

# Deep Generative Learning

Deep Learning for Computer Vision

Valeriya Strizhkova

15.03.23

# About myself

Valeriya Strizhkova

2nd year PhD student @ Inria & 3iA & UCA

[valeriya.strizhkova@inria.fr](mailto:valeriya.strizhkova@inria.fr)

Research topics:

- emotion detection
- computer vision
- deep learning



# Outline

- Introduction
  - Overview of Deep Generative Models
  - OpenAI's CLIP
- Variational Autoencoders
- Autoregressive Models
- Diffusion Models
- Generative Adversarial Networks
- Evaluation

“



There are many interesting recent development in deep learning...The most important one, in my opinion, is adversarial training (also called GAN for Generative Adversarial Networks). This, and the variations that are now being proposed, is the most interesting idea in the last 10 years in ML.

Yann LeCun

# The Landscape of Deep Generative Learning



Variational  
Autoencoders

Autoregressive  
Models

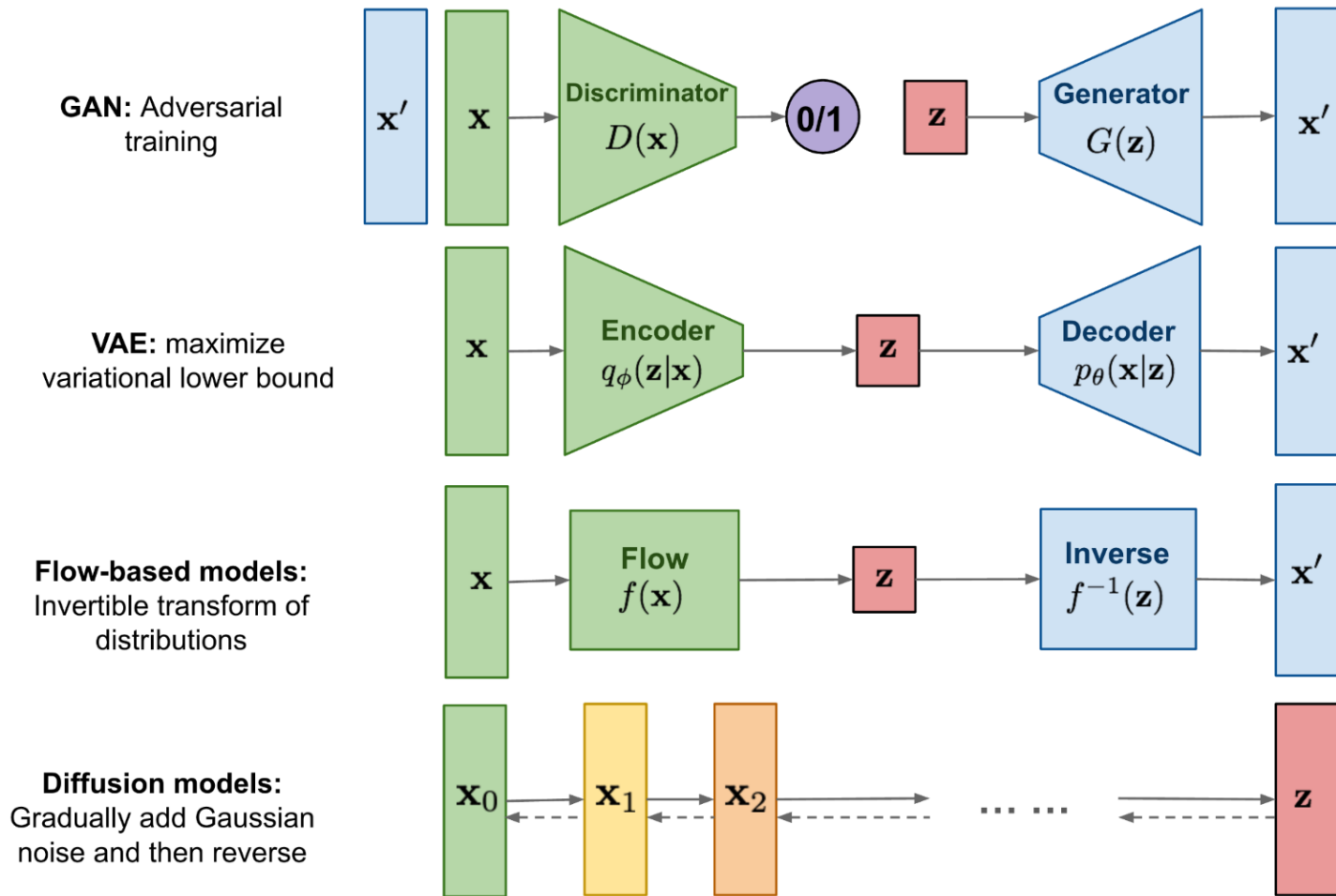
Normalizing  
Flows

Generative  
Adversarial Networks

Energy-based  
Models

Denoising  
Diffusion Models





# A question to ChatGPT

Q: When is it better to use **GAN** and when **diffusion models**?

