EEG source localization by best approximation of functions

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Context

EEG sources are modeled by K dipoles, with positions S_k and moments $\vec{p}_k$.

With a quasistatic approximation, the electric potential V obeys $\nabla\cdot\sigma\nabla V=\sum \vec{p}_k\cdot\nabla\delta_{S_k}$

The electric potential is measured on scalp electrodes, and the air is supposed non-conductive.

Classical methods to estimate source positions and moments are unstable with respect to source number. There is hence a need for robust methods for dipolar source estimation.

Proposed method

Sources are regarded as the poles/branchpoints of functions defined inside the domain.

The best rational approximation method allows to estimate these poles/branchpoints, from which dipole positions and moments can be recovered.

The data is assumed to be known on the boundary of the inner surface, and in practise, this implies a preliminary Cortical Mapping step.

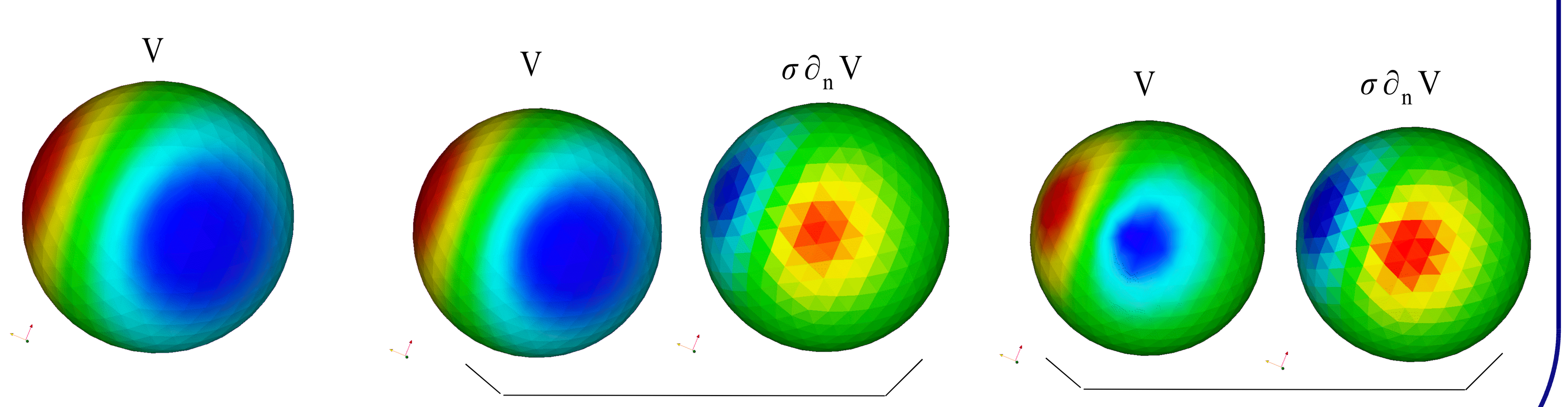
The method uses an analytical expression for the potential, hence is restricted to simple geometries (spheroidal).

Best rational approximation theory is based on complex analysis, and for this reason is applied on 2D slices. It has high robustness, leading to a well-posed 2D approximation problem.

EEG source localization

1 Cortical Mapping

Cortical mapping deals with the recovery of V and $\partial_n V$ on the cortical surface, from measurements of γ on scalp electrodes. Our Cortical Mapping solution uses the Symmetric Boundary Element Method, in which V (resp. $\sigma\partial_n V$) is discretized with piecewise linear (resp. constant) polynomials on each surface of the head model [1, 2].



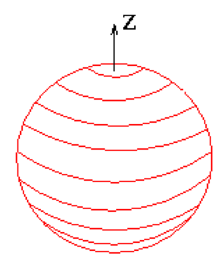
2 Outer source filtering with spherical harmonics

V may be corrupted by outside sources, hence it is considered to be known only up to a harmonic function.

From V and $\partial_n V$ on the boundary, filtering out the harmonic function coming from outside sources, by projection on spherical harmonics, yields f on boundary.

