APICS

Analysis and Problems of Inverse type in Control and Signal-processing

Proposal for a new project-team at INRIA Sophia-Antipolis

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2 Introduction

The present document is an abridged version of the "Document de proposition de création du projet APICS" (in French) [4] available from *http://www-sop.inria.fr/miaou*. The latter contains a comprehensive description of the research proposed by APICS and its foreseen applications. It also gives a thorough account of the connections with the former project MIAOU to which every member of APICS was taking part. However, two previous members of MIAOU are not participating in APICS.

With respect to INRIA's strategy plan, APICS will be a project-team in Applied Mathematics aiming, from the methodological point of view, at contributing to critical challenge number 4:

- Coupling data and models for simulation and control of complex systems.

[1] F. Ben Hassen, I. Fellah et M. Mahjoub are co-advised by the project-team and the ENIT (École N^{ale} d'Ingénieurs de Tunis) within the joint research project LAMSIN.

This makes APICS a natural candidate to join INRIA's scientific theme NUM A. However, the applications sought are geared a good deal towards the synthesis of analog-devices in telecommunications, and towards inverse source problems like those arising in Electro-Encephalography. Therefore, APICS would indirectly be a potential contributor to critical challenges number 1 and 7:

- To design and control the understructure of future networks and telecommunication services;
- To feed Information and Communication Sciences and Techniques into Medical Technology.

Marginally, APICS may also be concerned with critical challenge number 6:

- Modeling living material.

3 Overall description

The APICS project-team is the successor of MIAOU, whose main endeavor was to show that certain techniques from harmonic analysis, approximation theory, and differential geometry could be made effective in identification and control of dynamical systems. MIAOU mainly addressed identification issues for linear systems in the frequency domain (*i.e.* 1-D deconvolution), and stabilization issues for non-linear systems governed by ordinary differential equations; It also dealt with some structural aspects of control, like exact linearizability.

The scientific headlines of APICS are listed below. The list only features those topics that are among the main priorities of the project-team. They do not stand at the same stage of their development, and this is reflected by the exposition. For instance inverse problems is a relatively new field of investigation to the team that will require rather extensive theoretical developments, while frequency design is an older concern that was partly recast in light of the actual industrial applications of MIAOU. Nonlinear control stands somewhat half-way between theory and practice, in that some versatile techniques are available already but more specific tools must be developed to tackle the particular applications that APICS has in mind. Additional items of interest but less immediate concern to APICS may be found in [4].

Inverse potential problems (detailed in section 4). The new team will broaden its scientific scope in applied harmonic analysis, by considering inverse potential problems for elliptic equations in 2 and 3-D, with initial emphasis on the Laplacian. The thread is here to explore whether techniques developed by MIAOU in frequency identification, when the latter is regarded as an inverse problem for the Cauchy-Riemann equations, can be mimicked and adapted to other elliptic equations in dimensions 2 and 3. The applications that are sought comprise inverse source problems, like those arising in electro-encephalography, and inverse boundary problems as encountered in non-destructive testing from over-determined boundary data of diffusive phenomena.

APICS's approach rests on a blend of Harmonic Analysis and Approximation Theory which is fairly recent. This should be regarded as mid-to-long-term research on which some first progress is expected within three years or so. The ultimate goal is to built a constructive theory of (weak) recovery of potentials from the field they generate outside a neighborhood of their support, of which the geometric behaviour of best meromorphic approximants that we now begin to understand would be the 2-D (logarithmic) instance.

The team will collaborate with several academic groups on the subject, notably at ENIT (Tunis, Tunisia), CNRS-LENA (Paris, France), Univ. of Leeds (Leeds, UK), Univ. de Nice (Nice, France), Univ. de Provence (Marseille, France), Vanderbilt Univ. (Nashville, TN, USA), and also with the project-team ODYSSEE at INRIA (Sophia-Antipolis, France).

Frequency domain design and synthesis (detailed in section 5). APICS will still address deconvolution issues in the frequency domain, following the path opened up by MIAOU. This time, however, stress will be put on design rather than identification, up to the synthesis of physical parameters. As we mentioned already, these issues pertain to the field of elliptic inverse problems as well; but they involve in addition a substantial amount of system theory and matrix-valued function theory. The target applications lie with the design of certain telecommunications devices like output multiplexors and surface acoustic wave filters.
In this area, established techniques from Function Theory, Circuit Theory, and Optimization, should team up with Harmonic Analysis techniques and Computer Algebra methods of more fresh vintage. This is a short-to-mid-term objective whose feasibility could be assessed within one to two years, not withstanding the fact that new questions may arise. In the longer term, the ultimate goal is to capsulize a theory of frequency optimization into a numerical library that solves the main constrained extremal problems arising in band-limited design rational-exponential transfer (or scattering) matrices.

The team works jointly both with industrial and academic associates in the field, notably ALCATEL-SPACE (Toulouse, France), CNES (Toulouse, France), IRCOM (Limoges, France), LADSEB-CNR (Padova, Italy), Univ. of Maastricht and CWI (Netherlands).

Besides, the project-team looks forward to benefit from past experience in function theory to approach the issue of reconstructing planar domains from the sequence of their 2-D (complex) moments. Applications include inversion schemes of the (complex) Radon-transform which is relevant in tomography and geophysics.
Such moment problems have received growing attention in the past few years from the scientific community. They were included in APICS's research program because of tight connections with the techniques just mentioned, which lay hope for a contribution. As the project-team has no experience in this direction, it must be considered as a mid-term topic, on which contacts with academic partners was recently made (Univ. of California at Santa-Barbara, USA).