A: The choice between using GANs and diffusion models depends on the specific characteristics of the data being generated and the requirements of the generative modeling task at hand.

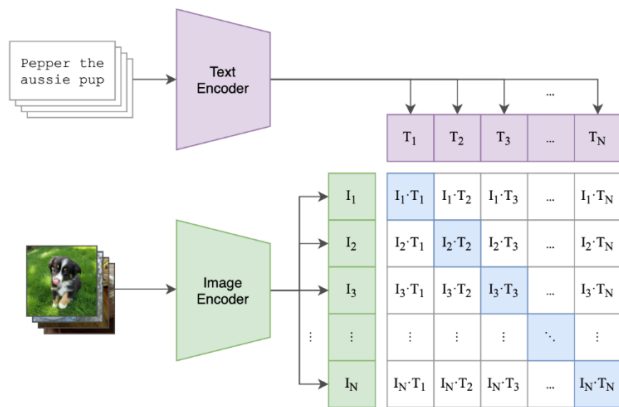
In general, GANs are often preferred for tasks that require generating **high-quality, visually realistic images**, as they have been shown to produce photorealistic images with fine details and textures. GANs are also well-suited for **image-to-image translation** tasks, such as style transfer or super-resolution, where the goal is to modify an existing image in a particular way while preserving its underlying structure and content.

On the other hand, diffusion models are often preferred for tasks that involve **modeling complex distributions or generating samples that exhibit long-range dependencies**. Diffusion models have been shown to be effective at modeling complex image and video datasets, and can generate high-quality samples that capture the underlying structure and dynamics of the data. Diffusion models can also be used for tasks such as **image denoising or inpainting**, where the goal is to remove noise or missing information from an image while preserving its structure and content.

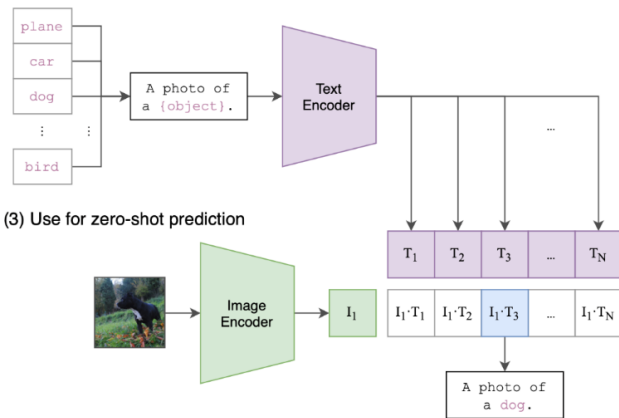
# OpenAI's CLIP

- encodes image, and text to similar embeddings
- is trained with contrastive learning, maximizing cosine similarity of corresponding image and text
- CLIP's output image embeddings contain both style and semantics
- is used in DALL-E 1, DALL-E 2, image-text classification

(1) Contrastive pre-training



(2) Create dataset classifier from label text



(3) Use for zero-shot prediction



# CLIP Architecture

- text and image have separate transformer encoders
- visual encoder is ViT (vision transformer)
- text encoder is GPT-2 transformer
- the fixed-length text embedding is extracted from [EOS] token position
- text token embeddings and image patch embeddings also available
- trained on 256 GPUs for 2 weeks

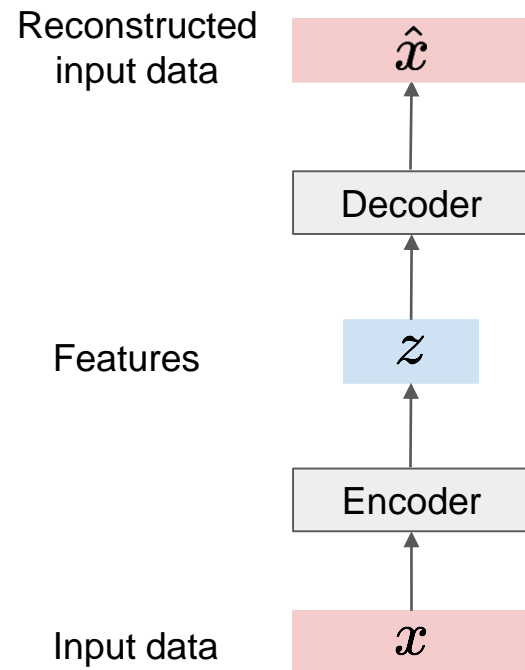
# Outline

- Introduction
- Variational Autoencoders
  - Architecture
  - Maximization of variational lower bound
  - dVAE
- Autoregressive Models
- Diffusion Models
- Generative Adversarial Networks
- Evaluation

