Parameter estimation for marked point processes: application to multidimensional object extraction from high resolution remote sensing images

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Introduction

Object extraction from high resolution remote sensing images

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Marked point process model

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Parameter estimation : ***Stochastic Expectation-Maximization*** algorithm (SEM)

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Ellipse model:
Pink flamingo extraction, tree crown extraction and boat detection

Rectangle model:
Building footprint extraction and tent detection

Plan

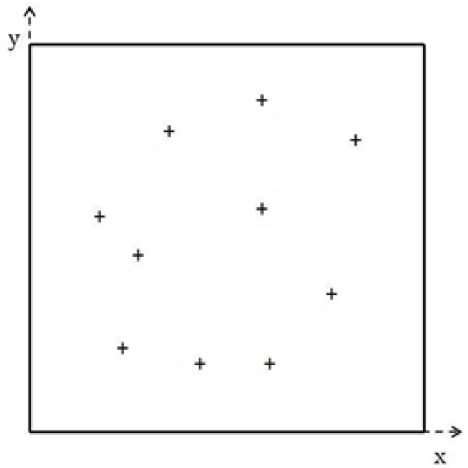
- Marked point process for object extraction
- Parameter estimation
- Ellipse process
- Rectangle process
- Conclusion and future work

Marked point process for object extraction

Point process

- Random set of points

  $\mathbf{x} = \{p_1, p_2, \ldots, p_n\}$.

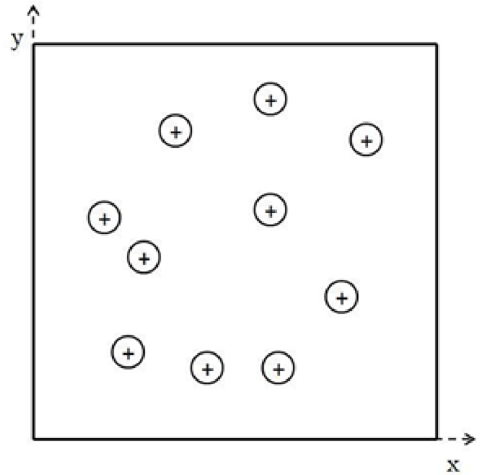


Marked point process

- Random set of objects

  $\mathbf{x} = \{(p_1, m_1), (p_2, m_2), \ldots (p_n, m_n)\}$.

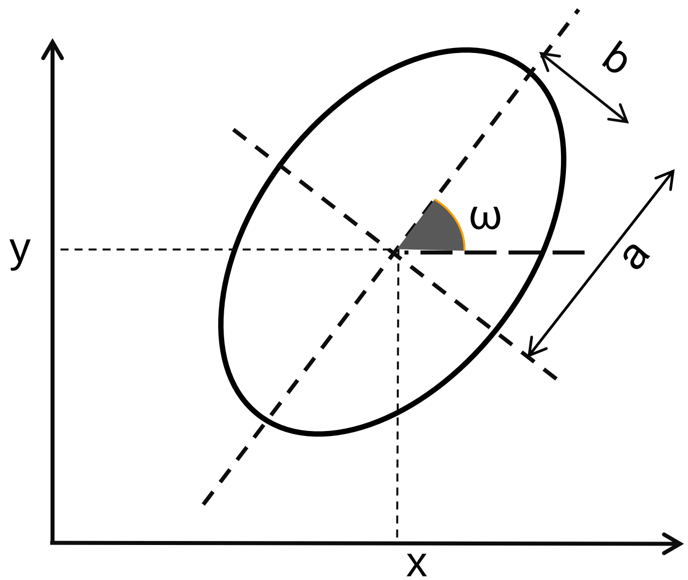
  - $\{p_1, p_2, \ldots, p_n\}$ is a point configuration.
  - $\{m_1, m_2, \ldots, m_n\}$ is the set of the associated marks.



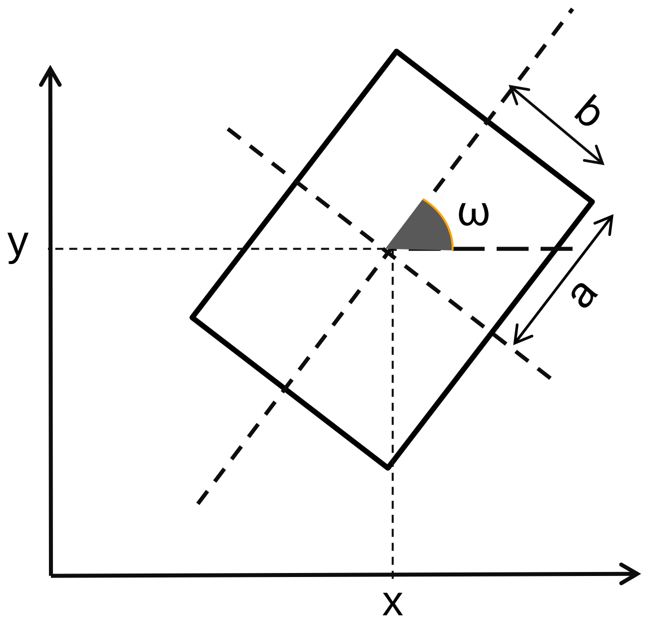
Marked point process for object extraction

Marked point process

- **Ellipse parametrization**



- **Rectangle parametrization**



Marked point process for object extraction

Object extraction problem

- Searching for the most likely object configuration **x** that fits the image **y**.

Solution

- **x** is a realization of a Gibbs process given by:

$$f_\theta(X = x/y) = \frac{e^{-U_\theta(x,y)}}{c(\theta/y)}$$

- The most likely configuration is given by:

$$\hat{x} = Arg \max_{x \in \Omega} f_\theta(X = x/y) = Arg \min_{x \in \Omega} U_\theta(x, y)$$

- The process energy is composed of two types of energies:

$$U_\theta(x, y) = U^d_{\theta_d}(x, y) + U^p_{\theta_p}(x)$$

Data energy term (external energy) → $U^d_{\theta_d}(x, y)$

Prior energy term (internal energy) → $U^p_{\theta_p}(x)$

Marked point process for object extraction

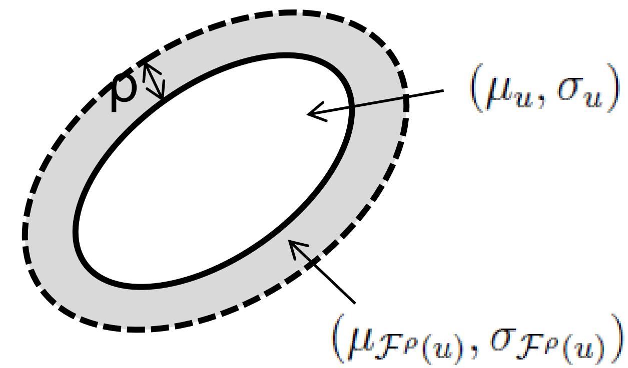
External energy or data energy

- The data energy of an **object configuration** $\boldsymbol{x}$:

$$U^d_{\theta_d}(\boldsymbol{x},\boldsymbol{y}) = \gamma_d \sum_{u \in \boldsymbol{x}} U^d_{\theta_d}(u)$$

