Spring school & workshop on inverse problems and approximation techniques in planetary sciences, 16-18 May 2018

Inria Sophia Antipolis

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Program, titles & abstracts, list of participants

	Wednesday, May 16		Thursday, May 17		Friday, May 18
		9:30	N. Schneider (30 + 5 min)	9:30	D. Ponomarev (45 + 5 min)
		10:05	B. Kretz (30 + 5 min)	10:20	*pause*
		10:40	*pause*	10:40	J. Heleine (30 + 5 min)
		11:10	F. Ferranti (45 + 10 min)	11:15	discussions closure session
		12:05	*lunch*	12:05	*lunch*
14:00	C. Gerhards (60 + 10 min)	14:00	E. Lima (45 + 10 min)		
15:10	Y. Quesnel (30 + 5 min)	14:55	K. Mavreas (45 + 10 min)		
15:45	*pause*	15:50	*pause*		
16:15	D. Clamond (45 + 10 min)	16:20	L. Baratchart (45 + 10 min)		
		19:30	*dinner*		

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informatics mathematics Inria

INVESTISSEMENTS D'AVENIR INITIATIVE D'EXCELLENCE

UCA J.E.D.I. UNIVERSITÉ CÔTE D'AZUR

Abstracts

Regularization issues for inverse magnetization problem

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We consider the problem of recovering a magnetization distribution supported in some known subset in 3-D space from knowledge of the field it generates in some disjoint subset thereof. We are interested in the case where the support is a piece of a plane, which arises naturally in scanning magnetic microscopy for rock samples, but we keep an eye on volumic samples as well. When the magnetization is modeled by a compactly supported measure, we present some regularization schemes which constrain the total variation of that measure, and point out at certain "sparse" cases where the recovery is theoretically possible, asymptotically when the measurement error goes to zero. The underlying machinery somewhat stands as a continuous analog of popular methods for sparse recovery in the discrete case. The nature of the kernel of the forward operator plays here a key role in the notion of sparsity that we set forth. We also discuss the more realistic case where the measurements are not exact. Finally, we shall mention the problem of recovering the net moment, which is less difficult but still substantial, and sketch some approaches to regularize this issue.

Water waves determination from pressure measurements at the seabed

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Water waves profile determination from pressure measurements at the seabed is of great practical interest. Mathematically, for the pressure, this problem leads to solving a nonlinear Poisson-like equation. The difficulty lies in that the source term of the Poisson equation is unknown and that the equation must be solved in a domain that is unknown. The laws of Physics provide additional relationships to close the problem, leading to a nonlinear system of PDEs.

Using the elementary complex analysis and a few calculation tricks, the problem can be completely solved analytically in an implicit form. Explicit solutions are then obtained numerically by fixed-point iterations, that are convergent to an unique solution.

Uncertainty Quantification of complex systems described by large-scale equations

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The efficient and accurate prediction (quantication) of the effects of uncertain parameters (e.g., design, environmental, and medical parameters) is of paramount importance in a lot of applications : e.g., the performance of integrated circuits and nanotechnologies, earthquake engineering, orbital mechanic, hydro-meteorological processes, the probability of failure of complex communication networks, the stability of smart grids using renewable energy sources, the behavior of Internet of Things devices in changing environments, and the accuracy prediction of medical models for reliable patient-specific medicine (clinical decision-making). Uncertainty quantication (UQ) techniques are meant to address these challenges !

Monte Carlo (MC) is a well-known approach for UQ. It is easy to implement but computationally expensive due to a very large number of simulations needed to achieve accurate results. Therefore, alternative approaches have been explored in the literature in order to reduce the needed computational resources while keeping the accuracy of the UQ results.

