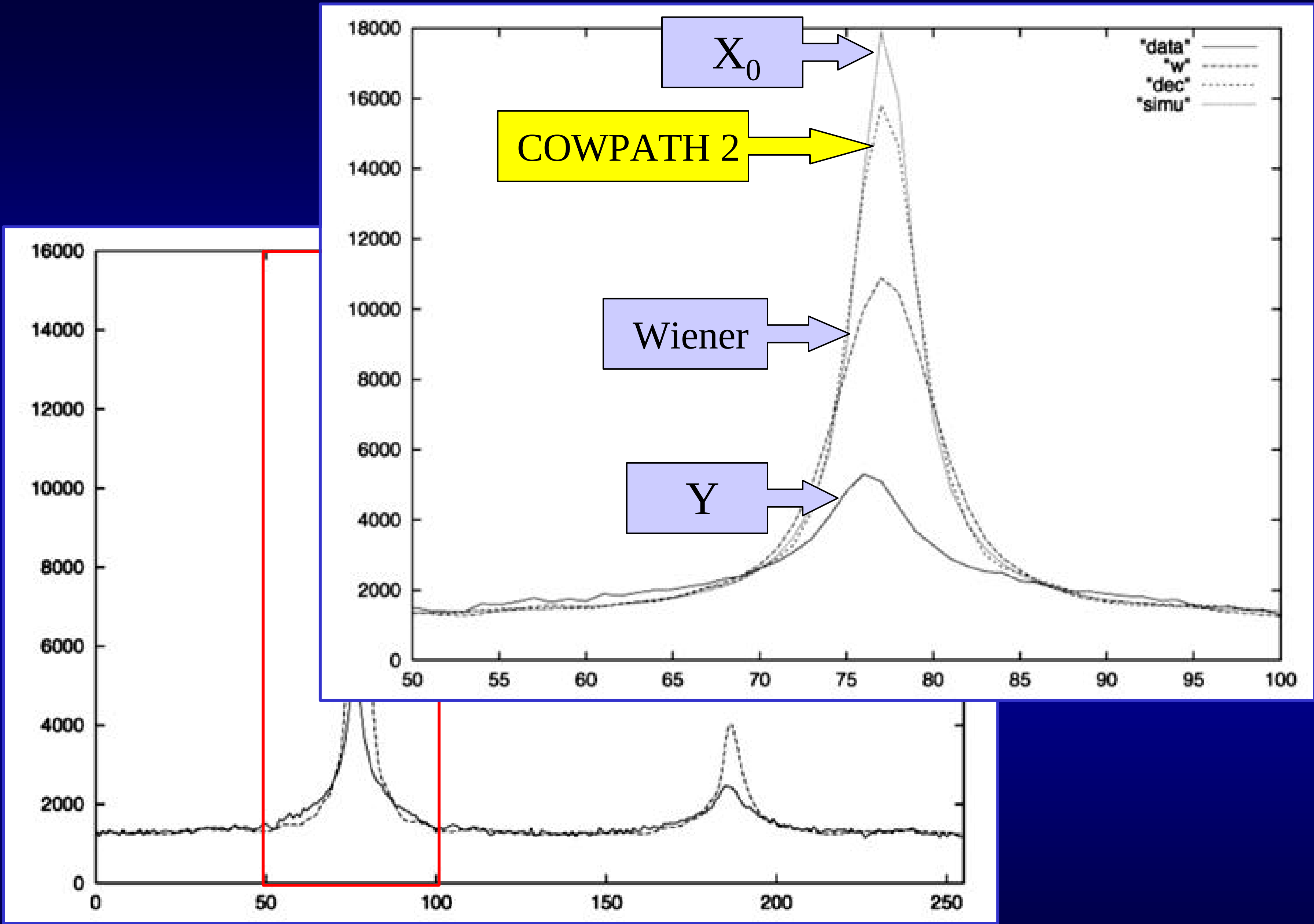




Deconvolution using quadratic regularization (~Wiener)



Deconvolution using CWP


X0
COWPATH 2
Wiener
Y
"data"
"w"
"dec"
"simu"

Conclusion and future work

Providing better results by

- **Adapting the structure of the tree to the problem**
  → taking into account images and PSFs
- **Better subband modeling**
  → inhomogeneous Generalized Gaussian model ?
- **More accurate data term**
  → noise transform coefficients not fully independent
- **Taking into account the interactions between scales**
  → Hidden Markov Trees [Nowak *et al.* 98]

Hybrid method : DEPA [Jalobeanu *et al.* 00]

Result of COWPATH → estimation of the parameters of an **adaptive** regularizing model