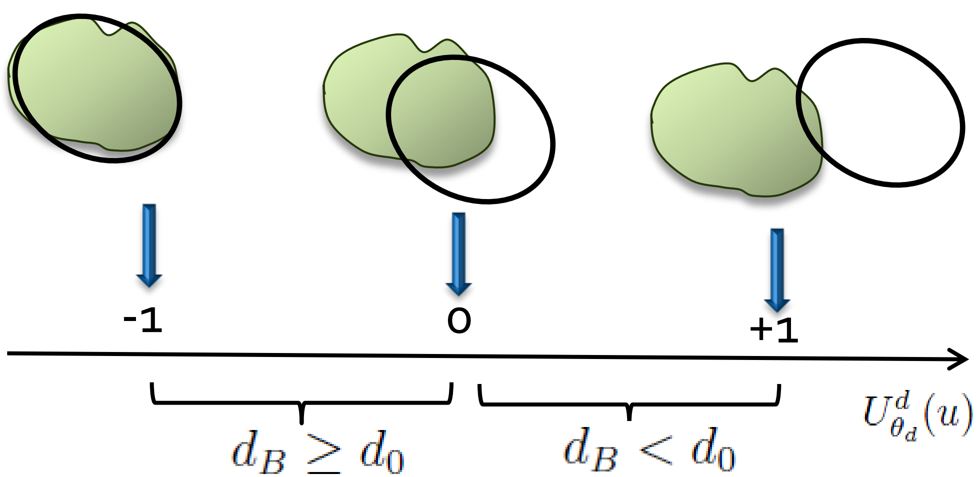
- The data energy of an **object** u:

$$U^d_{\theta}(u) = \mathcal{Q}_{d_0}(d(u, \mathcal{F}^{\rho}(u)))$$



- Quality function:

$$\mathcal{Q}_{d_0}(d) = \begin{cases} 1 - \left(\frac{d}{d_0}\right)^{\frac{1}{3}} & if\ d < d_0 \\ exp\left(-\frac{d - d_0}{3d_0}\right) - 1 & if\ d \geq d_0 \end{cases}$$



Marked point process for object extraction

Prior energy

- The overlap proportion between two objects x_i and x_j is given by:

$$A(x_i, x_j) = \frac{\text{Area}(x_i \cap x_j)}{min(\text{Area}(x_i), Aire(x_j))}$$

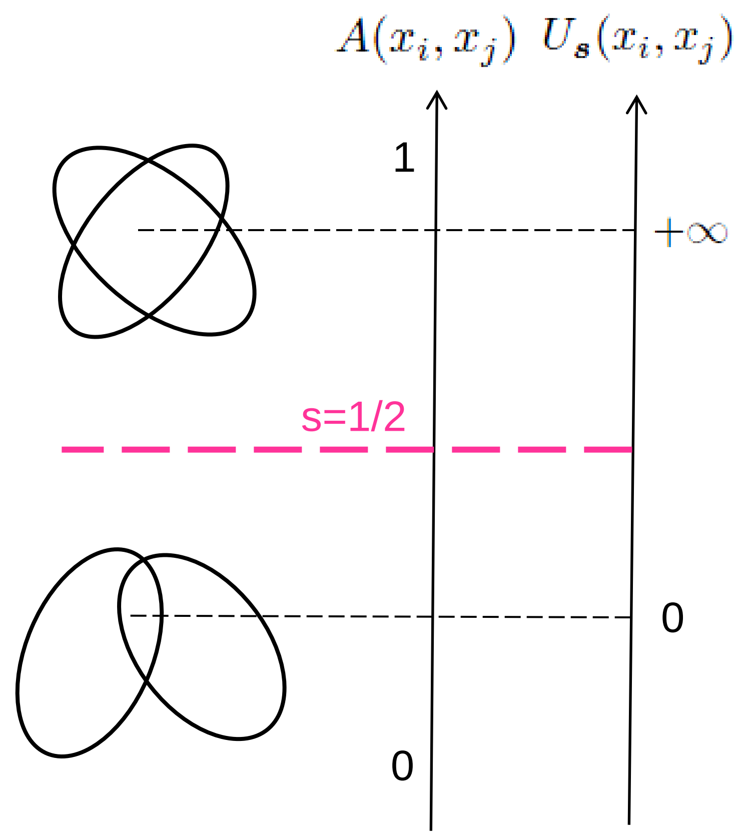
- Penalize the overlapping between objects by using a **Hard Core** process:

$$h_{\theta_p}(x) = \beta^{n(x)}\, e^{-U_s(x)}$$

- The overlapping potential of an object pair:

$$U_s(x_i, x_j) = \begin{cases} 0 & \text{if } x_i \sim x_j, \\ +\infty & \text{otherwise.} \end{cases}$$

- s in[0, 1]: the maximal overlapping ratio.



Marked point process for object extraction

Total energy and parameter identification

Total energy :

$$U_\theta(\boldsymbol{x}, \boldsymbol{y}) = U^p_{\theta_p}(\boldsymbol{x}) + U^d_{\theta_d}(\boldsymbol{x}, \boldsymbol{y})$$

$$\Longrightarrow \quad U_\theta(\boldsymbol{x}, \boldsymbol{y}) = -n(\boldsymbol{x}) Log\beta + \sum_{1<i<j<n(\boldsymbol{x})} t_s(x_i, x_j) + \gamma_d \sum_{u \in \boldsymbol{x}} U^d_{\theta_d}(u)$$

- The associated parameter vector θ is composed of:
  - $\beta > 0$: activity parameter,
  - $s \in [0, 1]$: maximal overlapping ratio,
  - $d_0 > 0$: contrast difference threshold,
  - $\gamma_d \geq 0$: data energy weight,
  - $\rho > 0$: width of an object boundary.

Plan

- Marked point process for object extraction
- **Parameter estimation**
- Ellipse process
- Rectangle process
- Conclusion and future work

Parameter estimation

Maximum Likelihood Estimator (MLE)

$$\hat{\theta} = arg \max_{\theta \in \Theta} f_\theta(\boldsymbol{y})$$

unknown

☺ **Solution:** maximize the extended likelihood: $\hat{\theta} = arg \max_{\theta \in \Theta} f_\theta(\boldsymbol{x}, \boldsymbol{y})$

☹ **Problem:** the object configuration $\boldsymbol{x}$ is unknown .