Nonlinear feedback control (detailed in section 6). The research of APICS in control dwells on that of MIAOU, but will focus on analyzing the performance of stabilizing feedback-laws as compared, say to optimal control. A test-case will be the orbit-transfer problem for satellites with low thrust, like those powered by ionic engines. This entails

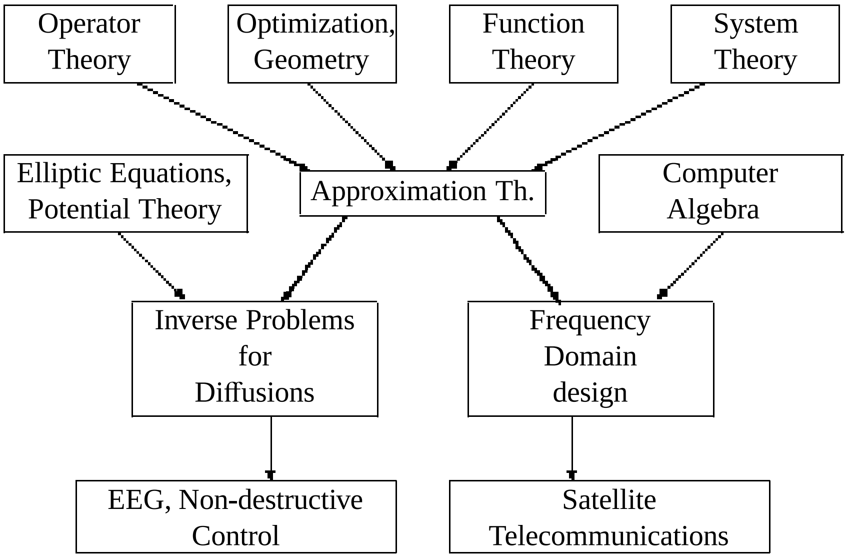

Figure 1: Analysis and Inverse Problems for Signal-processing

some involvement in celestial mechanics. Closed-loop stabilization begins to be understood in its relation to control Lyapunov functions, but the comparison with optimal control is more tentative. The test case of the orbit transfer problem should attend progress within two years, although there are many long-term issues. The ultimate goal would be to build a methodology of quantitative feedback design, in comparison with optimal control. The subject gives rise to cooperation both with industrial and academic partners, notably ALCATEL-SPACE (Cannes, France) and SISSA (Trieste, Italy).

In addition, APICS will pursue a long term study of local dynamical linearizability, including the so-called "flatness" property which is demonstrably useful, *e.g.* for motion planning. This piece of research however lies upstream with respect to applications, and aims at impinging on formal computation in connection with control. MIAOU recently made progress in a collaborative effort with some co-workers at Univ. Baumann (Moscow, Russia). This effort will be pursued in a long-range perspective.

The research in control conducted within APICS is expected to turn, after a few years, into an independent project, once some critical momentum is reached. The arrival of two doctoral and one post-doctoral fellows in the area should be appraised in light of this.

Software (detailed in section 7). For each item above, one explicit goal is to produce algorithms to be implemented as prototypical numerical codes. In addition, APICS wants to actualize the achievements of MIAOU in computational function theory by developing a toolbox that could be wrapped in standard mathematical software. This may offer the team an opportunity to contribute to the `Scilab` platform.

The two diagrams in figures 1 and 2 recap the main scientific connections on which the activity of APICS ultimately rests.

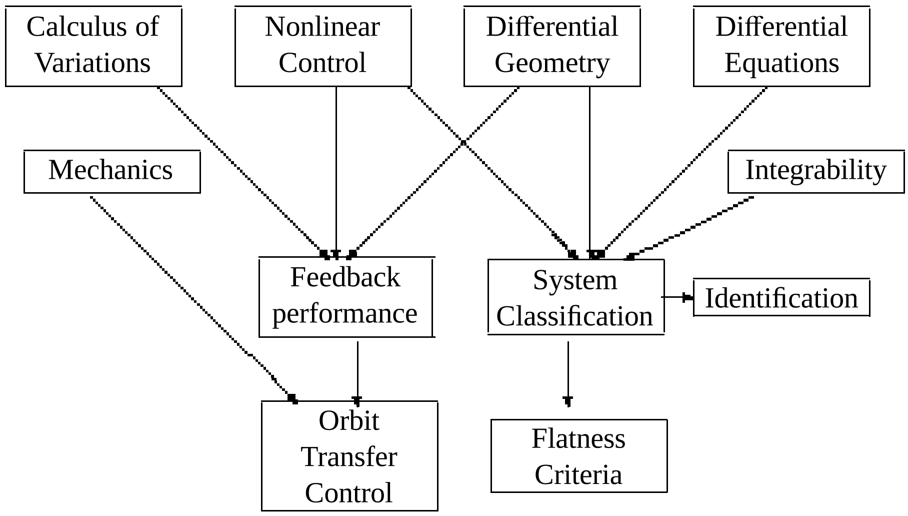

Figure 2: Analysis for control

4 Inverse potential problems

Participants: *L. Baratchart, F. Ben Hassen, M. Jaoua, I. Fellah, J. Leblond, M. Mahjoub, J.-P. Marmorat, J.R. Partington, E.B. Saff.*

Related sections of [4]: *3.5, 5.1, 5.2, 6.3.*

Recall that, given an elliptic operator, a potential is obtained by convolving some fundamental solution with a measure. The inverse potential problem consists in recovering the measure from the knowledge of its potential in some domain that does not contain the support. This kind of problem is very old as it dates back to Newton for gravitational fields. It appears naturally in geophysical or electro-magnetical settings [58, 57], and more generally in non-destructive control [51, 49] which is one of APICS's main application themes. Of course due to the phenomenon of *balayage* [73], the problem cannot be solved unless extra-assumptions are made on the measure.

For the Laplace operator in dimension 2, the complex derivative of a (logarithmic) potential is a Cauchy integral. This stresses strong links between potential theory and analytic functions [31, 58, 72], and indicates that inverse problems of the 2-D Laplacian may be recast in terms of complex analysis. In particular rational functions are derivatives of discrete potentials, so that rational (more generally meromorphic) approximation can be viewed as a means to discretize potentials with control on the Sobolev norms of the error. Likewise, approximation by functions analytic in a domain gives a way to identify potentials whose measure has prescribed support. These observations form the basis of APICS's approach to the subject.

A first instance of such a use of Approximation Theory already lies with MIAOU's approach to analytic continuation in a plane domain from *incomplete* boundary data, a most classical inverse problem for the Cauchy-Riemann equation [65]. It was extensively studied in the disk or the half-plane by the former project-team from the point of view of bounded extremal problems in Hardy spaces [16, 19, 20, 22, 41], see section 5 and [4, section 5.1.1] for a more precise description. Here it suffices to say that, given a function on a subset of the circle, the continuation issue can be formulated as the one of finding a best L^p-approximant on this subset out of the Hardy space H^p under some constraint (in norm or pointwise in modulus) on the complementary subset of the circle. In the context of inverse problems, which is presently our concern, Dirichlet and Neumann data furnish (an incomplete approximation of) the trace on the boundary of a domain of some analytic function to be recovered. The norm-constraint on that part of the boundary which is not accessible to measurements plays here the role of a regularization process (of Tikhonov type). This provides one with a constructive way of handling Cauchy extension in dimension 2. It is effective for instance to recover an exchange coefficient of Robin type (that models corrosion effects) from electrical measurements [39, 40]. Combined with the use of conformal maps, it also offers an approach to the geometrical inverse problem of recovering an unknown piece of boundary [48]. In the same vein, best uniform meromorphic approximation was proposed to extend incomplete boundary data while localizing a crack inside a domain [3, 32]. In this case, connecting the behaviour of the poles to the location of the crack is a non-trivial matter which is touched upon in the next paragraph.