In this talk, different UQ techniques based on data-driven (system identification-based UQ) and model-driven (model order reduction-based UQ) methodologies will be presented with a focus on electrical engineering applications, such as design and analysis of high-speed circuits and electromagnetic systems. Both frequency-domain and time-domain UQ will be discussed. The development of UQ techniques oriented towards the exploitation of parallel computing resources will be also discussed. Although the numerical examples are focused on specific electrical engineering applications, the use of the presented UQ methods is general-purpose and they can be applied to different real-life challenging problems.

Examples of Satellite Data Based Potential Field Problems and Approximation Methods on the Sphere

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We begin with a brief overview on different constituents of planetary gravity and magnetic fields (in particular, the Earth's gravity and magnetic field). But the main focus is on two problems addressing crustal magnetizations and magnetic induction by conductive ocean water. We provide some analysis of the problems (in particular, uniqueness issues) as well as different approaches to approximating the quantity of interest. The focus of the latter is on situations where either the data is only regionally available or the underlying source reveals some spatial localization. We present spherical multiscale methods as well as problem adapted sets of trial functions (e.g., by the inclusion of Slepian functions).

Sensitivity Analysis for Maxwell's Equations and Application to an Inverse Problem

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We are interested in time-harmonic Maxwell's equations for the electric field, as described below:

$$(\mathcal{M}) \qquad \begin{cases} \mathbf{curl}\,\mathbf{curl}\,\mathbf{E} - k^2\kappa\mathbf{E} = 0, & \text{in } \Omega, \\ \mathbf{curl}\,\mathbf{E} \times \mathbf{n} = \mathbf{g}, & \text{on } \Gamma, \end{cases}$$

where Ω is a bounded domain of $\mathbb{R}^3$, of smooth boundary Γ, with unit outward normal denoted by $\mathbf{n}$. Here, $\mathbf{g}$ is a given field and $\mathbf{E}$ is the electric field intensity. The parameter k is the wave number defined by $k = \omega\sqrt{\mu_0\varepsilon_0}$ with ω the frequency of the wave, μ_0 the magnetic permeability in vacuum and ε_0 the electric permittivity in vacuum. By denoting by ε and σ, respectively, the electric permittivity and conductivity in Ω, we define κ, the refractive index in the domain:

$$\kappa = \frac{1}{\varepsilon_0}\left(\varepsilon + i\frac{\sigma}{\omega}\right).$$

The aim of sensitivity analysis is to study how the electric field is affected by perturbations of small amplitudes induced in the electric permittivity and conductivity. Mathematically, we study the Gteaux derivative of the solution of $(\mathcal{M})$ in the direction corresponding to these perturbations. We show that this derivative is solution of Maxwell's equations with a source term and we study it numerically.

This study led to numerical results that gave us ideas to solve an inverse problem related to microwave imaging. In this inverse problem, we want to localize and characterize perturbations in the refractive index of the medium inside an object from boundary measurements. Our algorithm allows us to retrieve the center and volume of interior perturbations from boundary data.

Multiscale Modelling in Poroelasticity

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In geothermal research the aspect of poroelasticity is important to consider. Poroelasticity is placed in material research and describes the interaction between solids deformation and the fluid flow. The mathematical model and equations to describe this behaviour are dated back to Biot in the 1930s. Since we are interested in aquifers in geothermal research, the poroelasticity is one choice to combine and connect the manner between the solid and the fluid phase.
We regard the quasistatic equations of poroelasticity (QEP) with the unknown terms u (displacement) and p (pore pressure).

$$-\frac{\lambda+\mu}{\mu}\nabla_x(\nabla_x \cdot u) - \nabla_x^2 u + \alpha \nabla_x p = f,$$
$$\partial_t(c_0 \mu p + \alpha(\nabla_x \cdot u)) - \nabla_x^2 p = h,$$

Fundamental solutions -which we need for our approach- of these equations were derived for example by [3]. One problem is that they have singularities depending on the space or the time. Based on [2], we want to regularize these fundamental solutions concerning a parameter τ (called scale or scaling parameter) with the help of a Taylor approximation and construct wavelets by subtracting two regularized fundamental solutions with a different scale.
By the convolution of given data u and p e.g. from the method of fundamental solutions (cf. [1]) we want to decorrelate these data and obtain/extract more details of u and p. Furthermore we want to show some theoretical results, that hold true for the regularized fundamental solution.