☺ **Solution: Expectation-Maximization algorithm (EM)**

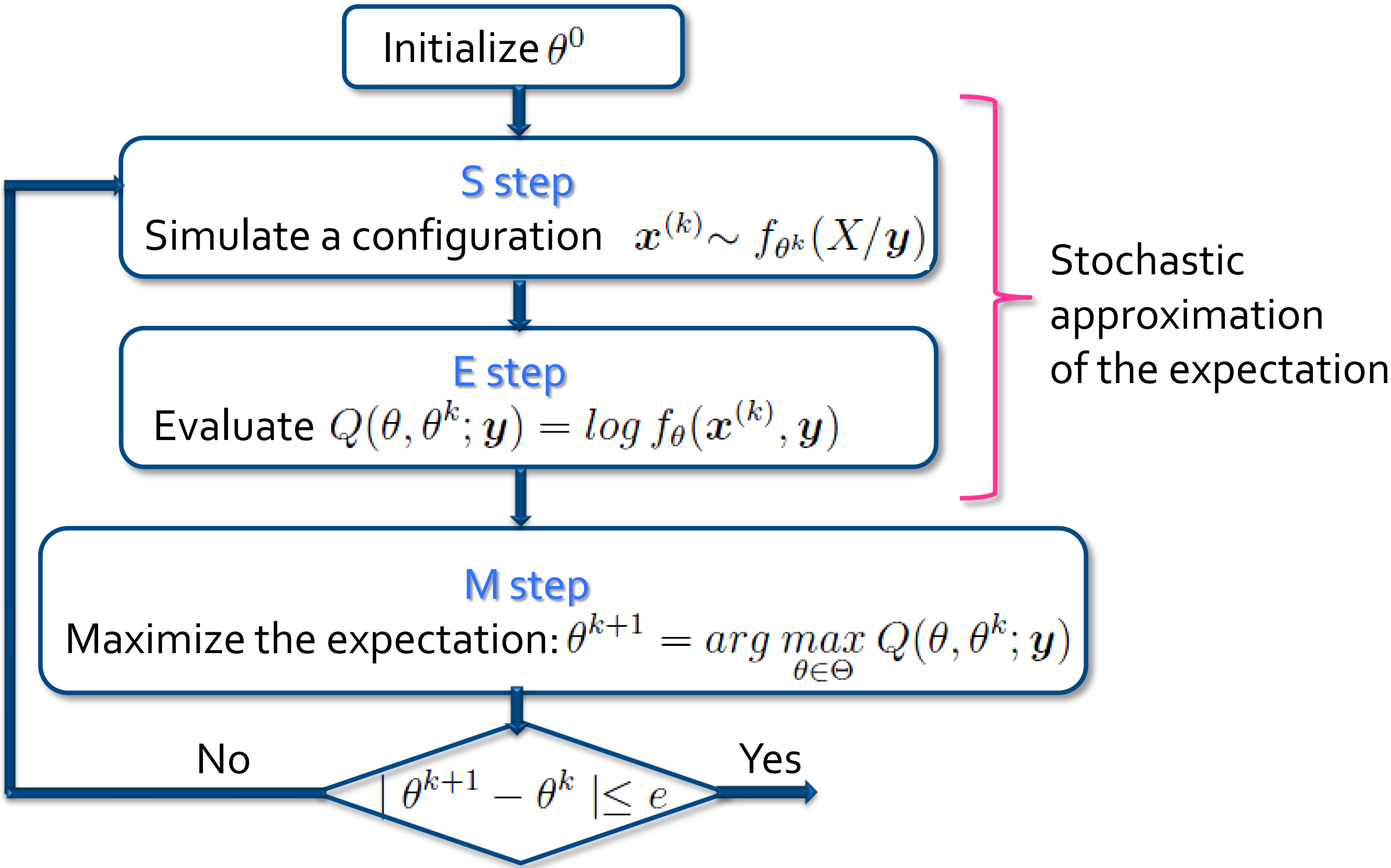
1. E step: evaluate the conditional expectation: $E_{\theta^k}\left[log\, f_\theta(\boldsymbol{x}, \boldsymbol{y})/\boldsymbol{y}\right]$
2. M step: maximize the expectation : $\theta^{k+1} = arg \max_{\theta \in \Theta} E_{\theta^k}\left[log\, f_\theta(\boldsymbol{x}, \boldsymbol{y})/\boldsymbol{y}\right]$

☹ **Problem:** there is no explicit expression of the conditional expectation.

☺ **Solution:** stochastic version of the EM algorithm (SEM).

Parameter estimation

- **Stochastic Expectation-Maximization algorithm (SEM)**



Parameter estimation

Stochastic Expectation-Maximization algorithm (SEM)

☹ **Problem:** there is no analytical or numerical expression for $f_\theta(\boldsymbol{x}, \boldsymbol{y}) = \frac{h_\theta(\boldsymbol{x}, \boldsymbol{y})}{c(\theta)}$ since $c(\theta) = \int \int h_\theta(\boldsymbol{x}, \boldsymbol{y})\, \mu(d\boldsymbol{x})\nu(d\boldsymbol{y})$ is not known

☺ **Solution :** approximation of the extended likelihood $f_\theta(\boldsymbol{x}, \boldsymbol{y})$ by the pseudo-likelihood:

$$PL_W(\theta; \boldsymbol{x}, \boldsymbol{y}) = \left[\prod_{x_i \in \boldsymbol{x}} \lambda_\theta(x_i; \boldsymbol{x}, \boldsymbol{y})\right] exp\{-\int_W \lambda_\theta(u; \boldsymbol{x}, \boldsymbol{y})\Lambda(du)\}$$

- Papangelou intensity:

$$\lambda_\theta(u; \boldsymbol{x}, \boldsymbol{y}) = \beta\, exp\left(-\gamma_d U_d(u) - \sum_{x_i \in \boldsymbol{x}/x_i \neq u} t_s(u, x_i)\right)$$

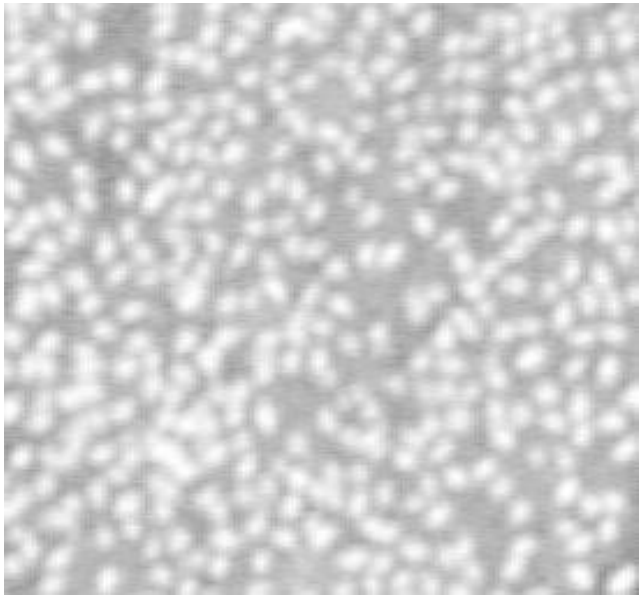
Plan

- Marked point process for object extraction
- Parameter estimation
- **Ellipse process**
- Rectangle process
- Conclusion and future work

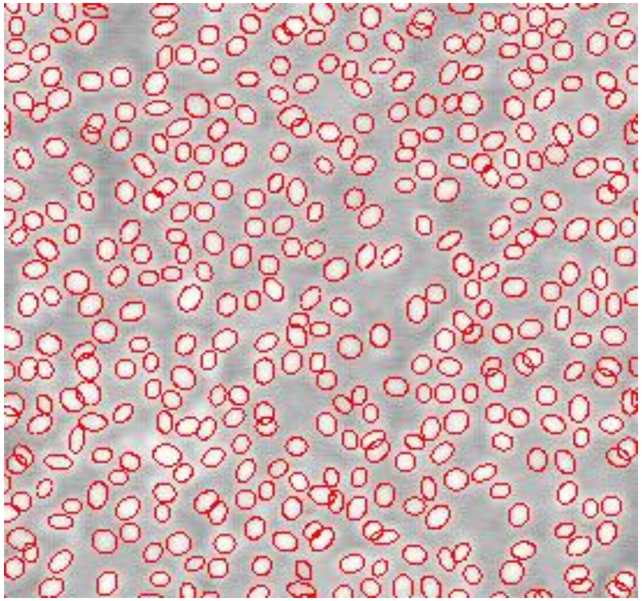
Ellipse process

Application to pink flamingo extraction

- Estimation using the SEM algorithm performed in 55 min and 39 s on 1.86 GHz processor.
- Extraction using simulated annealing: 398 ellipses