APICS plans to approach similar issues in multiply connected domains, a situation which arises when the domain interfaces several layers of different conductivities, as in brain and head modeling [43, 47], or when it posseses a natural inner boundary like tubes or cylindrical domains [33]. The use of numerical conformal mapping will here be necessary in order to consider realistic geometries. The project-team also intends to study various generalizations of the norm-constraints involved, as well as other families of approximants for bounded extremal problems (see section 5).

A second example of how APICS wants to apply approximation techniques to 2-D inverse potential problems arises from source detection for the Laplacian. The idea is to compute a best (say L^2 or L^∞) meromorphic approximant, with at most n poles in a domain, to the boundary values of the (complexified) solution of some overdetermined Dirichlet-Neumann problem in that domain. The poles of the approximant are then expected to furnish some sort of a discrete approximation to the measure generating the potential.

This point of view was originally taken in [17, 66] for crack detection, and raises two types of questions. The first one concerns the actual computation of best approximants, a subject which has been much studied by MIAOU [10, 23, 25, 26, 28, 27, 30, 52, 8] up to effective numerical codes [54, 67], and in the broader context of matrix-valued approximation (see section 5). APICS will dwell on this available stock of algorithms, although need may arise to supply some more and to advance certain fundamental issues further, *e.g.* more general conditions that guarantee unimodality in such problems. The second type of questions has to do with the relations between the measure generating the potential (*i.e.* the singularities of the function to be approximated) and the poles of the meromorphic approximants (*i.e.* the singularities of the approximating function). This relation is of course trivial (although numerically efficient) when the measure to be recovered is already discrete, for instance when dealing with point-wise dipolar sources [9, 37]. It ceases to be trivial as soon as the measure is lumped. For analytic measures with 1-D support, the weak-asymptotics of that relation have been obtained [61, 28] in a hyperbolic analog to the seminal work [76, 53] on Padé approximation: the poles converge to the Green equilibrium distribution of the geodesic arc linking the endpoints of the support. Moreover, quantitative (non-asymptotic) bounds on the geometry of the poles are also available [15]. From the point of view of nondestructive control, this case corresponds roughly speaking to two monopolar sources or to a "sufficiently analytic" crack. Because the equilibrium measure charges the endpoints of the support, it yields a constructive means to locate them by computing meromorphic approximants.

For piecewise analytic measures with 1-D support, corresponding to a finite but arbitrary number of sources or piecewise analytic cracks, the asymptotic behaviour of the poles has been established only recently [29], in terms of equilibrium distributions on certain extremal contours for the Green potential. The history of such contours, formally introduced in [75], can be traced back to problems of Chebotarev and Lavrentiev [62, 63].

APICS will be busy implementing and testing these methods (see section 7), and also studying extensions of them. One such extension, of great importance from the constructive viewpoint, is to obtain quantitative bounds in the piecewise analytic case as well. Another extension is to obtain sharp error bounds in approximation (this requires additional assumptions on the domain and on the boundary data), that would in turn quantify the speed of convergence of the poles. Yet another extension, and a most exciting one, is to handle 2-D singular sets for which hardly anything exists. A conjecture can be found in [4, section 5.1.4].

Although 2-D algorithms are useful when dealing with thin plates or in the presence of cylindrical symmetry, most inverse problems practically occur in 3-D Euclidean space. APICS will consider such situations, starting with possible extensions of the previously described approach to higher dimensions. In this connection, it is common belief that techniques from complex analysis cannot apply to 3-D problems. Actually it is not always so, and the "selected application" below describes a way to deal with inverse source problems in a 3-D ball *via* meromorphic approximation in 2-D slices. In fact, the method would formally extend to those volumes whose plane sections are quadrature domains [1]. Due to the density of quadrature domains in the Hausdorff metric, this looks like an interesting line of development and a major thrust in APICS's mid-term research will be put in this direction. In another connection, the question whether optimal discretization of a potential yields some information on the support of the underlying measure is still very valid in 3-D. Of course, one no longer has the computational power of complex analysis at hand, but it is possible to believe that weaker tools, for instance from quaternionic analysis, could be of some value in this higher dimensional context. APICS plans to explore such issues in the long-term.

It is natural to ask how the techniques we just sketched position with respect to the thriving variety of methods to tackle inverse problems. A discussion may be found in [4], the main points of which are put into perspective below.
First of all inverse problems are most often ill-posed, hence some kind of regularization is needed whose effect is usually to translate them into optimization problems. The most popular approaches then are the so-called *direct* ones, where the unknown gets parametrized by a finite-dimensional family (*e.g.* a crack by consecutive line segments, a hole by a polygon, sources by a distribution of points, measures by dicrete sums of atomic squares, functions by nodes and splines, *ad lib.*); subsequently, one tries to minimize some

distance between the observed data and the numerical simulation of the direct problem for current values of the parameters, using for instance descent algorithms. Here, automated differentiation may be used to compute the gradient. Among the many references, let us quote [34, 35, 74, 56, 68, 55, 47, 51] for an illustration (not necessarily related to inverse potential problems). Such methods tend to be precise and easily recycled, but they require extensive computation and may not converge if the initial guess is inappropriate.
In specific cases, dedicated algorithms sometimes called *semi-explicit* methods have been developed, that do not require repeated integration of the direct problem. These are usually proved convergent under rather strong smoothness assumptions, and their computational cost is comparatively low. Limiting ourselves to inverse source or crack problems from overdetermined data of diffusive phenomena, let us quote for example [3, 36, 43, 64]. Semi-explicit methods are natural candidates to proceed in real time or to initialize heavier direct methods. The approach taken up by APICS pertains to the semi-explicit type. In 2-D, it has been compared rather favourably, albeit on preliminary simulated data, to the methods in [3, 49, 43]. It also performed reasonably well on first experiments with 3-D source problems in spherical geometry (see the "selected application" below). There the data were numerically simulated by the project-team ODYSSEE at INRIA-Sophia. Of course, several theoretical deepenings and many more experiments– this time against real data– will be needed to assess the value of such algorithms. This is part of the research program to be carried out by APICS. One final comment on this approach is perhaps in order; whereas the algorithms just quoted proceed by approximating, in various ways, the *solution* to the equations involved in the mathematical model of the problem, the technique we propose is based on approximating the *boundary conditions* of these equations furnished by the experiments.