# Autoencoders (non-variational)

Unsupervised method for learning latent features from data without any labels.

$$L = ||\hat{x} - x||_2^2$$

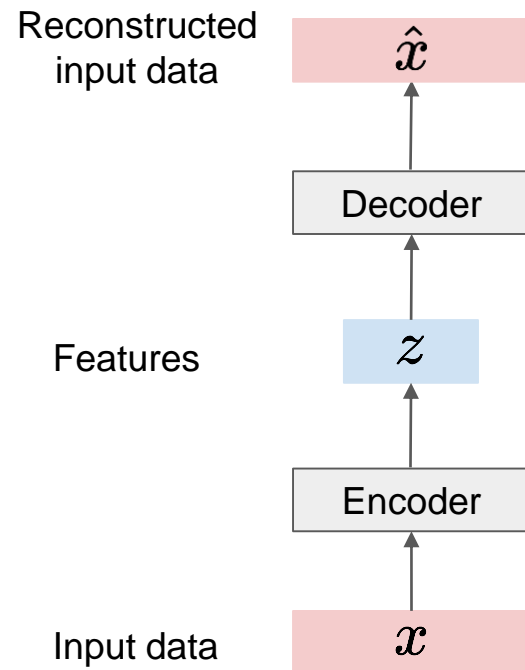


# Autoencoders (non-variational)

Unsupervised method for learning latent features from data without any labels.

Features need to be **lower dimensional** than the data.

$$L = ||\hat{x} - x||_2^2$$



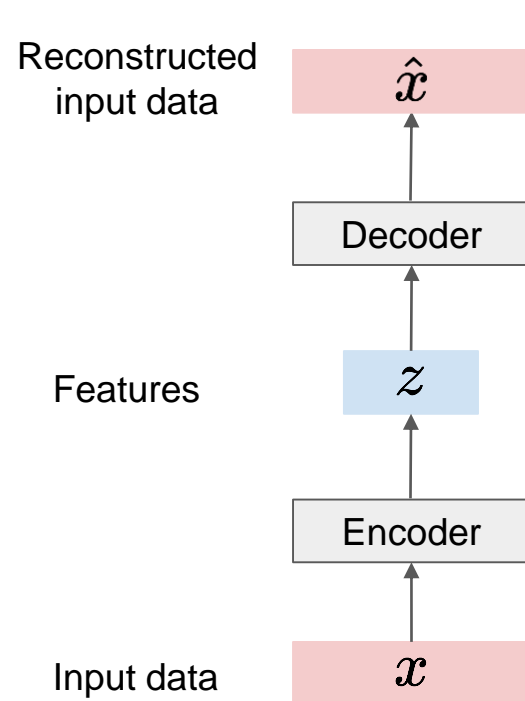
# Autoencoders (non-variational)

Unsupervised method for learning latent features from data without any labels.

Features need to be **lower dimensional** than the data.

**Limitation: no way to produce any new content**

$$L = ||\hat{x} - x||_2^2$$



# Variational Autoencoders (VAE)

- VAE is an autoencoder that learns **latent features** from data and enables **generative process**.

# Variational Autoencoders (VAE)

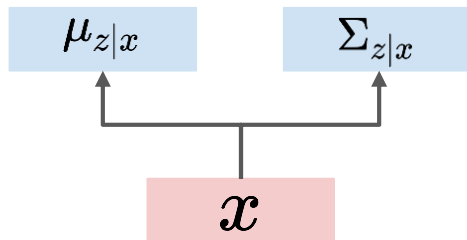
- VAE is an autoencoder that learns **latent features** from data and enables **generative process**.
- Instead of encoding an input as a single point, VAE encodes it as a distribution over the latent space.

# Variational Autoencoders (VAE)

**Encoder network** inputs data  $x$  and outputs distribution over latent codes  $z$

**Encoder Network**

$$q_{\phi}(z|x) = N(\mu_{z|x}, \Sigma_{z|x})$$





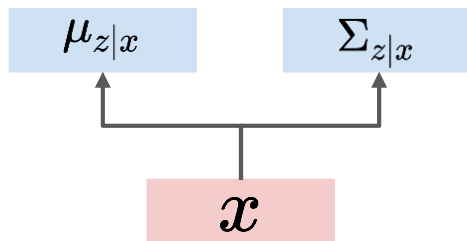
# Variational Autoencoders (VAE)