Pink flamingo colony in Camargue, France © Tour du Valat



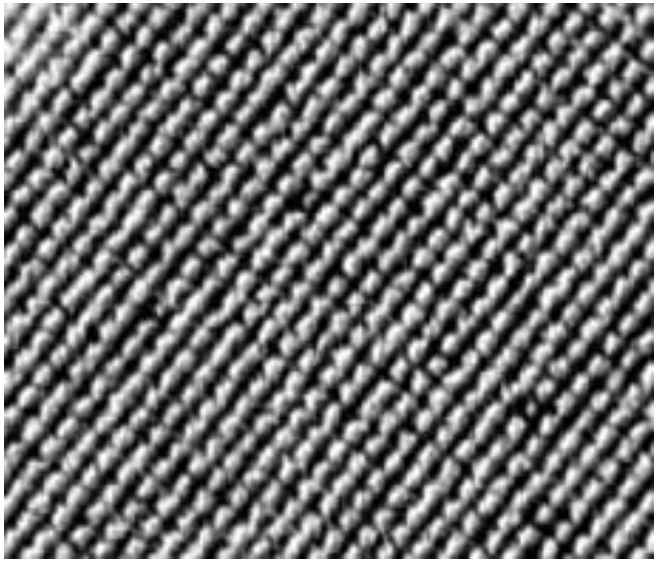
$$\begin{cases} s = 0.3 \\ \beta = 1000 \\ \gamma_d = 12.8839 \\ d_0 = 1.33 \end{cases}$$

Extraction results
Detection error < 5%

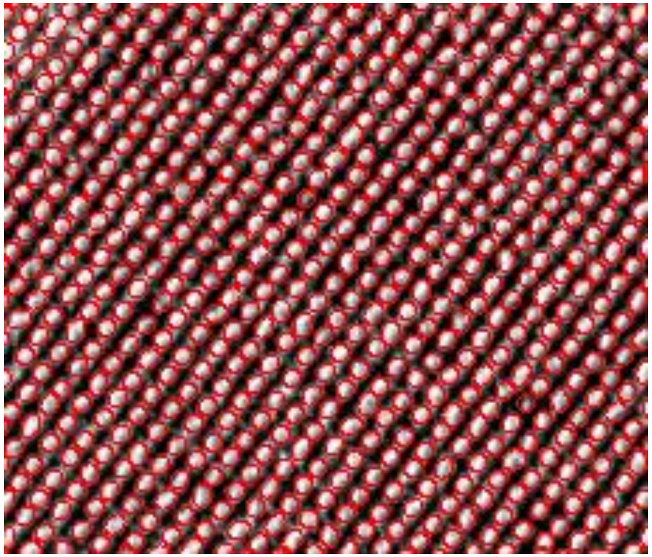
Ellipse process

Application to tree crown extraction

- Estimation using the SEM algorithm performed in 38 min and 29 s on 1.86 GHz processor.
- Extraction using simulated annealing: 606 ellipses.



Poplar plantation sample in Saone et Loire, France © IFN

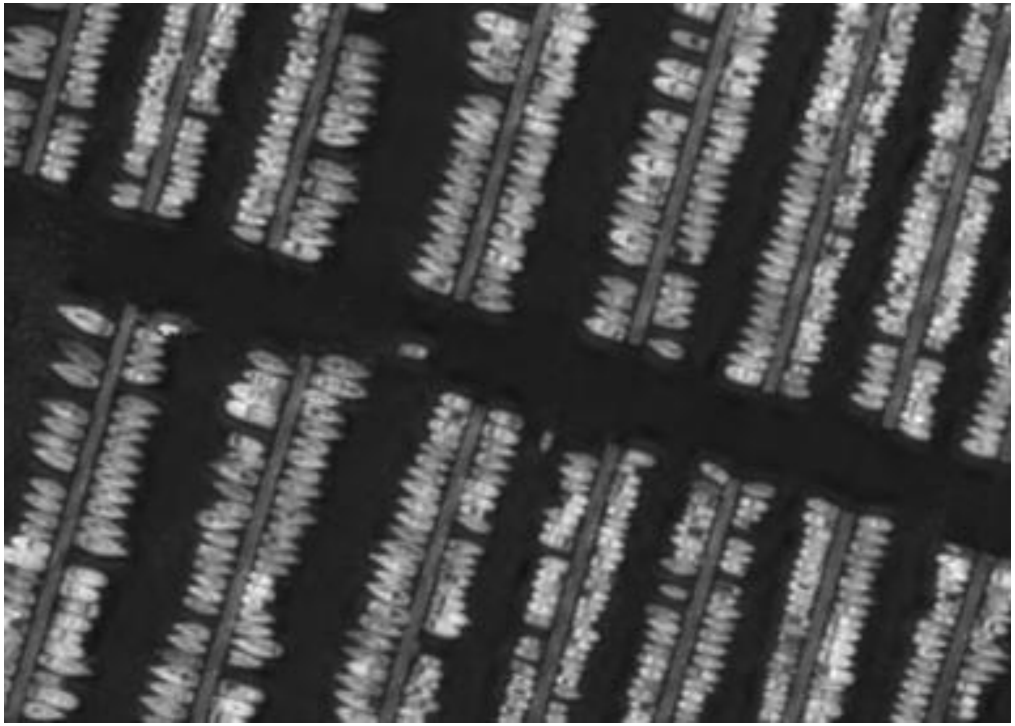


Extraction result
Detection error < 3%

$$\begin{cases} s = 0.3 \\ \beta = 1000 \\ \gamma_d = 9.2657 \\ d_0 = 2 \end{cases}$$

