Extraction of Coupling Parameters For Microwave Filters: Determination of a Stable Rational Model from Scattering Data

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Abstract — **We present a method to derive a rational model from scattering data for electrical parameter extraction. Unlike other methods, the stability and the MacMillan degree of the rational approximation are guaranteed. In order to improve the usability of our method in computer-aided tuning , we also present an algorithm performing automatic reference plane adjustment. Results obtained on a 10th order dual mode IMUX filter are presented.**

I. INTRODUCTION

Extracting coupling parameters from measured or simulated scattering data of filters can be very effective to reduce the cost of hardware and CAD tuning. However, the direct approach which consists in feeding to a generic optimizer the function evaluating the scattering matrix from the coupling parameters, in order to fit the data, often depends on a favorable initial guess and substantial efforts are currently being spent to design more robust methods.

Another approach consists in first deriving a rational model for the data. In a second step, the coupling parameters are extracted from this rational model using classical design methods [1]-[2]. Recent publications [3]-[4] have advocated the use of the Cauchy method to compute the rational model. Let us point out three major problems encountered in this direction:

- there is no guaranty on the stability of the rational model, i.e. the derived model can have unstable poles;
- there is no control on the MacMillan degree of the model (the number of circuits in the equivalent low-pass model), i.e. the residues of the corresponding rational matrix will usually not be of rank 1, whereas this is mandatory in the electrical model (see [2]);
- no constraint is imposed to the model outside the frequency band of measurement, which may result in unrealistic behavior there.

To overcome these difficulties, the usual trick consists in forming the stable lossless rational model matching the transmission and reflection zeroes computed by the Cauchy method. Specifically, this amounts to forget about the denominator computed by the Cauchy method and replace it by the one computed from the numerators using spectral factorization. Unfortunately, nothing ensures that the derived lossless rational model will fit our data, which in turn can lead to a loss of accuracy of the whole parameter extraction procedure.

We present here a method to derive a stable rational model of prescribed MacMillan degree from scattering measurements. As the determination of the delays caused by access devices can be quite a laborious task, our method also includes, as a preliminary step, an automatic reference plane adjustment.

II. AUTOMATIC REFERENCE PLANE ADJUSTMENT

We denote by $(w_i, S_{1,1}(w_i), S_{2,1}(w_i), S_{1,2}(w_i), S_{2,2}(w_i))$ the scattering measurements after low-pass transformation. The low-pass model, including delay components assumes the following form:

$$\frac{1}{q}\begin{pmatrix} e^{j a\, h(w)} p_{1,1} & e^{j\frac{(a+b)}{2}h(w)} p_{1,2} \\ e^{j\frac{(a+b)}{2}h(w)} p_{2,1} & e^{j b\, h(w)} p_{2,2} \end{pmatrix} \quad (1)$$

where h(w) is the transformation which maps normalized frequencies to high frequencies. The $p_{i,j}$ and q are polynomials defining the rational scattering matrix associated to the low-pass equivalent circuit. In order to reduce the problem to pure rational approximation, we first need to determine the real numbers α and β.

For this, let us select a subcollection of measurement indices according to the following rule:

$$\mathrm{I} = \{i, |\, w_i \,| > w_c\} \qquad (2)$$

where w_c is chosen sufficiently large for the moduli of $S_{1,1}$ and $S_{2,2}$ to behave smoothly when $|w| > w_c$; this of course entails that the broadband, where measurements are made, is somewhat larger than the equiripple bandwidth. Now, at higher frequencies, far off the pass-band, one can reasonably expect a good approximation of the rational components in (1) by the first few terms of their Taylor expansion at infinity. Letting n_c denote selected number of Taylor coefficients, which is to be viewed as a design parameter of the algorithm, we define the cost function:

$$y(t) = \min_{(a_0,..a_{n\_k}) \in C^{n_c+1}} \sum_{i \in I} | \sum_{k=0}^{n_c} \frac{a_k}{w_i^k} - S_{1,1}(w_i) e^{-jt h(w_i)} |^2 \quad (3)$$

and the number α we are seeking will be assigned to the value where y reaches its minimum.