Other inverse problems that the project-team may be led to consider concern the Beltrami equation (variable conductivity) in connection with quasi-conformal mappings, and the Helmholtz equation (inverse scattering) in connection with Hankel potentials. We refer the reader to [4] for more details.

Selected application: the inverse EEG problem

The inverse electro-encephalography (EEG) problem consists of localizing in the brain epileptic foci from electrical data measured on the scalp. A simplified spherical model will be used [43, 47], and the quasi-static approximation to Maxwell's equations will be made. Thus the head is assumed to be a ball in $\mathbb{R}^3$, made up of 3 disjoint homogeneous connected layers (for scalp, skull and brain) of known conductivity, interfaced by concentric spheres. In order to set up the source recovery issue as an inverse potential problem for the 3-D Laplacian, one needs first to solve two Dirichlet-Neumann problems, in order to propagate the data across the first two layers down to the innermost interface. This boundary problem is not so easy to solve in practice, but it is not our subject at present and we refer the reader to [60] for an algorithm. Assuming this step has been performed, we let Ω denote the unit ball in $\mathbb{R}^3$ and $\partial\Omega$ the unit sphere to face the following question :

given ϕ (current flux) and γ (measured potential) on the outer boundary $\partial\Omega$, find points $S_j\,,\ C_k \in \Omega$ and moments $\lambda_j \in \mathbb{R}$, $p_k \in \mathbb{R}^3$ such that the solution u to

$$-\Delta\, u = \sum_{j=1}^{m_1} \lambda_j\, \delta_{S_j} + \sum_{k=1}^{m_2} p_k \,.\, \nabla\, \delta_{C_k} \ \text{in}\ \Omega \quad \textit{satisfies} \quad \frac{\partial u}{\partial n}_{|\partial\Omega} = \phi\ , \quad u_{|\partial\Omega} = \gamma\,.$$

In the above equation, u is the electrical potential and the singularities $S_j\,,\ C_k$ are monopolar and dipolar point-wise sources.

Assuming for simplicity that the sources are in general position, namely none of them lies on the vertical axis and no horizontal plane contains more than one source. Then, it can be shown [18, 21] that the trace u_p of u on the circle T_p, cut out by the horizontal plane $\{x_3 = p\}$ on the sphere $\partial\Omega$, coincides with the trace on T_p of a function f_p which is analytic with branched singularities in that plane (although of course $f_p \neq u$ outside T_p since the restriction of u to the horizontal plane is not even harmonic in general). It turns out that there are as many singularities to f_p inside T_p as there are sources, and that exactly one of these singularities crosses a maximum of its modulus when the plane contains a source, in which case it coincides with that source lying in the plane. Therefore the 2-D techniques based on meromorphic approximation that were sketched above to spot the branched singularities of a holomorphic function (see *e.g.* [18]) can be used on a dicrete collection of horizontal planes to locate, by dichotomy, the height of the sources. Once this is done, the remaining parameters are easy to determine. Figure 3 displays the result of such a computation on simulated data [18]. Of course much work remains to be done in this connection both in 2 or 3-D : On the geometry of the poles for distributed sources, on the handling of more general geometries than the sphere in relation to quadrature domains, on “genuine 3-D” potential approximation (*e.g.* does a 3-D analog to the Adamjan-Arov-Krein Theory exist?), on variable conductivities in connection with the Beltrami equation, on incomplete data, magnetic data (from magneto-encephalography : MEG)... More details about these issues can be found in [4, sections 5.2,6.3].

5 Frequency domain design and synthesis

Participants: *L. Baratchart, P. Enqvist, A. Gombani, J. Grimm, J.-P. Marmorat, M. Olivi, F. Seyfert.*

Related sections of [4]: *3.1, 3.2 , 5.1, 6.1, 6.2.*

A substantial amount of the research effort by MIAOU was spent at the intersection between Approximation Theory and Linear System Identification. The key remark there was that the transfer function of a stable causal linear system can be regarded as a functional object with some analyticity properties. In the finite-dimensional case, the transfer function of a (causal) stable system is rational with poles in the open left half plane. In infinite-dimension, if stability is required to hold in the L^2 sense (L^2 inputs induce L^2 outputs) the corresponding functional space for the transfer function is the Hardy space H^∞ of the right half-plane. Likewise, $L^2 \to L^\infty$ stability means the transfer function is in H^2. In practice, one can estimate the value of a transfer function at the purely imaginary point iw from the steady state output of the system subject to a harmonic input of frequency