Consider a parallel slicing of the sphere, with an arbitrary normal direction z.

and f_m the restriction of f on a circle m.



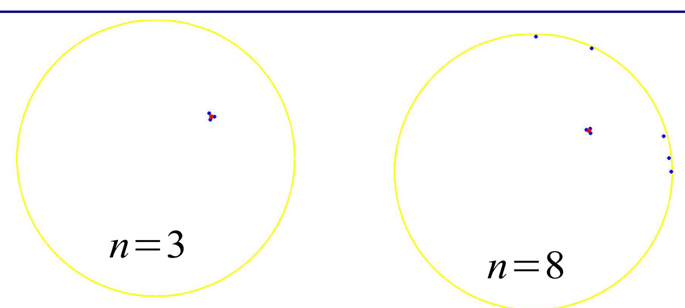
f_m^2 has
- a triple pole C_m in presence of one source.
- branchpoints $C_{k,m}$ with power 3/2 singularity in presence of several sources.

3 Best rational approximation in the disk

In a given slice, f has a pole C or branchpoints $C_1,\ldots,C_K$ which are to be recovered.

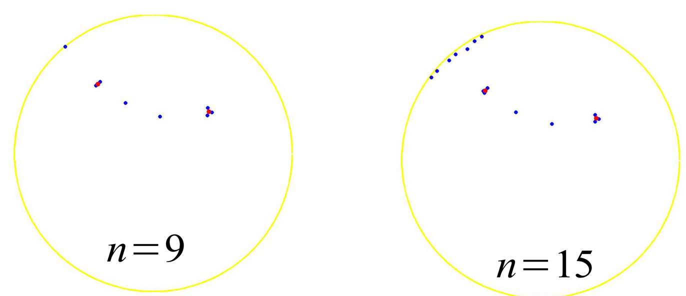
Best rational approximation minimizes the L^2 norm $\|f-p_n/q_n\|$ on the circle:

p_n, q_n are polynomials with $d^0 p_n \le d^0 q_n = n$, and the roots of q_n are inside the disk [3].



K = 1 source:

f has a triple pole (●) approximated by the first roots of q_n (●).



K = 2 sources:

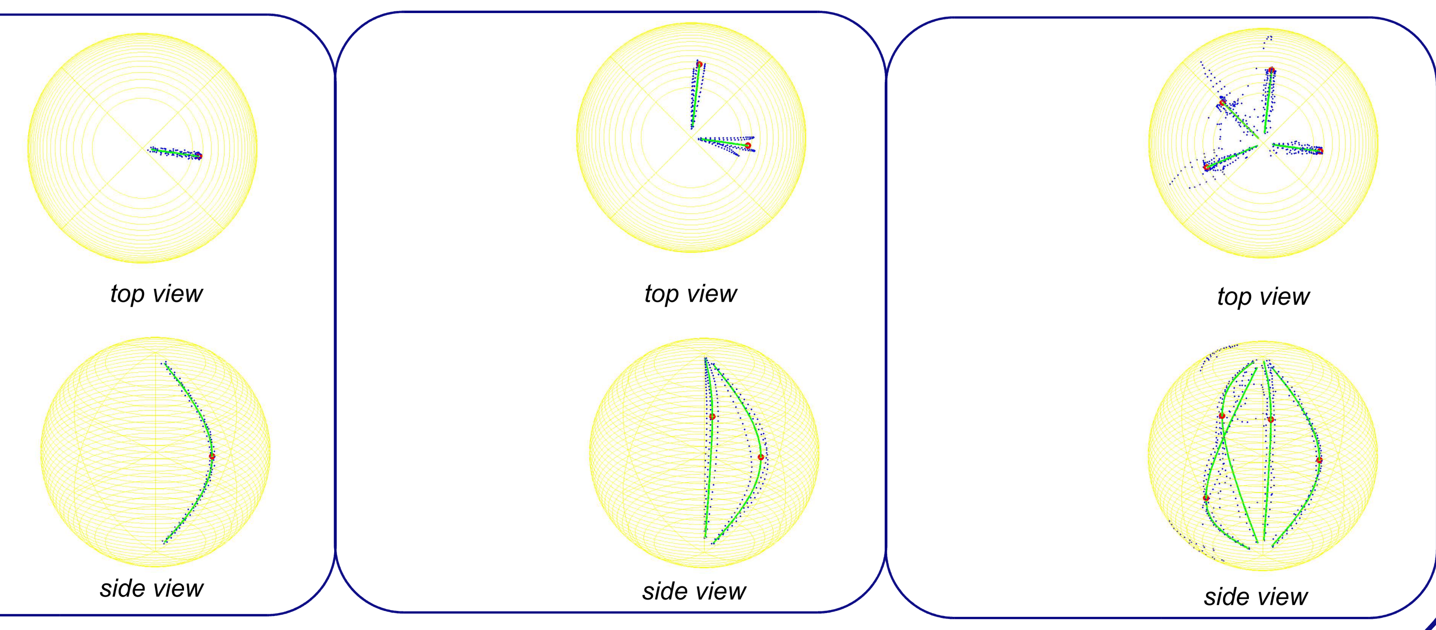
f has 2 branchpoints (●) linked by the first roots of q_n (●).