The underlying idea here is that, away from the pass band, the filter will be close to a polynomial in $1/w$ if, and only if, the delay components are properly compensated. In practice, the values w_c=2.5 (we work throughout with normalized frequencies) and n_c =4 seem to give very satisfying results when the broadband is three times bigger than the pass band.

For practical implementation of this method , note that evaluating y is a standard least squares problem, whose solution requires a matrix inversion of size $(n_c+1)x(n_c+1)$. This matrix does not depend on τ hence the inversion needs to be performed only once for. This allows us to perform the minimization in a brute force manner: we first select a priori bounds for the delay values (recall that α can be seen as the time it takes for the signal to travel from the source to the reference plane of the filter) and proceed by exhaustive evaluations leading to the determination of α within a prescribed tolerance. To determine β we proceed in the same manner using the measurements of $S_{2,2}$ instead of those of $S_{1,1}$.

III. Projection On A Set Of Stable Causal Systems

In addition to estimating the delay, the above method provides us, by means of the Taylor expansion it computes, with a completion of our data outside of the broadband, all the way to infinity. In what follows, we shall improve this completion by making use of two basic properties our identified model should possess, namely causality and stability.

In terms of rational functions, these properties are equivalent to the fact that poles are located at finite distance in the open left half plane. The latter is in turn equivalent to the fact that the rational function is analytic in the closed right half plane including at infinity. We now imbed such rational functions in a larger space of analytic function on the right half plane which allows us (thanks to its Hilbert space structure) to handle causality and stability in a convenient manner.

The space we have in mind is the Hilbert space of analytical functions on the open right half plane whose $L^2(dw/(1+w^2))$-norm remains uniformly bounded on vertical lines [5]. This space is one of the so-called Hardy spaces of the right half-plane, denoted by H^2. We also define the space L^2 of all complex functions defined on the imaginary axis whose modulus to the square is integrable against the measure $dw/(1+w^2)$, and we endow it with the the L^2 norm:

$$\| f \|^2 = \int_{-\infty}^{+\infty} \frac{| f(iw) |^2}{1+w^2} dw . \quad (4)$$

An important fact is that each member of H^2 can be identified with its trace on the imaginary axis (the trace exists as a non-tangential limit, see [5]). This allows one to consider H^2 as a subspace of L^2. We let G^2 be the orthogonal complement of H^2 in L^2. Note that, by construction each L^2 function can be decomposed as the sum of a function in H^2 (which is called its **stable part**) and a function in G^2 (that can be considered as its **unstable part**).

Consider now that our data have been compensated as explained in the previous section and that each $S_{i,j}$ is a function defined on the broadband J=[min(w_i),max(w_i)], its value between two measurement points being obtained using, say spline interpolation. Using again the hypothesis that our rational model should behave nearly like a polynomial of order n_c in $1/w$ for $|w|>w_c$, we consider the following optimization problem:

$$\min_p \sum_{k \in I} | S_{1,1}(w_k) - p(\frac{1}{w_k}) |^2$$

$$\begin{cases} p \in C_{n_c}[x] \\ \| P_{G^2}(S_{1,1} \vee p) \|^2 \leq E_c \\ \forall w \in J_c \quad | p(\frac{1}{w}) |^2 \leq 1 \end{cases} \quad (5)$$

In what precedes, J_c is the complement of J and ∨ denotes the concatenation operator, so that $S_{1,1} \vee p$ is the function defined by $S_{1,1}$ on J and by p(1/w) on J_c. Moreover P_{G^2}, denotes the orthogonal projection from L^2 onto G^2 and $C_{nc}[x]$ is the set of polynomials with complex coefficients whose degree is less or equal to n_c.

In words, our optimization problem (5) reads: find the polynomial completion which best fits the data on I under the constraint that:

- the complemented data have an anti-stable part whose L^2-norm to the square is less than Ec
- the modulus of the completion remains bounded by 1 on J_c.

