# RARL2 : Realizations and Rational Approximation in $L^2$ norm

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# A Matrix Rational Approximation Problem

Given

- $F(z) \in L^2(\mathbb{T})^{p \times m}$ matrix-valued function, with nul Fourier coefficients of positive index

$$F(z) = \sum_{i=0}^{\infty} F_i z^{-i}$$

- $n$ positive integer

find $H$ rational, stable of McMillan degree $\leq n$ which minimizes

$$\|F - H\|^2 = \frac{1}{2\pi} \mathbf{Tr} \left\{ \int_0^{2\pi} (F - H)(e^{it})(F - H)(e^{it})^* dt \right\}$$

stable: poles inside the unit disk

# Stable Rational Functions

A rational matrix function $W(z)$, finite at infinity, admits a realization

$$W(z) \;=\; C\,(z\,I - A)^{-1}\,B \;+\; D$$

- easy to built one, not unique : for all $T$ invertible, $(TAT^{-1}, TB, CT^{-1}, D)$
- minimal realization: size $n$ of $A$ minimal
  McMillan degree= $n$
- minimal realization : poles of $W(z)$ = eigenvalues of $A$

Stable rational irreducible fraction :

$$f(z) = \frac{p(z)}{q(z)} \Leftrightarrow \deg p < \deg q = n, \quad |\text{roots of } q| < 1$$

# In practice

The function $F(z)$ can be given in one of the following forms:

- a realization

$$F(z) = D + C(zI - A)^{-1}B, \quad A \; N \times N$$

- a finite number of Fourier coefficients

$$F(z) = \sum_{i=0}^{N} F_i z^{-i}$$

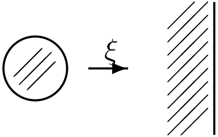
- some pointwise values on the unit circle.

$$F(e^{i\theta_k}), \; k = 1, \ldots, N$$

# Applications

- Model Reduction
  data = a realization of a finite order LTI system
- Identification from frequency data :

finite order LTI system $\leftrightarrow$ rational transfer function $H(z)$
input: $e^{i\omega}$ $\longrightarrow$ output: $H(i\omega)e^{i\omega t}$




$$\xi(z) = s = \frac{1+z}{1-z}$$

- Source detection

Big Issue : complete the band-limited data!

# Main advantages of the software

- it works for *matrix-valued* functions,
- a separation of the variables which allows to work with a compact set of parameters.
- a nice parametrization of inner functions represented by unitary realization matrices, which presents a lot of advantages:
  - it takes into account the *stability constraint*
  - it ensures identifiability
  - it is well-conditionned
- a recursive search on the degree which improves the chances to reach the global minimum.