4 Dipole parameter estimation

$C_{k,1}\ldots,C_{k,M}$ and S_k are in a plane also containing the z axis, which allows to sort out the poles and branching points according to source number k = 1...K (see *top views* in figures below).

For fixed k, the source S_k is estimated by the pole/branchpoint at the slice m at which $\|C_{k,m}\|$ is maximal (see *side views* in figures below).

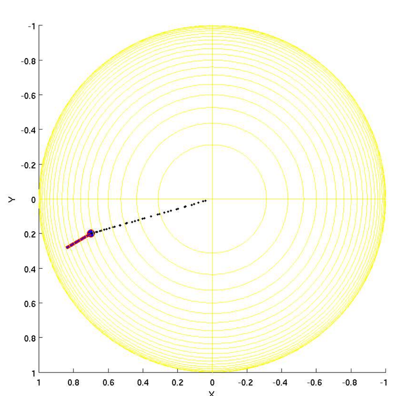
The moments are estimated by least squares, by using f , $C_{k,m}$ and the polynomial p_n [4].

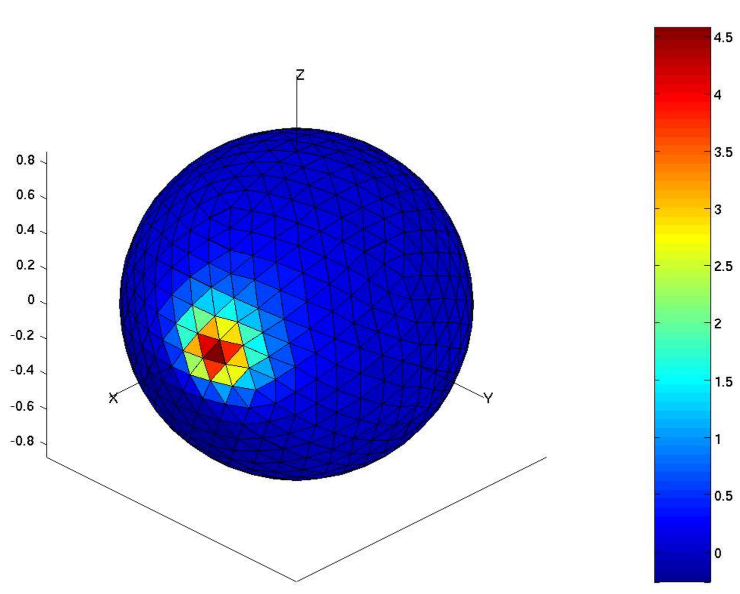


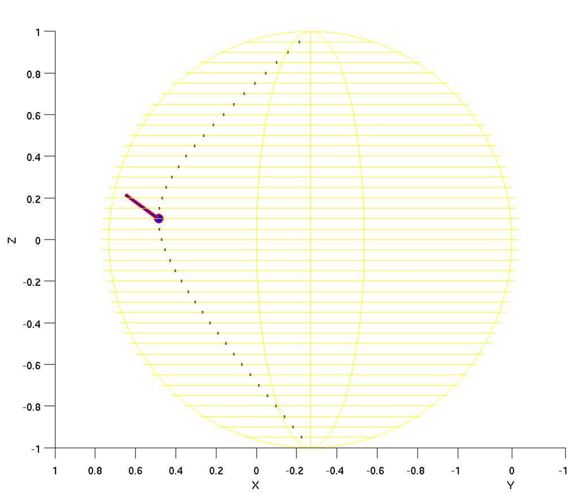
Results

with **analytic data** on the cortex

Real dipole ( ) and Estimated dipole ( ) coincide.

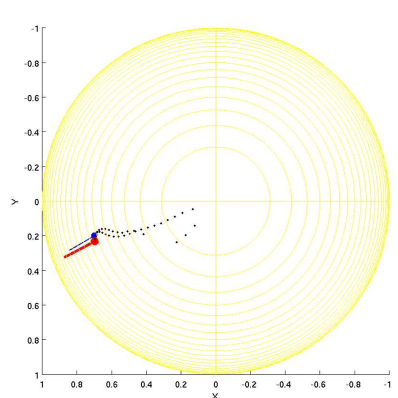


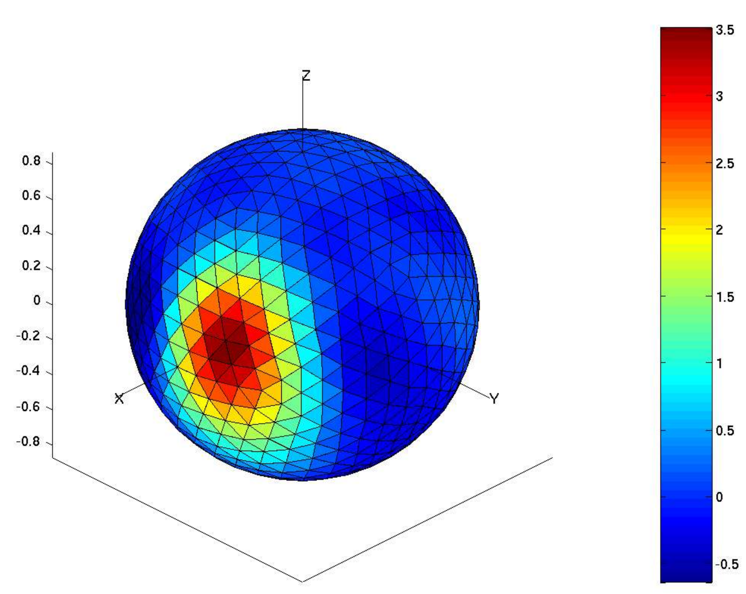


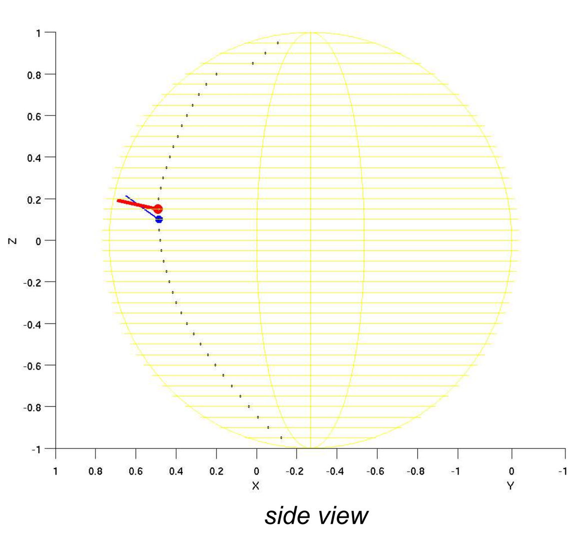


with **BEM cortical mapping data**

Real dipole ( ) and estimated dipole ( ) have good agreement.







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