**Encoder network** inputs data  $x$  and outputs distribution over latent codes  $z$

**Decoder network** inputs latent code  $z$  and outputs distribution over data  $x$

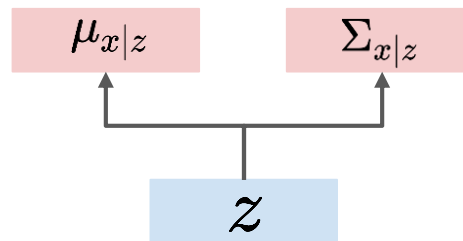
**Encoder Network**

$$q_{\phi}(z|x) = N(\mu_{z|x}, \Sigma_{z|x})$$



**Decoder Network**

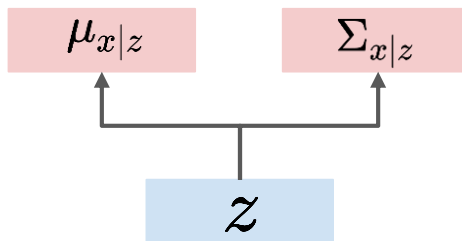
$$p_{\theta}(x|z) = N(\mu_{x|z}, \Sigma_{x|z})$$



# Variational Autoencoders (VAE)

Decoder neural network represent  $p(x|z)$  where  $x$  is an image,  $z$  is latent features to generate  $x$ .

$$p_{\theta}(x|z) = N(\mu_{x|z}, \Sigma_{x|z})$$

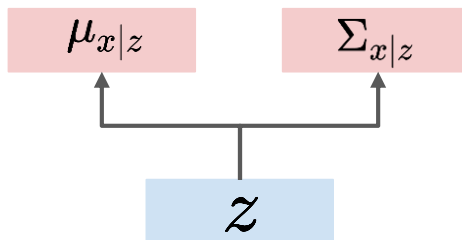


# Variational Autoencoders (VAE)

Decoder neural network represent  $p(x|z)$  where  $x$  is an image,  $z$  is latent features to generate  $x$ .

Assume prior  $p(z)$  is standard unit diagonal Gaussian.

$$p_{\theta}(x|z) = N(\mu_{x|z}, \Sigma_{x|z})$$



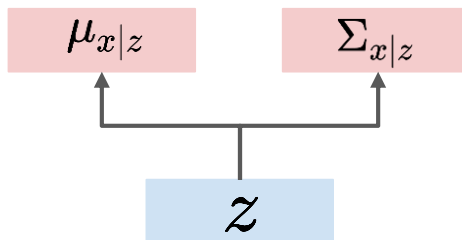
# Variational Autoencoders (VAE)

Decoder neural network represent  $p(x|z)$  where  $x$  is an image,  $z$  is latent features to generate  $x$ .

Assume prior  $p(z)$  is standard unit diagonal Gaussian.

How to train this model?

$$p_{\theta}(x|z) = N(\mu_{x|z}, \Sigma_{x|z})$$



# Variational Autoencoders (VAE)

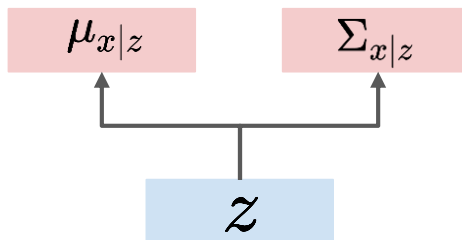
Decoder neural network represent  $p(x|z)$  where  $x$  is an image,  $z$  is latent features to generate  $x$ .

Assume prior  $p(z)$  is standard unit diagonal Gaussian.

How to train this model?

**Maximize likelihood of data**

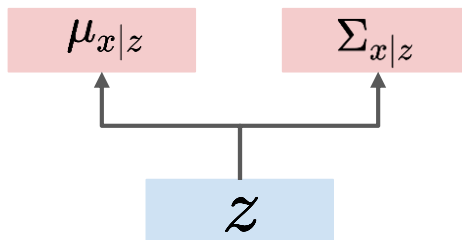
$$p_{\theta}(x|z) = N(\mu_{x|z}, \Sigma_{x|z})$$



# Variational Autoencoders (VAE)

**Maximize likelihood of data**

$$p_{\theta}(x) = \int p_{\theta}(x, z)dz = \int p_{\theta}(x|z)p_{\theta}(z)dz$$

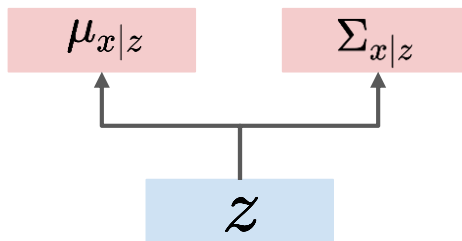


# Variational Autoencoders (VAE)

**Maximize likelihood of data**

$$p_{\theta}(x) = \int p_{\theta}(x, z)dz = \int p_{\theta}(x|z)p_{\theta}(z)dz$$

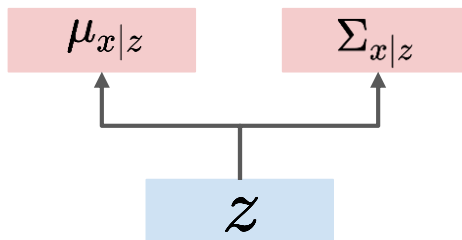
**Problem: impossible to integrate over all  $z$**