The latter constraint is meaningful as our filter is passive. In addition for the diagonal terms $S_{1,2}$ and $S_{2,1}$

one can prescribe zeros at infinity by imposing zeros at 0 for the polynomial p.

As to the solution of problem (5), it can be shown that as soon as the number of measurement points in I exceeds *(n_c+1)* the cost function is strictly convex. The admissible set defined by the constraints in (5) is easily shown to be convex. This two remarks leads two the fact that (5) has a unique optimal solution unless its admissible set is empty. Let us mention here without proof that the following family of functions

$$\left(\frac{s-1}{s+1}\right)^k, k>0 \qquad (6)$$

is an orthogonal basis of G^2 that can be extended to an orthogonal basis of L^2 by letting k range over negative integer as well, so the matrix of the orthogonal projection P_{G2} is easily expressed. Concerning the last constraint in (5) we choose do discretise it, which of course entails an approximation that can, however, be controlled by classical theorems. All this allows one to regard (5) as a convex quadratic optimization problem (in the coefficients of p) which can be tackled by classical Lagrangian techniques [6]. For n_c=4 its resolution (Pc, 600 Mhz Pentium) takes less than 2 seconds.

In order to ensure that the admissible set of (5) is not empty we simply solve in the same manner the following problem:

$$\min_p \left\| P_{G^2}(S_{11} \vee p) \right\|^2 \qquad (7)$$
$$\begin{cases} p \in C_{n_c}[x] \\ \forall w \in J_c \; |\, p(\frac{1}{w}) \,|^2 \le 1 \end{cases}$$

and denote by E_{min} its optimal value (note that the admissible set for this problem is never empty as it contains p=0). This gives an easy characterization of the solvability of (5), namely: $E_c \ge E_{min}$.

If $p_{i,j}$ are the polynomial completions computed by the latter method we define

$$F_{i,j} = P_{H^2}(S_{i,j} \vee p_{i,j}) . \quad (8)$$

Those functions can be seen as the causal, stable projections of our initial data; note that, by construction, their L^2 distance to the data is less than the square root of E_c.

IV. Stable Rational Approximation Of Given MacMillan Degree

If n_f is the order of the filter (i.e the number of coupled circuits considered in the low pass equivalent circuit) we now consider the following rational approximation problem:

$$\min_H \||| F - W |||^2 \; (= \left\| \mathrm{trace}(F-W)(\overline{F-W})^t \right\|) \qquad (9)$$
$$\begin{cases} W \text{ rational 2x2 matrix} \\ \mathrm{Deg}(W) \le n_f \end{cases}$$

where the degree of a matrix is the Mc-Millan degree. We denote by W_{opt} the optimal solution of (9). Note first that, thanks to the fact that F is defined on the whole imaginary axis, W_{opt} cannot have purely imaginary poles. Suppose now that W_{opt} has some strictly unstable pole. Its partial fraction expansion decomposes as the sum of a stable part W_s (corresponding to the stable poles) and an unstable one W_u (corresponding to the unstable poles). But the orthogonality of W_u with H^2 (to which the elements of F do belong, thanks to the preceding step) indicates that W_s would be a better rational approximation to F than W_{opt}. This implies that:

- W_{opt} is stable
- In any algorithm constructing a minimizing sequence for problem (9) (typically a gradient algorithm) it is never favourable to let one or several poles become unstable. In other words, for such algorithms, we can restrict quite naturally to stable rational functions (the constraint of stability is never active).

For the practical implementation of such an algorithm there remains to find a tractable parametrization for the rational matrices of given MacMillan degree. This problem has been widely studied in [7]-[8]. Getting into details here would deserve a paper on its own but what can be said briefly on this topic is:

- Schur parameters allow one to nicely parametrize stable rational matrices of given MacMillan degree (such parameters are interpolation values from which the rational matrix is recovered explicitly).
- This in turn offers a way to parametrize all stable rational matrices of given degree in connection with (9).

Those ideas were implemented in two gradient based rational approximation engines called hyperion [9] and RARL2 [10] (the two software differ in their choice of Schur parametrization).