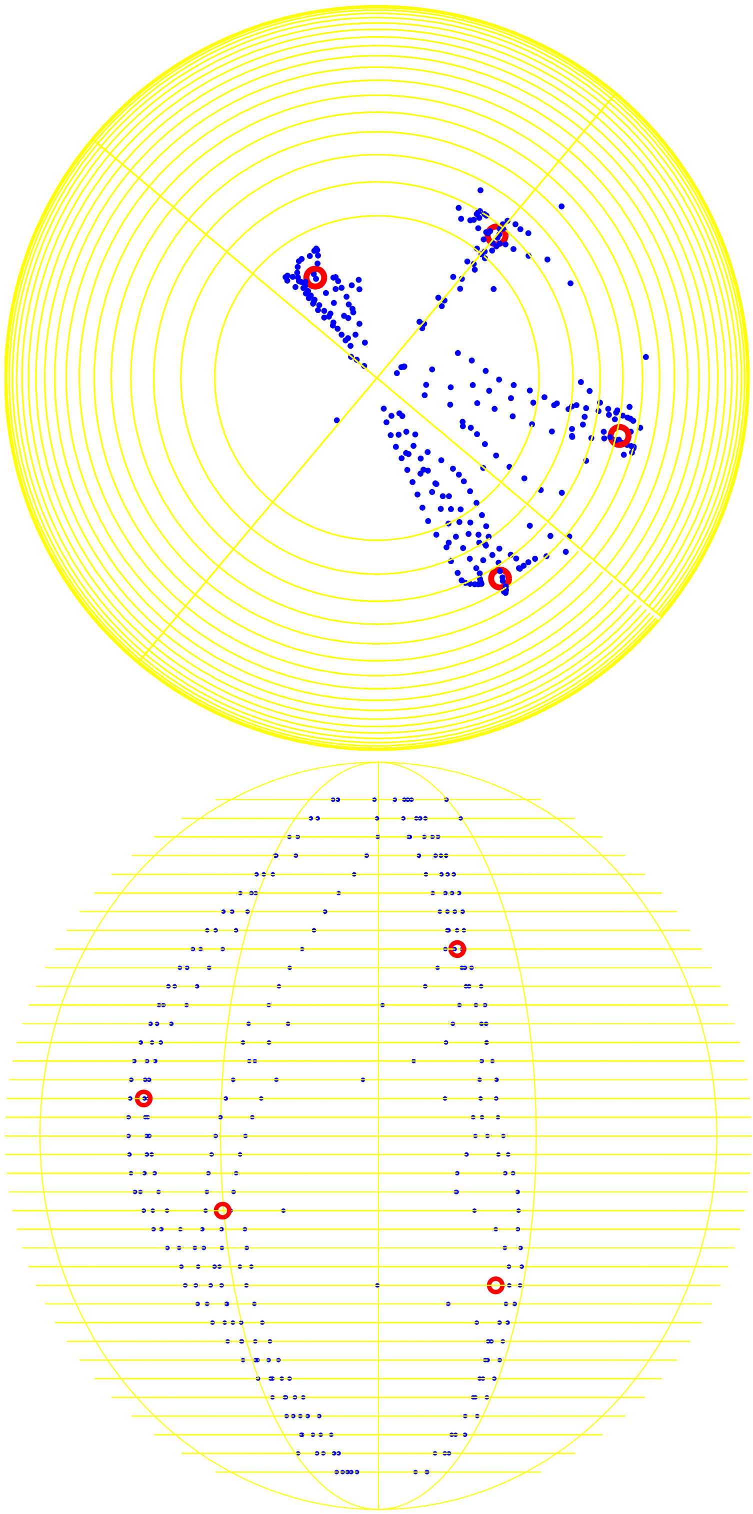

Figure 3: 4 dipoles (large bullets) in a 3-D ball, viewed from the north pole above and the equator below. In each horizontal plane, the poles (dots) of a best meromorphic approximant on the sliced circle, of degree 8, to the potential. Their clusters reach maxima in planes containing a source.

w. MIAOU's work resulted in a reconstruction process of the transfer function of a stable system from such frequency experiments.

In a first step the transfer function is sought as a member of a Hardy space H^p; in a second step, a stable (generally matrix-valued) rational approximation of fixed MacMillan degree is computed in order to obtain a finite-dimensional model. The first step led to the analysis of bounded extremal problems already mentioned in section 4 [16, 19, 20, 22, 28], where one typically looks for the best approximation of an L^p function defined on a subinterval I of the imaginary axis (the bandwidth) by H^p functions whose norm remains bounded on the complement of I in $i\mathbb{R}$. Constructive solutions were obtained in H^2 and H^∞, leading to software implementations in *C++* and `matlab`, see section 7.

The second step, namely matrix-valued rational approximation of given Mc-Millan degree, was mainly considered in the L^2-sense because of the underlying stochastic interpretation and the Hibertian framework it makes for. It led MIAOU to develop an optimization algorithm over the manifold of inner matrices of given Mc-Millan degree, which seems to be first of this kind. Charts were obtained from Schur analysis [2, 69], while the recursive structure of the criterion on the boundary of the manifold, consisting of inner matrices of lower degree, ensures convergence to a local minimum [10, 52] which is global if the data are near-rational of the prescribed degree [24]. Two versions of this algorithm were implemented in the `hyperion` software and `RARL2` software. The `hyperion` software is a *C++*-based platform with a powerful arithmetic and a Lisp-like command interpreter in which the function *arl2* performs the rational approximation, handling systems at the polynomial level. The `RARL2` software is a dedicated `matlab`-based program, in which the state-space description is used instead (see section 7).

The benchmark chosen by MIAOU to demonstrate the feasibility of this function-theoretic approach has been the identification of hyperfrequency filters, made of coupled resonant cavities, that are used in telecommunication satellites for channel multiplexing [14]. A longstanding cooperation with the CNES (Toulouse branch) resulted in the dedicated software `PRESTO-HF` that wraps both `hyperion` and `RARL2` into a package performing additional steps like delay compensation. `PRESTO-HF` was transfered to the Alcatel-Space company in Toulouse, and is now fully integrated in the design and tuning process there.

In the course of this mathematical technology-transfer, the need to recover coupling parameters between resonators from frequency measurements caused rational approximation to be followed by a constrained realization step. The latter was the starting point of the team's involvement into some convex optimization and computer algebra techniques. This brings APICS today to envisage synthesis problems for such frequency devices, with the function-theoretic techniques of MIAOU on the one hand (to perform frequency design), and the optimization and algebraic tools on the other hand (to compute the physical parameters). We will get a little more into details about this program in what follows.

Rational approximation

Matrix rational approximation will remain an important topic for APICS as it should be an essential component of frequency design. Although the algorithms developed by MIAOU may in part be considered today as tools, several generalizations need to be performed for the purpose of synthesis. Among them we quote the adjunction of pointwise constraints, (a necessary ingredient due to the passivity of the devices), and approximation in the hyperbolic distance (useful to handle chain scattering for output multiplexor design). Moreover, one also has to deal with exponential factors to account for the delays. This may require a deeper understanding of the geometry of inner transfer functions, in particular to improve the behavior of the approximation algorithm and the strategy used to chase the global minimum. In such issues, the influence of the parameters's choice should also be decisive.