# Variational Autoencoders (VAE)

**Maximize likelihood of data**

$$\text{Bayes' Rule: } p_{\theta}(x) = \frac{p_{\theta}(x|z)p_{\theta}(z)}{p_{\theta}(z|x)}$$



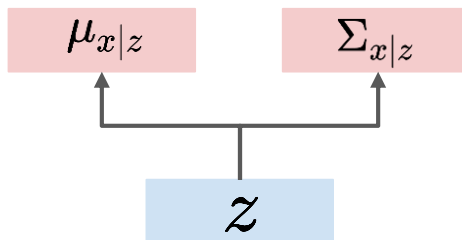


# Variational Autoencoders (VAE)

## Maximize likelihood of data

Bayes' Rule:  $p_{\theta}(x) = \frac{p_{\theta}(x|z)p_{\theta}(z)}{p_{\theta}(z|x)}$

**Problem:** no way to compute  $p_{\theta}(z|x)$



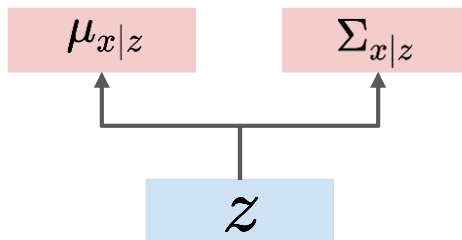
# Variational Autoencoders (VAE)

## Maximize likelihood of data

Bayes' Rule:  $p_{\theta}(x) = \frac{p_{\theta}(x|z)p_{\theta}(z)}{p_{\theta}(z|x)}$

**Problem:** no way to compute  $p_{\theta}(z|x)$

**Solution:** train another network (encoder) that learns  $q_{\phi}(z|x) \approx p_{\theta}(z|x)$



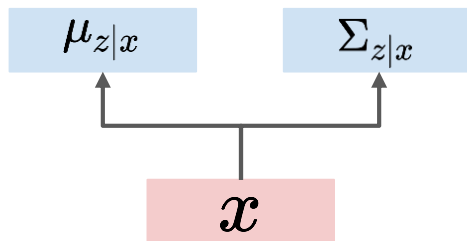
# Variational Autoencoders (VAE)

**Encoder network** inputs data  $x$  and outputs distribution over latent codes  $z$

**Decoder network** inputs latent code  $z$  and outputs distribution over data  $x$

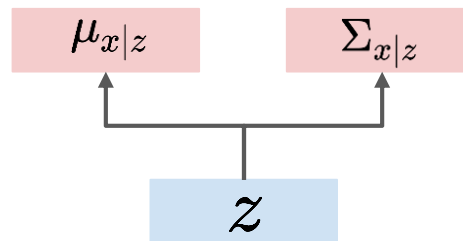
**Encoder Network**

$$q_{\phi}(z|x) = N(\mu_{z|x}, \Sigma_{z|x})$$



**Decoder Network**

$$p_{\theta}(x|z) = N(\mu_{x|z}, \Sigma_{x|z})$$

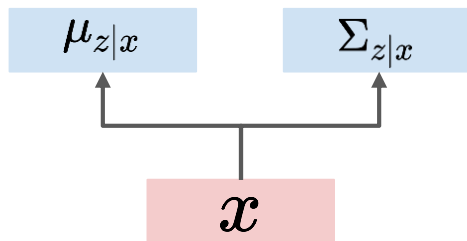


# Variational Autoencoders (VAE)

Jointly train **encoder**  $q$  and **decoder**  $p$

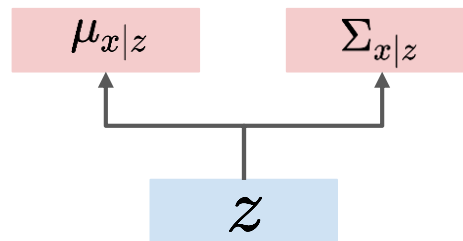
## Encoder Network

$$q_{\phi}(z|x) = N(\mu_{z|x}, \Sigma_{z|x})$$



## Decoder Network

$$p_{\theta}(x|z) = N(\mu_{x|z}, \Sigma_{x|z})$$



# Variational Autoencoders (VAE)

$$\log p_{\theta}(x) = \log \frac{p_{\theta}(x|z)p(z)}{p_{\theta}(z|x)}$$

# Variational Autoencoders (VAE)

$$\log p_{\theta}(\mathbf{x}) = \log \frac{p_{\theta}(\mathbf{x}|z)p(z)}{p_{\theta}(z|\mathbf{x})} = \log \frac{p_{\theta}(\mathbf{x}|z)p(z)q_{\phi}(z|\mathbf{x})}{p_{\theta}(z|\mathbf{x})q_{\phi}(z|\mathbf{x})}$$