Ellipse process

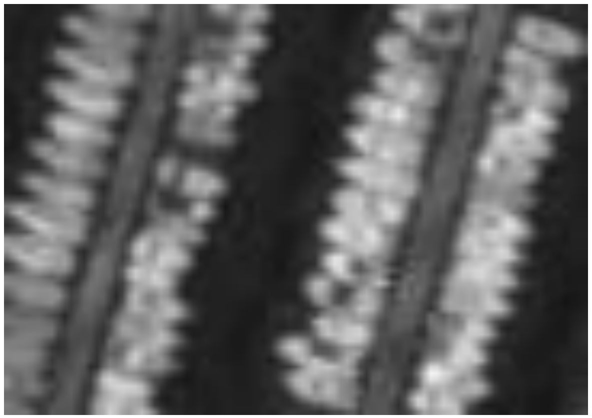
- **Application to boat detection**



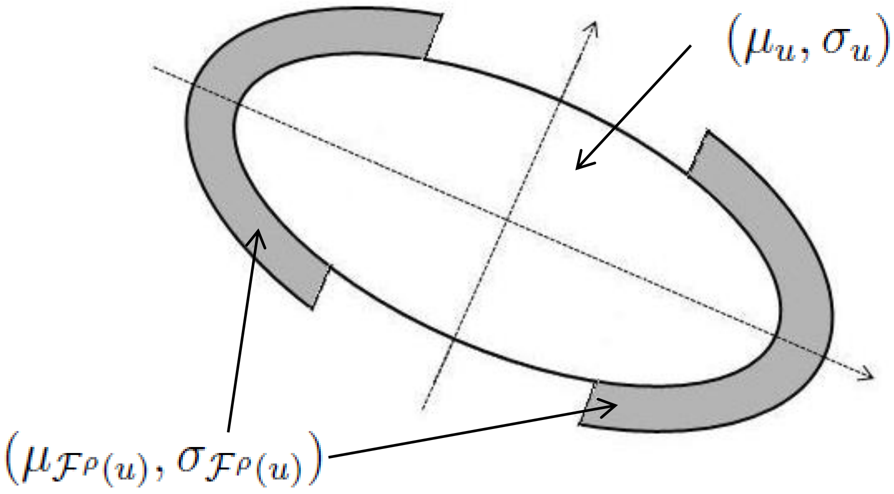
Boat image © CNES

Ellipse process

Modification of the data term for boat detection



- The boats are very close and contiguous.
- Considering the complete surrounding crown of an ellipse is not very effective. It is better to choose:



Ellipse process

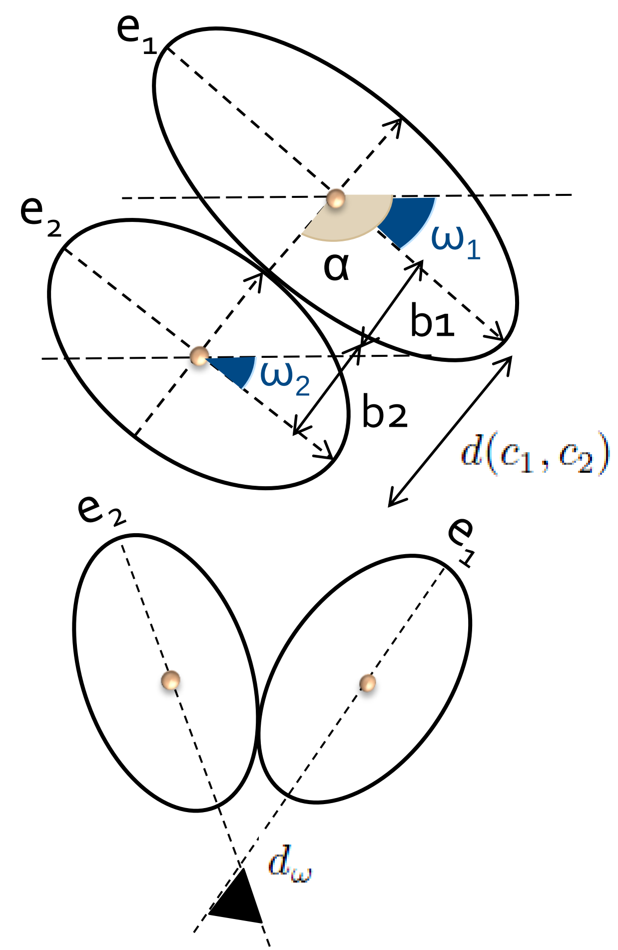
Modification of the prior term for boat detection

- Favor close and aligned ellipses
  - Two ellipses are said to be aligned ($e_1 \sim_{alig} e_2$) if:

1. $$d_\alpha(e_1, e_2) \equiv \mid \alpha - (\frac{\omega_1 + \omega_2}{2} + \frac{\pi}{2}) \mid [\pi] \leq d_{\alpha_{max}}$$

2. $$d_C(e_1, e_2) = \mid d(c_1, c_2) - (b_1 + b_2) \mid \leq d_{C_{max}}$$

3. $$d_\omega(e_1, e_2) = \mid \omega_1 - \omega_2 \mid \leq d_{\omega_{max}}$$



Ellipse process

Modification of the prior term for boat detection

- The prior energy of an **object pair** that favors aligned ellipses is given by:

$$\begin{cases} U_{alig}(e_1, e_2) \in [-1, 0[& si\ e_1 \sim_{alig} e_2 \\ U_{alig}(e_1, e_2) = 0 & sinon \end{cases}$$

- The prior energy of an **object configuration** that favors aligned ellipses is as follows:

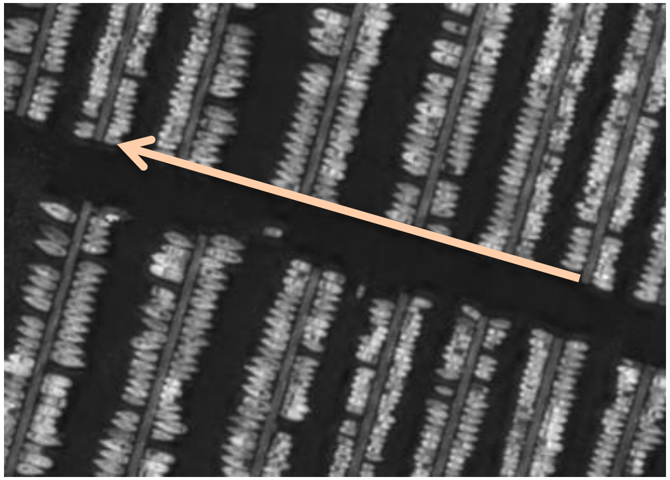
$$U^p_{alig}(\boldsymbol{x}) = \underbrace{\gamma_{alig}} \sum_{1<i<j<n(\boldsymbol{x})} U_{alig}(x_i, x_j)$$

Prior energy weight
(alignment constraint)

- γ_{alig} is estimated by the SEM algorithm.

Ellipse process

Modification of the prior term for boat detection

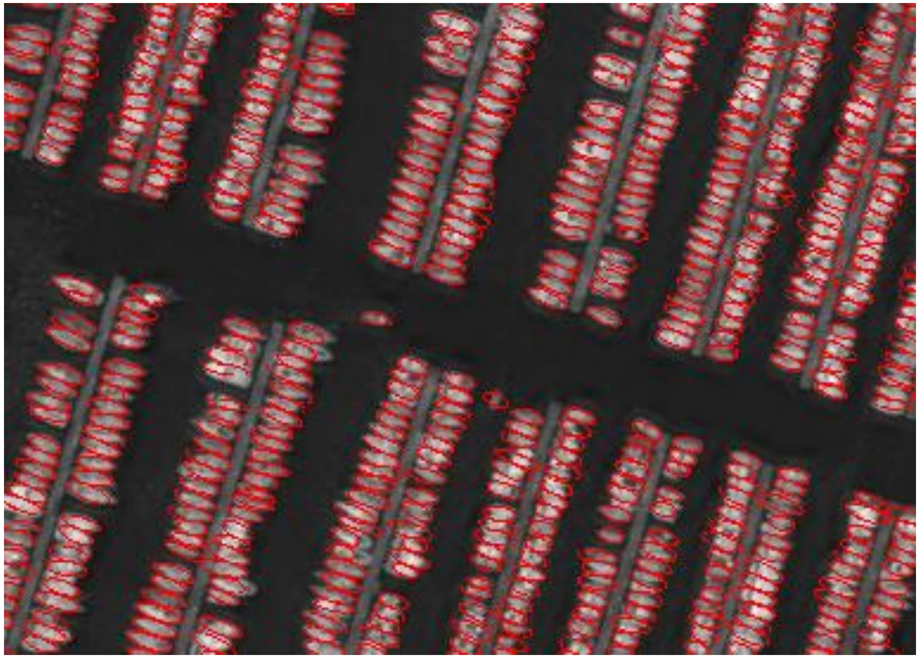


- Empirically define the boat orientation ω_N .
- The prior energy that favors aligned ellipses having a privileged orientation ω_N is:

$$\begin{cases} U_{alig_\omega}(e) = U^p_{alig}(x \cup \{e\}) - U^p_{alig}(x) & si \ \mid \omega_e - \omega_N \mid \leq d_{\omega_{max}} \\ U_{alig_\omega}(e) = 0 & sinon \end{cases}$$