Approximation problems with point-wise constraints - links with convex optimization

Filter design often faces rational or polynomial approximation problems where the admissible set is defined through a series of point-wise constraints on the modulus of the approximant. One aim of APICS is to tackle the resolution of such problems using at selected places some recent techniques from convex optimization. For example a new kind of extremal problems is now being studied by the project-team where the approximation is in L^2-norm while the constraint outside the bandwidth is in L^∞-norm [22]. This type of question is typically relevant when dealing with modulus-optimization of scattering functions of dissipative systems. When working on a finite-dimensional subspace of H^2 (*e.g.* in the polynomial case), it can be recast as a semi-infinite convex optimization problem, for which the use of interior point methods seems well-suited. However, the degree of the polynomials involved (several hundreds) is a major obstacle to this approach and APICS believes that the function-theoretic structure of this approximation problem (*e.g.* the duality in moment problems) might be decisive here in order to derive efficient algorithms. Conversely, various rational approximation problems occurring in filter design (*e.g.* those related to the famous Zolotariov problems [73]) can be recast as convex or quasi-convex problems where the admissible set is defined by an infinite number of point-wise constraints on the modulus of a polynomial. In this context, interior point methods exploiting the underlying structure of such problems seem promising. Let us also mention that the design of output multiplexers can be approached with similar ideas, in that it amounts to decide whether a scattering matrix obtained by chaining several filters in parallel can be construed to verify a series of point-wise constraints in modulus. The latter problem is no longer convex, but a relaxed version of it shows interesting connections with stable rational approximation under Schur constraints.

Parameterization issues

We mentioned already the role of parameters defining an atlas of charts in optimization problems on a manifold. Whereas such parameterizations are available for inner transfer functions after MIAOU's work, other classes must be considered. Indeed, the physical laws of energy conservation and reciprocity introduce subclasses of transfer functions which play an important role in filter design. These include J-inner, Schur (or contractive), positive real, and symmetric functions. From an algorithmic viewpoint, the parameterizations must take into account implementation facilities, numerical behavior and provide a strategy to choose an adapted chart for a given system.

In another connection, one must be able to handle systems having a particular state space form to account for the coupling geometry of the filters. However, the specifications concerning the device are typically expressed in the frequency domain (*e.g.* the return loss and group delay of the scattering matrix). It is therefore natural to split the design process into two steps; the first would consists in solving a rational approximation problem (for ex. of Zolotariov type) so as to determine a valid scattering matrix, the second would compute all possible "physical" realizations of the filter. For this splitting to work one needs to be able to derive a handy description of the set of admissible scattering matrices, and to solve a constrained realization problem. Past experience with coupled resonators has prompted an approach to this problem based on computer algebra. In this setting, the admissible set becomes an irreducible variety, and the constrained realization step amounts to find the zeros of some zero dimensional algebraic system. In collaboration with the INRIA team SPACES (Rocquencourt branch), APICS intend to make progress on this topic for which promising preliminary results concerning the exhaustive computation of feasible realizations have recently been obtained.

As should transpire from the above, parameterization issues will be a central topic to APICS, at least in the short-to-mid term. In our view, as far as frequency design is concerned, computer-algebra methods should team up with Schur analysis and classical tools from System Theory in order to pave the bridge between the frequency domain (where specifications are made) and the state-space domain (where the design parameters live).

Selected applications

Design of SAW filters

A surface acoustic wave (SAW) filter is made of electrical circuits printed on a piezzo-electric substrate; it transfers electrical power by means of acoustic waves propagation on the substrate. In this setting, two distinct types of energy are involved : the electric and the acoustic ones. In mobile and wireless communications, internal reflectors are used to reduce the losses inherent to this technology. It turns out that the design of such filters is quite challenging. The transfer function to be designed is a Schur function but it has a highly constrained structure. It is symmetric and its entries satisfy certain parity conditions that are still not well-understood. A realization can be obtained from the physical parameters, but this description doesn't help too much the optimization step. What is needed is a characterization, independent from the physical parameters, of the tranfer functions involved. In particular the electric transfer matrix of the filter, which is to be optimized, imbeds into a twice bigger lossless electro-acoustic matrix with an increase by 2 of the Mc-Millan degree. This gives rise to the following interesting mathematical question, connected to classical circuit theory :

What is the minimal increase of the Mc-Millan degree which is incurred when imbedding a symmetric contractive rational matrix S into a bigger symmetric lossless one?

From Darlington synthesis it is known that such an imbedding into a twice bigger symmetric lossless matrix is always possible at the expense of doubling the degree. This result is generically optimal, but it only indicates in the present case that electro-acoustic matrices are non-generic. In [13], necessary and sufficient conditions are given, in terms of the zeros of $I - SS^*$, for the imbedding to hold without increasing the degree. It is to be hoped that suitable refinements of that result will help characterizing the minimal Mc-Millan degree of the extension, and to parametrize in the frequency domain the electric transfer-function of a SAW filter as those that can be symmetrically embedded with an increase by 2 of the Mc-Millan degree.

A toolbox dedicated to filter synthesis

The upcoming work of APICS in frequency design and constrained realization theory will lead to a filter synthesis software toolbox for the industrial and academic public. Part of this toolbox will be developed in collaboration with the project SPACES, whereas potential users are companies like Alcatel-Space, Comdev or Tesat, as well as our academic partners CNES and IRCOM.

6 Nonlinear feedback control

Participants : *D. Avanessoff, L. Baratchart, A. Bombrun, J. Grimm, J.-B. Pomet, M. Sigalotti.*

Related sections of [4]: *3.6, 3.7, 5.3, 6.4.*

6.1 Optimal control and stabilization

Optimal control is a well established branch of the calculus of variations. What makes it an applied discipline is the need to actually compute the controls that produce an optimal behavior. However, computing one single optimal trajectory can be a tough numerical and conceptual problem, while the dependence of optimal controls on the state — known as optimal (feedback) synthesis— bears no regularity a priori and requires an even more difficult mathematical analysis, which is different for each system.

"Modern" engineering textbooks in Nonlinear Control Systems (e.g. [59], often considered as a reference) hardly mention optimal control, which is sometimes even not considered as part of automatic control. The emphasis is rather put on designing a **feedback control**, as regular and smooth as possible, satisfying some qualitative objectives : stabilization, disturbance attenuation, reaching a point or a target, in finite time or asymptotically...

Minimizing a cost should not be an obsession, but it is *very* relevant in some engineering problems. Since the robustness properties of a continuous feedback are also desirable, a link between the above two points of view is of considerable importance in engineering, while at the same time a challenging issue. Evaluating a posteriori a cost within a class of feedback strategies in order to perform some optimization on them is often known as "direct optimization" or sub-optimal control. Another approach is to get good enough an understanding of the optimal synthesis to attempt approximating it by a continuous feedback.