## Variational Autoencoders (VAE)

$$\begin{aligned}\log p_{\theta}(\mathbf{x}) &= \log \frac{p_{\theta}(\mathbf{x}|z)p(z)}{p_{\theta}(z|\mathbf{x})} = \log \frac{p_{\theta}(\mathbf{x}|z)p(z)q_{\phi}(z|\mathbf{x})}{p_{\theta}(z|\mathbf{x})q_{\phi}(z|\mathbf{x})} \\ &= \log p_{\theta}(\mathbf{x}|z) - \log \frac{q_{\phi}(z|\mathbf{x})}{p(z)} + \log \frac{q_{\phi}(z|\mathbf{x})}{p_{\theta}(z|\mathbf{x})}\end{aligned}$$

# Variational Autoencoders (VAE)

$$\begin{aligned}\log p_{\theta}(\mathbf{x}) &= \log \frac{p_{\theta}(\mathbf{x}|z)p(z)}{p_{\theta}(z|\mathbf{x})} = \log \frac{p_{\theta}(\mathbf{x}|z)p(z)q_{\phi}(z|\mathbf{x})}{p_{\theta}(z|\mathbf{x})q_{\phi}(z|\mathbf{x})} \\ &= \log p_{\theta}(\mathbf{x}|z) - \log \frac{q_{\phi}(z|\mathbf{x})}{p(z)} + \log \frac{q_{\phi}(z|\mathbf{x})}{p_{\theta}(z|\mathbf{x})} \\ &= E_z[\log p_{\theta}(\mathbf{x}|z)] - E_z\left[\log \frac{q_{\phi}(z|\mathbf{x})}{p(z)}\right] + E_z\left[\log \frac{q_{\phi}(z|\mathbf{x})}{p_{\theta}(z|\mathbf{x})}\right]\end{aligned}$$



# Variational Autoencoders (VAE)

$$\begin{aligned}\log p_{\theta}(x) &= \log \frac{p_{\theta}(x|z)p(z)}{p_{\theta}(z|x)} = \log \frac{p_{\theta}(x|z)p(z)q_{\phi}(z|x)}{p_{\theta}(z|x)q_{\phi}(z|x)} \\ &= E_z[\log p_{\theta}(x|z)] - E_z\left[\log \frac{q_{\phi}(z|x)}{p(z)}\right] + E_z\left[\log \frac{q_{\phi}(z|x)}{p_{\theta}(z|x)}\right] \\ &= E_{z \sim q_{\phi}(z|x)}[\log p_{\theta}(x|z)] - KL(q_{\phi}(z|x), p(z)) + KL(q_{\phi}(z|x), p_{\theta}(z|x))\end{aligned}$$

**Kullback-Leibler Divergence:**  $KL(p, q) = E_{x \sim p}[\log \frac{p(x)}{q(x)}]$

# Variational Autoencoders (VAE)

$$\begin{aligned}\log p_{\theta}(x) &= \log \frac{p_{\theta}(x|z)p(z)}{p_{\theta}(z|x)} = \log \frac{p_{\theta}(x|z)p(z)q_{\phi}(z|x)}{p_{\theta}(z|x)q_{\phi}(z|x)} \\ &= E_z[\log p_{\theta}(x|z)] - E_z\left[\log \frac{q_{\phi}(z|x)}{p(z)}\right] + E_z\left[\log \frac{q_{\phi}(z|x)}{p_{\theta}(z|x)}\right] \\ &= E_{z \sim q_{\phi}(z|x)}[\log p_{\theta}(x|z)] - KL(q_{\phi}(z|x), p(z)) + KL(q_{\phi}(z|x), p_{\theta}(z|x))\end{aligned}$$

$KL \geq 0 \Rightarrow$  dropping the last term gives a lower bound on the data likelihood

**Kullback-Leibler Divergence:**  $KL(p, q) = E_{x \sim p}\left[\log \frac{p(x)}{q(x)}\right]$

## Variational Autoencoders (VAE)

$$\begin{aligned}\log p_\theta(x) &= \log \frac{p_\theta(x|z)p(z)}{p_\theta(z|x)} = \log \frac{p_\theta(x|z)p(z)q_\phi(z|x)}{p_\theta(z|x)q_\phi(z|x)} \\ &= E_z[\log p_\theta(x|z)] - E_z\left[\log \frac{q_\phi(z|x)}{p(z)}\right] + E_z\left[\log \frac{q_\phi(z|x)}{p_\theta(z|x)}\right] \\ &= E_{z \sim q_\phi(z|x)}[\log p_\theta(x|z)] - KL(q_\phi(z|x), p(z)) + KL(q_\phi(z|x), p_\theta(z|x))\end{aligned}$$
  
$$\log p_\theta(x) \geq E_{z \sim q_\phi(z|x)}[\log p_\theta(x|z)] - KL(q_\phi(z|x), p(z))$$