References

[1] M. Augustin: A Method of Fundamental Solutions in Poroelasticity to Model the Stress Field in Geothermal Reservoirs, PhD Thesis, University of Kaiserslautern, 2015.
[2] C. Blick, W. Freeden, H. Nutz: Feature extraction of geological signatures by multiscale gravimetry. Int. J. Geomath. 8(1): 57-83, 2017.
[3] A.H.D. Cheng and E. Detournay: On singular integral equations and fundamental solutions of poroelasticity. Int. J. Solid. Struct. 35, 4521-4555, 1998.

The inverse problem in magnetic imaging of geological samples

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High-resolution magnetic field imaging of geological samples is a powerful tool for studying records of ancient planetary magnetic fields preserved in earth rocks, lunar rocks, and meteorites. However, magnetization cannot be directly measured with existing instruments except on a very thin surface layer of the sample. Owing to this difficulty, these imaging techniques rely instead on measurements of the sample's external magnetic field to infer the distribution of magnetic sources within the specimen, which typically leads to an ill-posed inverse problem. Several techniques and strategies in the spatial domain and in the Fourier domain have been proposed to regularize and solve the inverse problem for magnetization. For small weakly magnetized samples, alternative techniques for addressing the more tractable problem of estimating the sample's net magnetic moment (i.e., the integral of its magnetization distribution) have also been proposed. Understanding the strengths and limitations of each technique is critical not only for choosing the best approach when examining a particular rock sample but also for developing new analytical tools.

Rational approximation of a magnetic dipole from sparse field measurements

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Over the last few years, there is an effort to restudy Moon rock samples from Apollo missions (available at NASA). This re-examination is motivated from recent Paleomagnetic studies, which revealed evidences for an ancient, strong and global Moon dynamo field [5] that no longer exist. Planetary scientists, could recover valuable information for the duration and the evolution of this field from its residuals which preserved in the Moon rocks. Those residuals in the form of remanent magnetization, contribute to the generated magnetic field of the rock. The issue of recovering the remanent magnetization of a rock from magnetic field measurements, is the inverse problem that we address. In contrast to other paleomagnetic studies, where the sample was sliced or remagnetized, here the sample must be protected from any kind of damage. For this nondestructive inspection purpose, a special magnetometer has been constructed by scientists at CEREGE[1] which they call "lunometer", [4] . This device encloses the rock sample in a nonmagnetic cubic box and isolates it from external electromagnetic fields with a mu-metal shield. The lunometer's technical characteristics create the sparse measurement geometry of our study.

The underlying magnetic phenomenon is modeled by Maxwell equations in the magnetostatic and macroscopic framework [3]. We preliminary assume that the sample contains a unique pointwise dipolar unknown magnetic source located at X_d, with moment M_d. The expression of the magnetic field at $X \neq X_d$ (where μ_0 is the permeability of the free space) has the form:

$$B(X) = -\frac{\mu_0}{4\pi} \frac{|X - X_d|^2 M_d - 3\left[M_d \cdot (X - X_d)\right](X - X_d)}{|X - X_d|^5}\,.$$

From measurements of the magnetic field B at sensors positions, we want to recover the moment M_d and the location X_d of the dipole. Our strategy is to decompose this inverse problem into two subproblems and search firstly for the location X_d of the dipole (which is a nonlinear problem) and secondly for its moment M_d (which becomes a linear problem when the location is known). For the location recovery X_d, we use best quadratic rational approximation techniques, together with geometrical and algebraic properties of the poles of the approximants [1, 2]. During the talk, these techniques will be explained and numerical results will be discussed.