A **Control Lyapunov function (CLF)** is a (smooth) function that can be made a Lyapunov function (roughly speaking, a function that decreases along all trajectories, some may call this an "artificial potential") for the closed-loop system corresponding to *some* feedback law. This can be translated into a partial differential relation sometimes called "Artstein's (in)equation", see [6]. There is a definite parallel between a CLF that stabilizes a system[2], solution of this differential inequation on the one hand, and the value function of an optimal control problem for the system, solution of a HJB equation on the other hand. Now, optimal control is a quantitative objective while stabilization is a qualitative objective; it is not surprising that Artstein (in)equation is very under-determined and has, for instance, many more (if any) smooth solutions than HJB equation.

Designing a stabilizing feedback control from a smooth CLF can be made in a systematic way, but a CLF is also useful in itself, even if a stabilizing control law is known by other means, to study its robustness. Finally, it is quite tempting to use some knowledge on the value function of an optimal control problem to design a CLF that would be close to this value function so as to design sub-optimal feedback control laws in a more methodological way.

Objectives for a near future. Research in control conducted by MIAOU did not cover optimal control *per se*. However, in [44, 45], systematic ways to deform an object that is "almost" a CLF into a CLF were studied bearing in mind a parallel with optimal control. This emphasis will come to the fore in APICS. Let us draw more precisely two objectives.

1. *Quantifying feedback laws' performances.* If one has obtained, by a method that is in no way related to the cost to minimize, a (family of) feedback law(s) that meet some qualitative objective, one then has to evaluate, maybe optimize, its performance with respect to the cost. This can be done by rather empirical methods, based on simulations, but it is highly desirable to have some more systematic way to do it, if possible in a manner which is not too dependent on initial conditions.

2. *Characterizing optimal control problems whose value function is C^0-close to a smooth CLF.* This was the idea behind the work [44, 46] that studies uniform limits of CLFs, but it is only preliminary and needs to confront the study of the value function, either on generic optimal problems or in particular cases that are relevant to applications and display some additional structure, see below. The question as stated is very ambitious; APICS does not hope for a general answer, but believes it provides an adequate framework. Any piece of answer would be a significant advance in control already.

These questions are of general interest to nonlinear control, and their relevance is further evidenced by the importance they have in the application that we describe below. APICS's research on these matters will be both inspired by, and applied to this particular problem.

A space engineering problem: low thrust satellite orbital transfer. Space engineering and satellite guidance are a basic need of advanced telecommunication and networking. A crucial point in satellite engineering is to minimize the mass of fuel (ergol) to be taken on board in order to devote a larger proportion of the total mass to the payload,

[2] We take "stabilization" in a broad sense: reaching a possibly moving target, either asymptotically or in finite time, with some stability, i.e. little sensitivity to initial conditions or perturbations

like the hyper-frequency filters described in section 5. Electro-ionic engines use a magnetic field to expel particles (in this case, these are the fuel) at a very high speed. This much higher speed allows for a much higher efficiency than classical “chemical” engines but, due to the limited electric power on board, the thrust they deliver is much smaller. The very low magnitude of the control and the fact that it cannot be concentrated on small “burns” makes the design of the transfer a much harder control problem. In fact, no really satisfactory solution (at the same time robust, implementable and reasonable in transfer-time) is known, despite active research in the area.

This problem has a lot of structure. We hope to go as far as possible in the program described above on this case. Although it is unlikely that the optimal synthesis can be described explicitly, more can certainly be said than is presently known, and question 2 above makes a lot of sense.

APICS will devote a significant energy to this problem. It is the subject of a research contract with *Alcatel Space* (Cannes). The arrival of A. Bombrun (PhD student) and M. Sigalotti (Post-doc), both involved in this research, is boosting this topic in APICS. Note finally that there is ongoing research on low thrust orbital transfer in other institutions with which APICS will collaborate: *CAS, École des Mines*, and *Univ. de Bourgogne*.

6.2 (Dynamic) equivalence and linearization of nonlinear models

Motivations. They are two-fold.
- One is “nonlinear identification”. Linear identification became a prominent area of control science because the structure of linear models has been well-understood (although it is *not* a linear structure). These models suffice for many control applications, because first order approximations capture much of the local behaviour, but when a nonlinear model is needed no clear methodology is available for the set of nonlinear models is considerably larger... all known approaches are somehow heuristic. Studying equivalence of nonlinear models under suitable equivalence relations is at least one step towards a theory of nonlinear identification proper.
- From the point of view of control, assuming a nonlinear model is known (for instance based on the laws of physics) and used for control design, it is a recurrent question whether it can be “transformed” into a “simpler” model; everyone experienced that, very often, a clever change of variables enlightens a problem and leads to a solution.
The equivalence relations correspond to different transformations, i.e. more or less intricate “changes of variables” on both inputs and state, that may involve some additional dynamics to the system. Their degree of smoothness also plays a role [11, 12].

One objective: characterization of dynamic feedback linearizability, or “differential flatness”. This is a difficult mathematical problem, which is at the same time relevant to control theory. Original studies date back to the beginning of the 20th century (see [38] and others), and have been revived in the framework of control theory by authors who introduced the notion of “differential flatness” [50]. We refer to [4, sections 3.6 & 5.3.2] for a detailed explanation of the flatness property and of the problems arising when trying to decide whether a system is “flat”. The main difficulty is to decide whether a system of PDEs *that potentially depend on infinitely many variables* has a solution : no a priori bound is know in general on the number of variables which is needed.

Contributions by MIAOU to this issue may be found in [70, 5, 71], where a geometric formulation of the problem and a study in small dimensions is conducted. More recently, a notion of formal integrability in infinitely many variables was proposed in [7]. In a near future, it is reasonable to expect advances by combining this last notion with some new results by V. Chetverikov (U. Baumann, Moscow) [42]. These works are remarkably complementary. A collaboration has recently started, between these authors in a joint proposal to the Lyapunov Institute program. In parallel, a study in small dimension is still going on with the doctoral research of D. Avanessoff.

