Moment estimation of very weak magnetizations
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Abstract
We estimate the net magnetic moment of a thin rock sample from the only measures of the vertical component of the weak magnetic field induced near the sample. This problem comes from paleomagnetism, where planetary scientists try to understand how rocks record the Earth’s magnetic field history. This comes to be an ill-posed inverse problem, which is solved via a bounded extremal problem regularization. We present here numerical results.

1. Mathematical setting

2. Moment estimation problem

Given a magnetization vector field \( \vec{m}(s) \), \( s \in S \), for \( q \in Q \) let \( \vec{r} = q - s \), then magnetic field \( \vec{B} \) on \( Q \) is given by:

\[
\vec{B}(q) = \frac{\mu_0}{4\pi} \int_{S} 3(\vec{m}(s) \vec{r} - |\vec{r}|^2 \vec{m}) \, ds
\]  

(1)

From sole measure of the vertical component \( B_z \) of \( \vec{B} \) on subset \( Q \), estimation of the whole vector field \( \vec{m} \) on \( S \) is a difficult ill-posed inverse problem, which suffers non-uniqueness due to the existence of silent sources [1].

We focus to an easier problem: estimation of the total sum of \( \vec{m} \), or net magnetic moment:

\[
\langle \vec{m} \rangle = \int_{S} \vec{m}(s) \, ds \in \mathbb{R}^3
\]  

(2)

As shown in [2], uniqueness is now granted, but problem is still ill-posed.

3. Mathematical setting

Squid geometry: given three dimensions \( d_2, d_3, h \), then in the \( x_1, x_2, x_3 \) reference space, domains \( Q \) and \( S \) are the subsets:

\[
Q = \{ |x_1| \leq d_1, |x_2| \leq d_2, |x_3| \leq d_3, x_3 = 0 \}
\]

Functional spaces: \( L^2(Q) \) (resp. \( L^2(S) \)) is the classical Hilbert space of square-integrable real-valued \((x_1, x_2)\)-functions on \( Q \) (resp. \( S \)). Measurements on \( Q \) are functions of \( W^{1,2}(Q) \), functions in \( L^2(Q) \), with gradient in \( L^2(Q) \). Magnetizations \( \vec{m} \) on \( S \) are elements of \( [L^2(S)]^3 \). Three constant functions in this space, with values the three unit-vectors of the \( R^3 \) basis, are noted \( \vec{e}_1, \vec{e}_2, \vec{e}_3 \) so we have \( \langle \vec{m} \rangle = \sum \langle \vec{e}_i \rangle \) \( \vec{m} \) so we have \( \langle \vec{m} \rangle = \sum \langle \vec{e}_i \rangle \) \( \vec{m} \) and \( \vec{m} \) \( h \), very close to the sample \( S \).

5. Reliability

6. Build a magnetometer: choose \( \lambda \)

Choosing \( \lambda \) is a trade-off between error on \( S \) and gradient norm on \( Q \).

Reasonable choices lie near the maximum curvature zone of the L-curve [3].

7. Example

8. Relative error for various choices of \( \lambda \)

Total integral[2] is used to compute true moment \( \langle \vec{m} \rangle \).

Purley random gaussian noise is added to synthetic data \( B_3 \) (std=2%).

Estimated moments are given by formula(3) for different values of \( \lambda \).

9. Conclusion

Measures in a small domain of only one component of a magnetic field are sufficient to estimate the underlying magnetization net moment. Linear estimators are obtained via direct resolution of a bounded extremal problem with careful choice of the Lagrange multiplier. Current numerical experiments on a standard PC suffer limitation of the discretization grid size. Further works will concern recursive resolution scheme, noise influence study, parallelization.

10. References