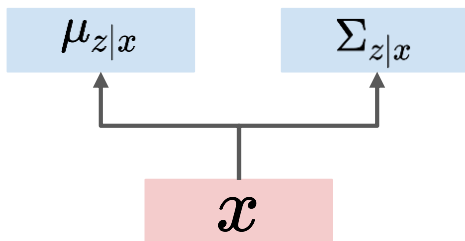
# Variational Autoencoders (VAE)

Jointly train **encoder**  $q$  and **decoder**  $p$  to maximize the **variational lower bound** on the data likelihood

$$\log p_{\theta}(x) \geq E_{z \sim q_{\phi}(z|x)} [\log p_{\theta}(x|z)] - KL(q_{\phi}(z|x), p(z))$$

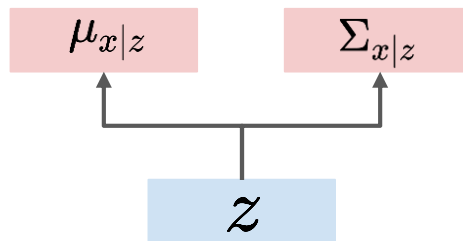
**Encoder Network**

$$q_{\phi}(z|x) = N(\mu_{z|x}, \Sigma_{z|x})$$



**Decoder Network**

$$p_{\theta}(x|z) = N(\mu_{x|z}, \Sigma_{x|z})$$



# Variational Autoencoders (VAE)

Closed form solution when  $q_\phi$  is diagonal Gaussian and  $p$  is unit Gaussian:

$$\log p_\theta(x) \geq E_{z \sim q_\phi(z|x)} [\log p_\theta(x|z)] - \boxed{KL(q_\phi(z|x), p(z))}$$

$$-KL(q_\phi(z|x), p(z)) = \int_Z q_\phi(z|x) \log \frac{p(z)}{q_\phi(z|x)} dz$$

$$= \int_Z N(z, \mu_{z|x}, \Sigma_{z|x}) \log \frac{N(z, 0, I)}{N(z, \mu_{z|x}, \Sigma_{z|x})} dz$$

$$= \sum_{j=1}^J (1 + \log((\sum_{z|x})_j^2) - (\mu_{z|x})_j^2 - (\Sigma_{z|x})_j^2)$$

# Variational Autoencoders (VAE)

Closed form solution when  $q_\phi$  is diagonal Gaussian and  $p$  is unit Gaussian:

$$\log p_\theta(x) \geq \boxed{E_{z \sim q_\phi(z|x)} [\log p_\theta(x|z)]} - KL(q_\phi(z|x), p(z))$$

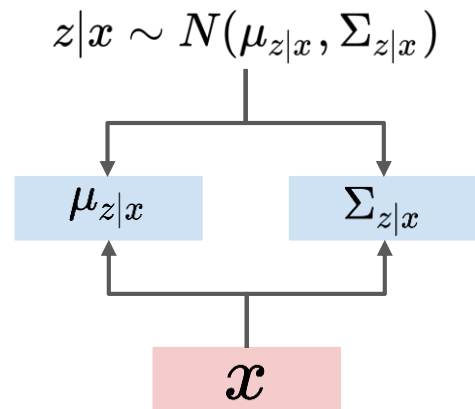
$E_{z \sim q_\phi(z|x)} [\log p_\theta(x|z)]$  is data reconstruction term

# Variational Autoencoders (VAE)

Train by maximize the **variational lower bound**.

$$E_{z \sim q_\phi(z|x)} [\log p_\Theta(x|z)] - KL(q_\phi(z|x), p(z))$$

1. The input is **encoded** as distribution over the latent space

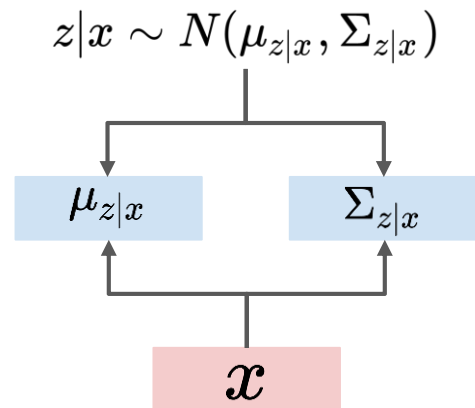


# Variational Autoencoders (VAE)

Train by maximize the **variational lower bound**.