7 Software Policy

Participants: *J. Grimm, J.-P. Marmorat, M. Olivi, F. Seyfert.*

Related sections of [4]: *4.1, 4.3.*

Like most research teams in Applied Mathematics, APICS will develop numerical codes for at least two reasons : on one hand there is a need to validate algorithms derived from theoretical work, and to feed the theory back with the results of numerical experiments; on the other hand, just like theorems and algorithms are made public via papers in conferences and journals, programs have to be distributed to the outer world through external collaborations. While a program code may be born for the first reason, it may evolve because of the latter, and the programming activity is not the same in both instances.

The first instance partakes of the natural go-between relating the mathematical description of an algorithm and its coded implementation, which gives theory its proper role. The goal is then to develop performing, yet rather prototypical code. The second instance, where the goal is to develop an application intended for export, requires an extra-amount of discipline, follow-up and end-user assistance, implying some standardization and documentation, and often the addition of specific functionalities that are not necessarily by-products of the initial research program.

APICS will undergo both production types. Indeed, the work on Function Theory by MIAOU gave birth to several algorithms dealing with interpolation and extrapolation of frequency-based data through the solution of bounded extremal and rational approximation problems. This led to numerical codes, the *C++*- based `hyperion` software, (exported to IRCOM, the CNES (Centre National d'Études Spatiales), some universities, and sold to Alcatel-Space in Toulouse), or else the `RARL2` software (`matlab` based, delivered to Alcatel-Space in Toulouse and to IRCOM), and finally `PRESTO-HF`, a code integrating both `RARL2` and `hyperion`.

The issue facing APICS is now two-fold : how to capitalize on the algorithmic achievements of MIAOU on the one hand, and which software development policy to adopt for the future on the other hand. Moreover, since numerical experiments take a long time, several algorithms designed by the MIAOU team are still to be implemented. In this connection, certain topics must be clarified regarding complexity, precision, and their mutual links (in some cases, a slower, but more precise algorithm can be faster than a theoretically fast algorithm). Also, many computations can be performed in bases that are numerically unstable (so that high precision libraries are required) but for which few coefficients are required in order to represent the objects of interest, so that the program can be globally efficient. On the other hand, optimizing on a non-trivial manifold like `hyperion` and `RARL2` do is rather unusual, and APICS could provide template codes for this task.

In another connection, a number of new prototypical codes will be needed to handle APICS's developments in inverse problems and satellite control. Some will deal with conformal mapping and spherical harmonics expansions, other with optimization and numerical integration of differential equations. Altogether, the long-term strategy is to develop mainly small programs, interfaced with `matlab`, and possibly with `Scilab`.

8 Collaborations

8.1 Academic Partners

Within INRIA: project-teams CAFÉ, CAIMAN, COPRIN, GALAAD, ICARE, and ODYSSÉE at Sophia-Antipolis; at other branches project-teams SPACES.

Regionally: CMA (École de Mines, Sophia-Antipolis), UNSA (Math. dept.), Observatoire Nice Côte d'Azur, Univ. de Provence (LATP, Marseille), CEMAGREF (Montpellier).

Nationally: CAS (École de Mines, Fontainebleau), IRCOM (Limoges), UTC (Compiègne), Univ. de Lille I, Univ. de Bourgogne (Dijon), Univ. de Besançon, Univ. de Bordeaux I.

Internationally: LAMSIN-ENIT (Tunis, Tu.), T.F.H. Berlin (Ger.), Univ. Szeged (Hung.), LADSEB-CNR (Padova, It.), Vanderbilt Univ. (Nashville, USA), Michigan State Univ. (East Lansing, USA), Univ. Beer Sheva (Isr.), Univ. Leeds (U.K.), Univ. Maastricht and CWI (Neth.), Polish Ac. Sc. (Warsaw, Pol.), SISSA (Trieste, It.).

8.2 Grants

1. ACI *Masse de données* "OBS-CERV," jointly with the project-teams CAIMAN, ODYSSÉE (INRIA-Sophia Antipolis, ENPC), UNSA (lab. Dieudonné), CEA, CNRS-LENA (Paris), and several Hospitals, 2003-2006 (inverse problems in EEG).

2. Regional council PACA: postdoctoral grant, exchange support with SISSA (Trieste, It.).

3. NATO CLG (Collaborative Linkage Grant), PST.CLG.979703, "Constructive approximation and inverse diffusion problems," with Vanderbilt Univ. (Nashville, USA) and LAMSIN-ENIT (Tunis, Tu.), 2003-2004.

4. Member of the NSF EMSW21 Research Training Group comprising INRIA-Sophia Antipolis and Vanderbilt University (Nashville, USA).

5. Marie-Curie EIF (Intra European Postdoc. Fellowship) FP6-2002-Mobility-5-502062, (24 mois, 2003-2005).

6. Marie Curie Multi-partner Training Site HPMT-CT-2001-00278 “Control Training site,” 2001-2005.

7. STIC: INRIA-Universités Tunisiennes, with LAMSIN-ENIT (Tunis, Tu.), 2004.

8.3 Industrial grants

Currently running:

- CNES Toulouse (Microwave filters),
- Alcatel-Space Toulouse (Microwave filters),
- Alcatel-Space Cannes (orbital satellite control).

Previously running and which may be reconducted:

- Thalès (surface acoustic waves filters),
- Alcatel CIT Marcoussis (control of signals regeneration devices in optical fibres, licence deposit in Sep. 2003).

9 Knowledge transfer

9.1 Teaching and training

In 2003-2004, course at the “DEA Géométrie et Analyse”, LATP-CMI, Univ. of Provence (Marseille).

Mathematics teaching at secondary school (cycle 12-15 years, Montessori school “les Pouces Verts”, Mouans-Sartoux).

Member (correspondant: J.B. Pomet) of the Marie Curie Multi-partner Training Site HPMT-CT-2001-00278 “Control Training site”, 2001-2005

Member (correspondants: L. Baratchart and B. Mourrain) of the NSF EMSW21 Research Training Group formed by INRIA-Sophia Antipolis and Vanderbilt University (Nashville, USA), 2003–2005.

9.2 Activities in the Scientific Community

The members of the project-team sat on the Scientific and Organisation committee of the Ecole thématique d’été CNRS-INRIA, *Analyse Harmonique et Approximation Rationnelle: leurs rôles en théorie du signal, du contrôle et des systèmes dynamiques*, Porquerolles, september 2003.

L. Baratchart is on the Editorial Board of *Computational Methods and Function Theory*.

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