References

[1] L. Baratchart, J. Leblond and J. P. Marmorat, *Inverse source problem in 3D ball from best meromorphic approximation on 2D slices*, Electronic Transactions on Numerical Analysis, 25, 41-53, 2006.

[2] S. Chevillard, J. Leblond, K. Mavreas, *Dipole recovery from sparse measurements of its field on a cylindrical geometry*, International Journal of Applied Electromagnetics and Mechanics, Proceedings of ISEM 2017, `http://www.isem2017.org/`, to appear.

[3] J. D. Jackson, *Classical electrodynamics*, third edition, John Wiley & Sons, 2001.

[4] M. Uehara, J. Gattacceca, Y. Quesnel, C. Lepaulard, E. A. Lima, M. Manfredi, P. Rochette, *A spinner magnetometer for large Apollo lunar samples*, Review of Scientific Instruments, American Institute of Physics, 88 (10), 2017.

[5] B. P. Weiss, S. M. Tikoo, *The lunar dynamo*, Science 346, 6214, 2014.

[1] Centre Europen de Recherche et d'Enseignement des Gosciences de l'Environnement, CNRS, France.

Reconstruction of obstacles from partial boundary for wave equation with finite measurement time

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We consider a problem of determining unknown interior boundaries in a regular (Lipschitz-smooth) domain in the case of wave equation. The ill-posed Cauchy problem is treated by quasi-reversibility method (in particular, its mixed formulation that is more adapted for finite-element discretization) which would allow its solution at once if the full boundary of the domain was known. In case of reconstruction of the interior boundary this procedure has to be combined with a level set method. We show that iterative repetition of such combined technique yields very decent recovery of the unknown boundaries (interior obstacles) providing the measurement time is sufficiently large.

Some magnetic field data processing on terrestrial and planetary cases

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After 1950, the mining, gas and oil exploration - as well as military applications - led to the rise of the airborne magnetic field mapping, where small variations of the Earth's magnetic field are measured using magnetometers. However, unveiling anomalies only due to the crustal magnetization requires a lot of processing, since there are multiple sources that contribute to the Earth's magnetic field. Here I will shortly detail some of these processing methods typically applied to magnetic field observations. Then, a last step consists in the numerical modeling of buried magnetized sources using forward and inverse approaches. Such applications on terrestrial, lunar and martian cases will be introduced.

Modelling Earths gravitational potential with a learned best basis

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The model of the Earths gravitational potential provides us with a description of its shape, the geoid. Further, using time variable data, we can determine the mass distributions and, hence, visualize the effects of the climate change. To construct the potential, we can theoretically use an infinite set of trial functions. Practically, we have to make a choice which trial functions are qualified for our needs. Traditionally, this choice limits us to only one type of trial functions, e. g. spherical harmonics or radial basis functions (RBFs). In the last decade, algorithms have been developed that make use of both the local and global structures of different trial functions. These algorithms are called the (Regularized) Functional Matching Pursuit ((R)FMP) and the (Regularized) Orthogonal Functional Matching Pursuits ((R)OFMP). The idea is to use an overcomplete but in practice naturally finite dictionary to iteratively build a best basis and compute a (e. g. gravity field) model in this best basis. The question at hand is, whether there is an optimal strategy for choosing a finite dictionary from the infinite set of trial functions. We present a learning strategy which enables us to find an optimized finite subset of certain RBFs (Abel-Poisson kernels) and spherical harmonics. With respect to the kernels, this approach includes solving a nonlinear constrained optimization problem in every iteration step of the matching pursuit. The problem is solved using the Ipopt software package with an HSL subroutine and yields a new candidate for being an element of an optimized dictionary. We decide whether it is inserted into the optimized dictionary by considering its effect on the decrease of the Tikhonov functional. In our presentation, we explain the idea of our learning algorithm and demonstrate numerical examples with respect to the EGM2008.

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