V. Parameter extraction of an Imux filter of order 10

We implemented this three steps approximation algorithm (delay detection, completion, rational approximation) as a matlab toolbox. We present here the results obtained on a 10th order Imux filter realized using 5 dual mode cavities (f_0 = 3.727 GHz, Bw= 44.5 Mhz).

Figure.1 shows the data, the compensated data, as well as the polynomial completion. Ec, the maximal norm of the anti-causal part (5), was set here to 0.3% of the L^2 norm of the data. Figure.2 shows the computed rational approximation. Note that the theoretical filter which couplings are shown in Table.1 has 6 transmission zeros: 4 at the border of the pass-band and 2 real opposite zeros used to adjust the group delay. Finally the extracted couplings are shown in Table.1.

The later measurements arise from a tuning session at the laboratory of Alcatel Space. After each parameter extraction phase, corrections were applied to the filter's tuning screws and irises. Convergence to the desired filter response was obtained in five such iterations.

VI. CONCLUSION

A complete strategy for deriving a stable rational model of given MacMillan degree from scattering data has been presented. The derived computer aided tuning method has shown to be very effective in practice, in particular when dealing with high order filters.

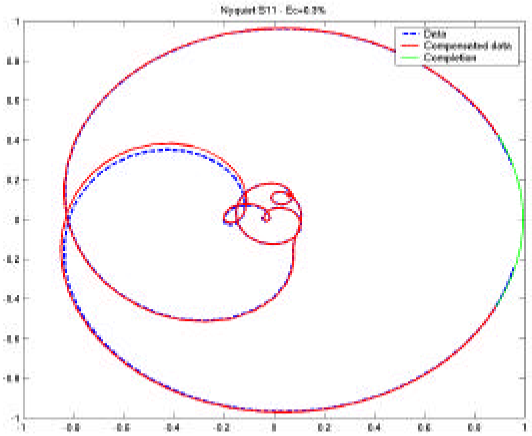

Fig.1: Data, Compensated Data, Completion

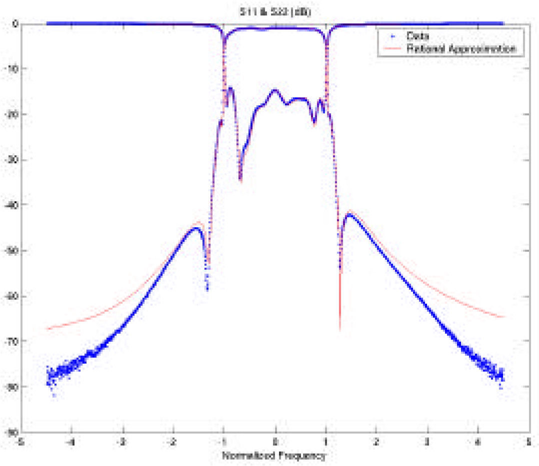

Fig.2: Compensated Data, Rational Approximation

	Theo.	Extract.		Theo.	Extract.
Z_{in}	1.075	0.98	$M_{1,2}$	-0.845	-0.79
Z_{out}	1.075	1.08	$M_{2,3}$	-0.538	-0.53
$M_{1,1}$	0	-0.06	$M_{3,4}$	-0.591	-0.56
$M_{2,2}$	0	-0.06	$M_{4,5}$	0.521	0.52
$M_{3,3}$	0	-0.01	$M_{5,6}$	-0.516	-0.50
$M_{4,4}$	0	-0.00	$M_{6,7}$	-0.546	-0.53
$M_{5,5}$	0	0.02	$M_{7,8}$	0.351	0.22
$M_{6,6}$	0	-0.02	$M_{8,9}$	-0.903	-0.96
$M_{7,7}$	0	0.01	$M_{9,10}$	-0.664	-0.52
$M_{8,8}$	0	-0.07	$M_{1,4}$	-0.080	-0.06
$M_{9,9}$	0	0.08	$M_{5,8}$	-0.029	-0.01
$M_{10,10}$	0	0.06	$M_{7,10}$	-0.532	-0.67

Table 1 : Theoretical and extracted couplings

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