Ellipse process

- **Simulation result favoring aligned ellipses with a privileged direction**
  - Estimation using the SEM algorithm performed in 1 h, 38 min and 32 s on 1.86 GHz processor.
  - Extraction using simulated annealing: 523 ellipses.



$$\begin{cases} \beta = 1000 \\ \gamma_d = 27.56 \\ \gamma_{alig} = 9.18 \\ s = 0.3 \\ d_0 = 6 \end{cases}$$

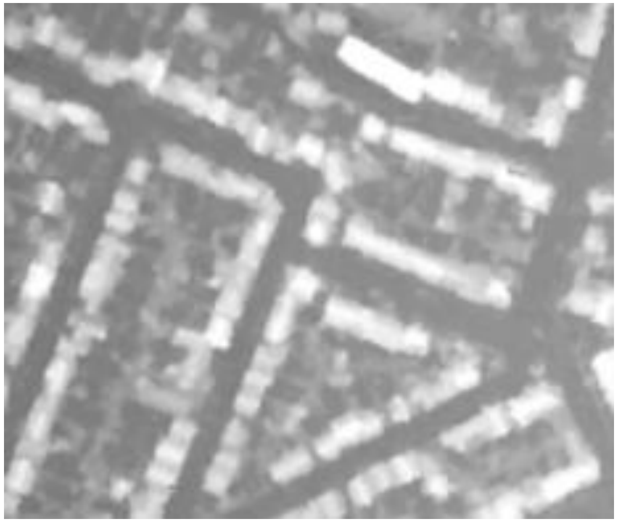
Detection error < 9.5 %

Plan

- Marked point process for object extraction
- Parameter estimation
- Ellipse process
- **Rectangle process**
- Conclusion and future work

Rectangle process

Application to building footprint extraction

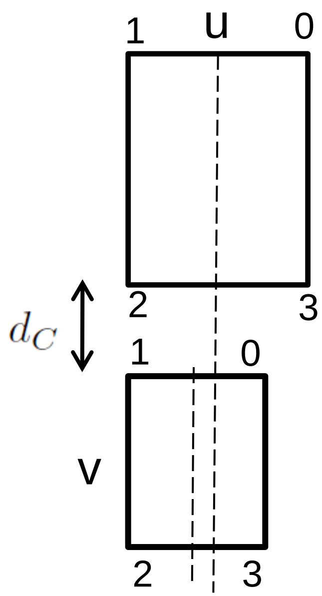


Digital Elevation Model of a part of Amiens, France © IGN

- Favor aligned rectangles,
- Favor orthogonal rectangles.

Application to building footprint extraction

- Alignment interaction between two rectangles:



$$u \sim_{alig_i} v,\ \forall i \in \{0,\ 1,\ 2,\ 3\} \iff \begin{cases} d_C(i,\ j) \leq d_{C_{max}} \\ d_\omega(u,\ v) \leq d_{\omega_{max}} \\ i + j = 3 \end{cases}$$

- Favor aligned rectangles via the following energy:

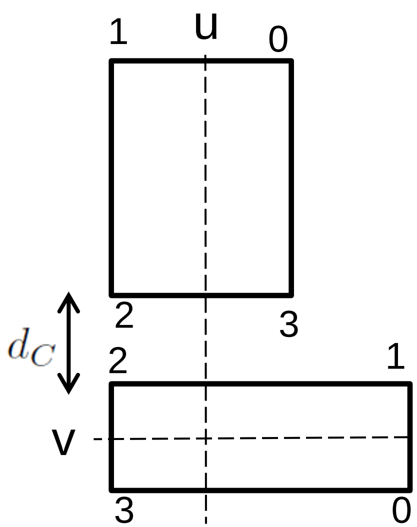
$$U_{alig}(x) = \underbrace{\gamma_{alig}} \sum_{1<j<k<n(x)} \sum_{0\leq i\leq 3} U_{alig_i}(x_j, x_k)$$

Prior energy weight
(alignment constraint)

Rectangle process

Application to building footprint extraction

- Orthogonal interaction between two rectangles:



$$u \sim_{orth_i} v,\ \forall i \in \{0,\ 1,\ 2,\ 3\} \Longleftrightarrow \begin{cases} d_C(i,j) \leq d_{C_{max}} \\ \mid d_\omega(u,v) - \frac{\pi}{2} \mid \leq d_{\omega_{max}} \\ i+j = 0,\ 2,\ 4,\ ou\ 6 \end{cases}$$

- Favor orthogonal rectangles via the following energy:

$$U_{orth}(\boldsymbol{x}) = \underbrace{\gamma_{orth}}_{\text{Prior energy weight (orthogonality constraint)}} \sum_{1<j<k<n(\boldsymbol{x})} \sum_{0 \leq i \leq 3} U_{orth_i}(x_j, x_k)$$

Prior energy weight
(orthogonality constraint)

Rectangle process

Application to building footprint extraction

- Prior energy which reflects building interactions:

$$U^{p}_{\theta_{int}}(\boldsymbol{x}) = \gamma_{int} \left[U_{alig}(\boldsymbol{x}) + U_{orth}(\boldsymbol{x})\right]$$

➡ Three new parameters :

- γ_{alig} : the prior energy weight (alignment constraint),
- γ_{orth} : the prior energy weight (orthogonality constraint),
- γ_{int} : the prior energy weight (building interaction).

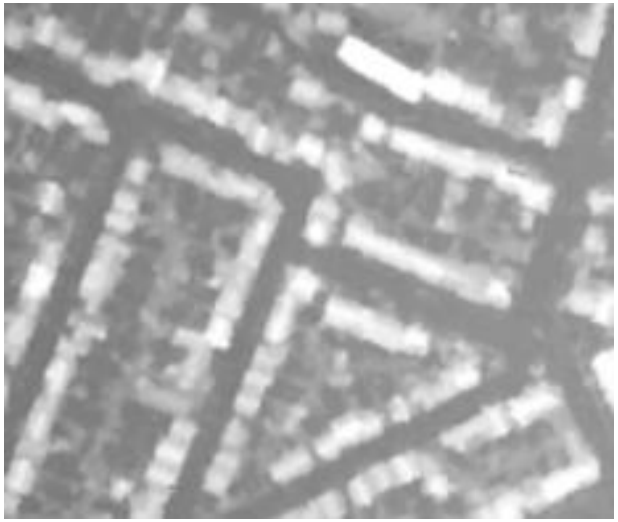
} Set to 1 (γ_{alig}, γ_{orth})

➡ Estimated by the SEM algorithm (γ_{int})

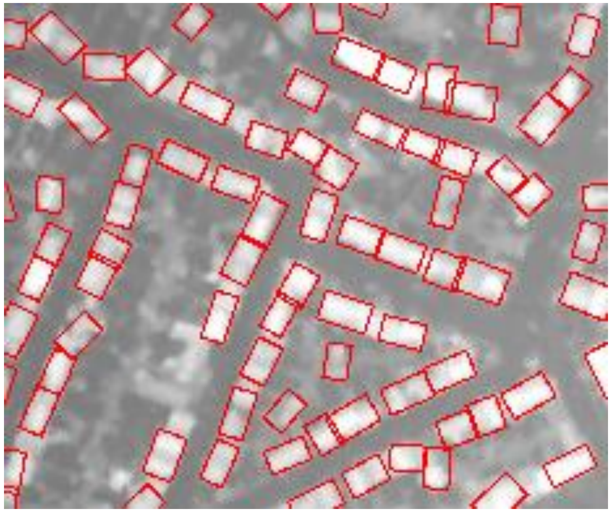
Rectangle process

Simulation result after modification of the prior energy

- Estimation using the SEM algorithm performed in 6 h, 21 min and 2 s on 1.86 GHz processor.
- Extraction using simulated annealing: 83 rectangles



Digital Elevation Model of a part of Amiens, France © IGN



Extraction result

$$\begin{cases} \beta = 500 \\ \rho = 1 \\ d_0 = 4 \\ \gamma_d = 30.059 \\ \gamma_{int} = 16.849 \\ \gamma_{alig} = 1 \\ \gamma_{orth} = 1 \end{cases}$$

Application to tent detection

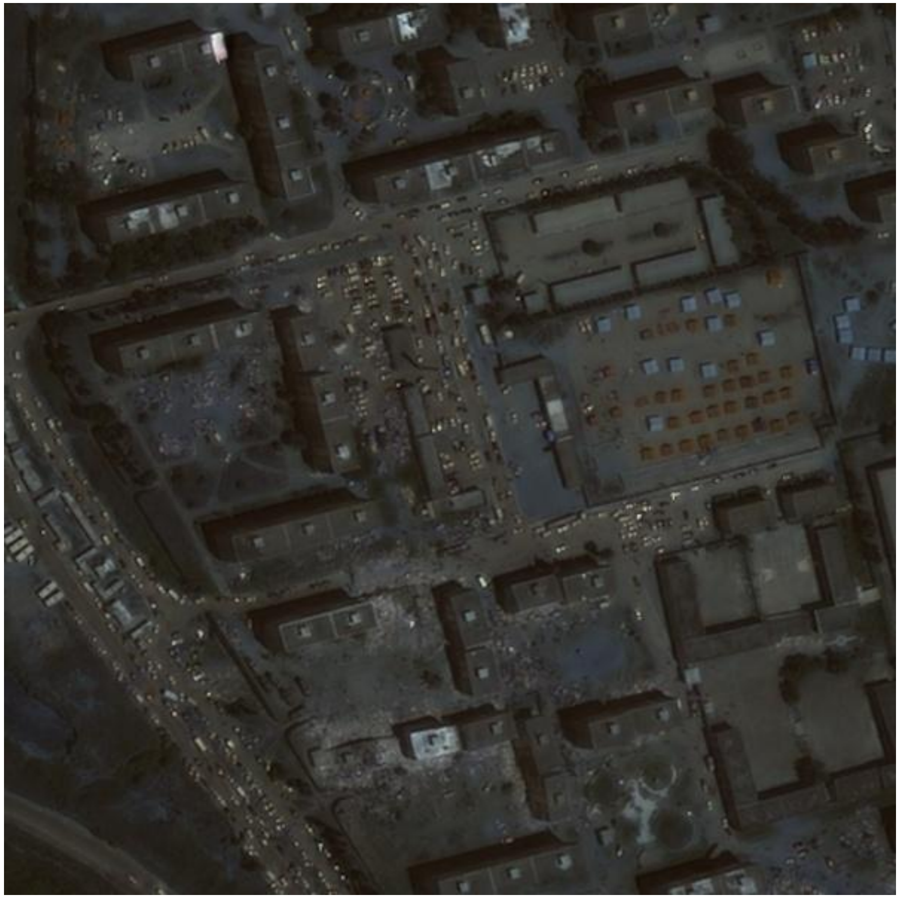
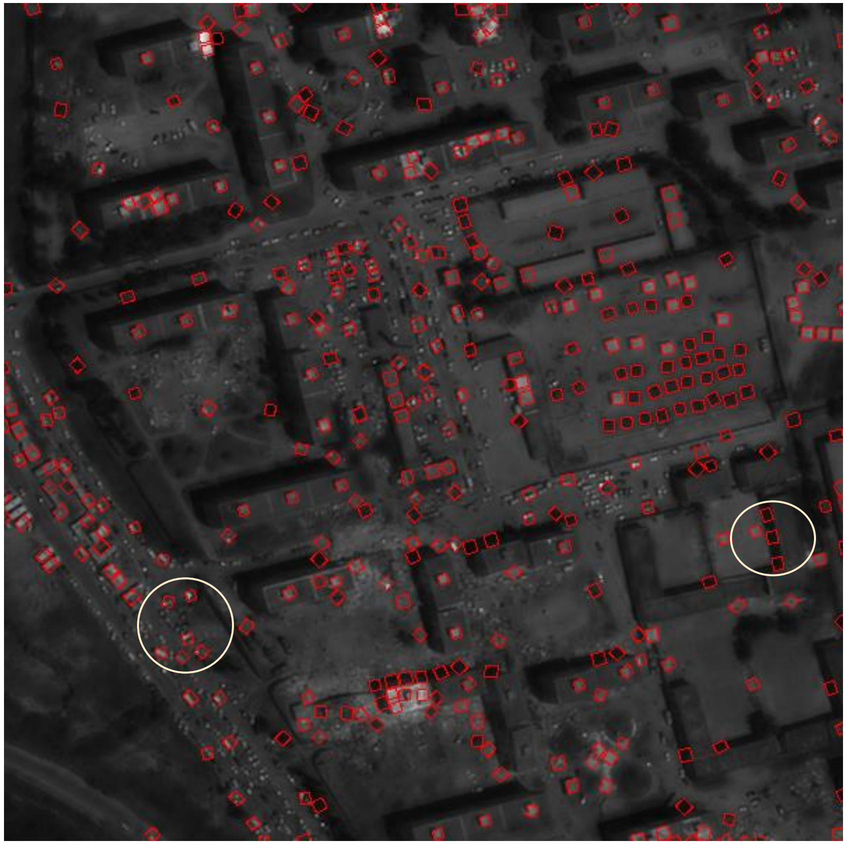


Image of tents in Boumerdes, Algeria
© CNES

Rectangle process

Application to tent detection

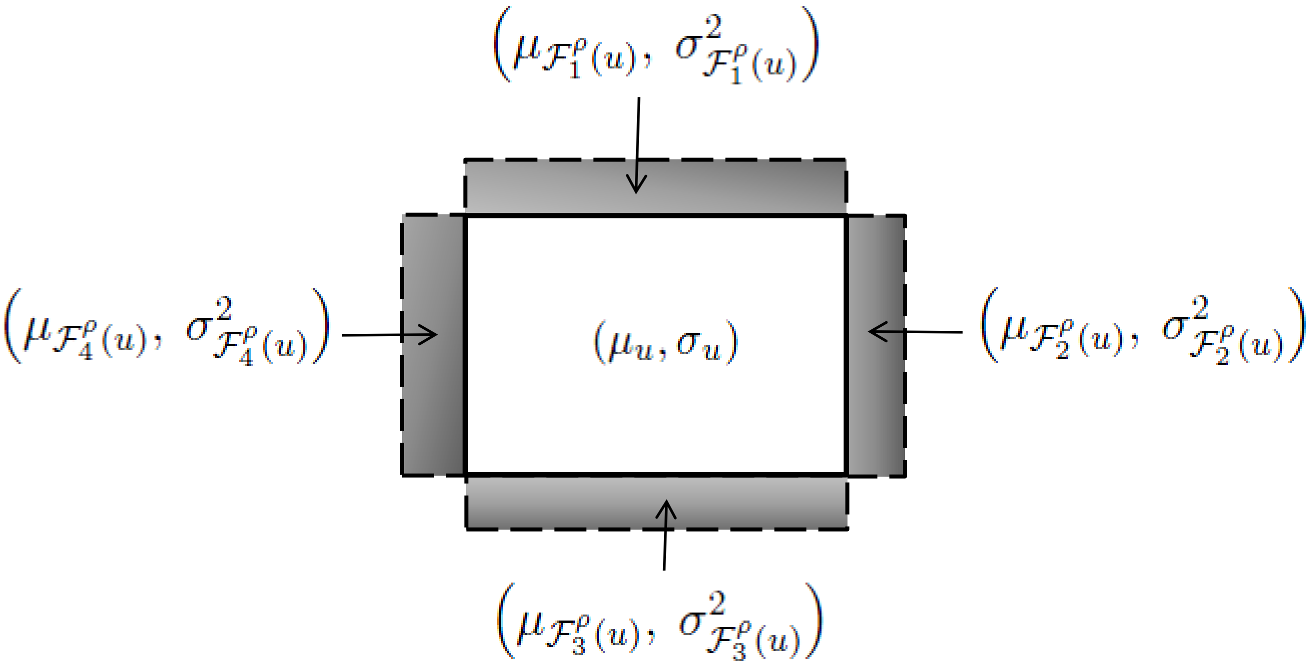
- Estimation using the SEM algorithm performed in 2 h, 17 min and 48 s on 1.86 GHz processor.
- Extraction using simulated annealing: 398 rectangles



$$\begin{cases} \rho = & 1 \\ d_0 = & 2.6 \\ \beta = & 500 \\ \gamma_d = & 32.5248 \end{cases}$$

Rectangle process

- **Modification of the data term for tent detection**



$$d_B(u, \mathcal{F}^\rho(u)) = \min_{1 \leq i \leq 4} d_B(u, \mathcal{F}_i^\rho(u))$$

Rectangle process

- **Proposed post-processing for tent detection**
  - Remove objects that do not fit well the data according to the second data energy definition,
  - Remove heterogenous objects.

Rectangle process

Simulation result

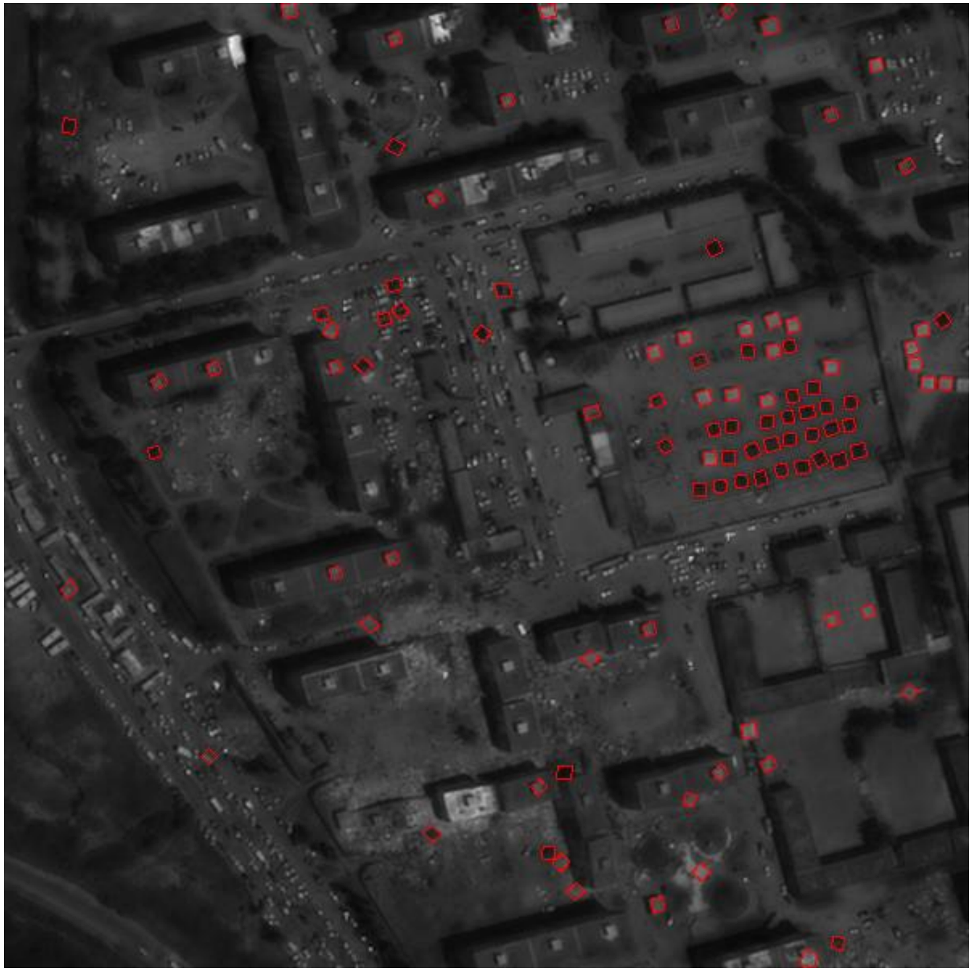
- Remove objects that do not fit well the data according to the second data energy definition.

Number of detected objects = 153

Rectangle process

Simulation result

- Remove heterogenous objects.



Number of detected objects= 98

Plan

- Marked point process for object extraction
- Parameter estimation
- Ellipse process
- Rectangle process
- **Conclusion and future work**

Conclusion and futur work

Conclusion

- **Relevance** of the SEM algorithm.
- **Effeciency of the ellipse model** for pink flamingo and tree crown extraction.
- Boat detection required the **introduction of a regularizing term** that favors aligned ellipses with a **privileged direction**.
- The building footprint extraction was performed by favoring aligned and orthogonal rectangles.
- The proposed approach failed to detect tents since **the color is not taken into account**.

☹ **Significant computational cost** mainly due to the **high dimensionality** of the used objects.

Conclusion and futur work

Future work

- **Optimisation strategy** for the **estimation** method.
- **Automatic road and river** extraction by using line segments.
- **Hybrid model** to detect **different forms**.
- **Automatically** estimate boat orientation in order to promote after this estimation the ellipses oriented in the specified direction.
- **Exploit the color information for** tent detection.