$$E_{z \sim q_\phi(z|x)} [\log p_\Theta(x|z)] - KL(q_\phi(z|x), p(z))$$

1. The input is **encoded** as distribution over the latent space
2. **Encoder output should match prior  $p(z)$**



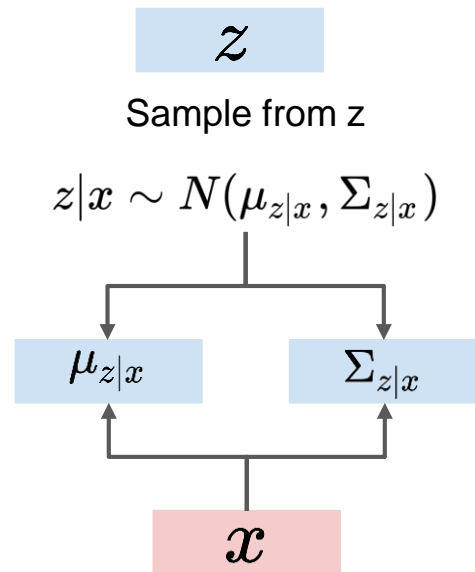


# Variational Autoencoders (VAE)

Train by maximize the **variational lower bound**.

$$E_{z \sim q_\phi(z|x)} [\log p_\Theta(x|z)] - KL(q_\phi(z|x), p(z))$$

1. The input is **encoded** as distribution over the latent space
2. **Encoder output should match prior  $p(z)$**
3. A point from the latent space is sampled from that distribution

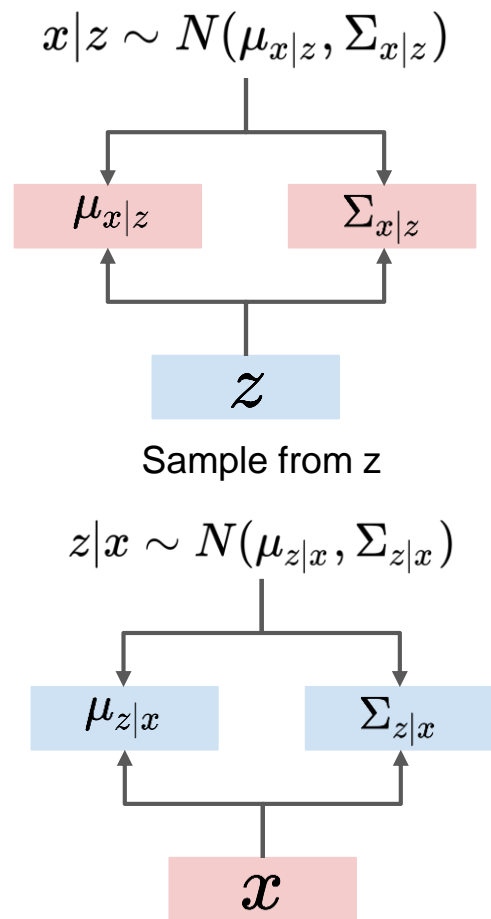


# Variational Autoencoders (VAE)

Train by maximize the **variational lower bound**.

$$E_{z \sim q_\phi(z|x)} [\log p_\Theta(x|z)] - KL(q_\phi(z|x), p(z))$$

1. The input is **encoded** as distribution over the latent space
2. **Encoder output should match prior  $p(z)$**
3. A point from the latent space is sampled from that distribution
4. The sampled point is **decoded**

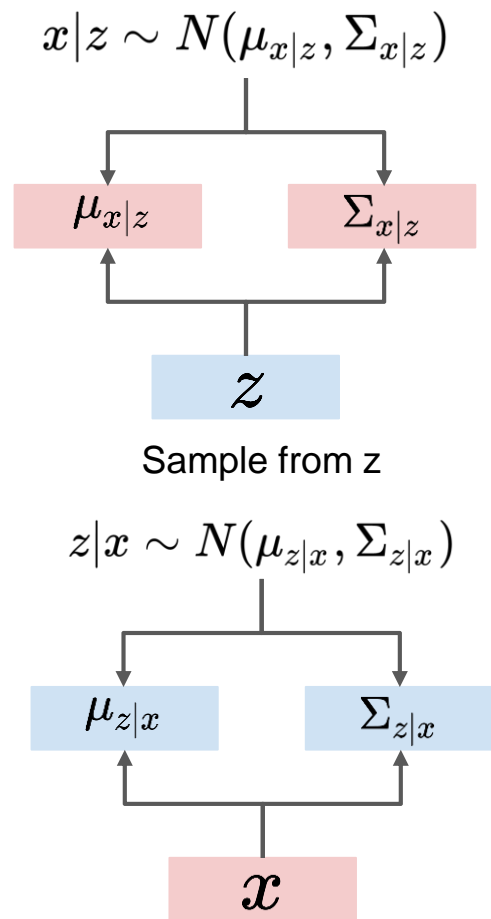


# Variational Autoencoders (VAE)

Train by maximize the **variational lower bound**.

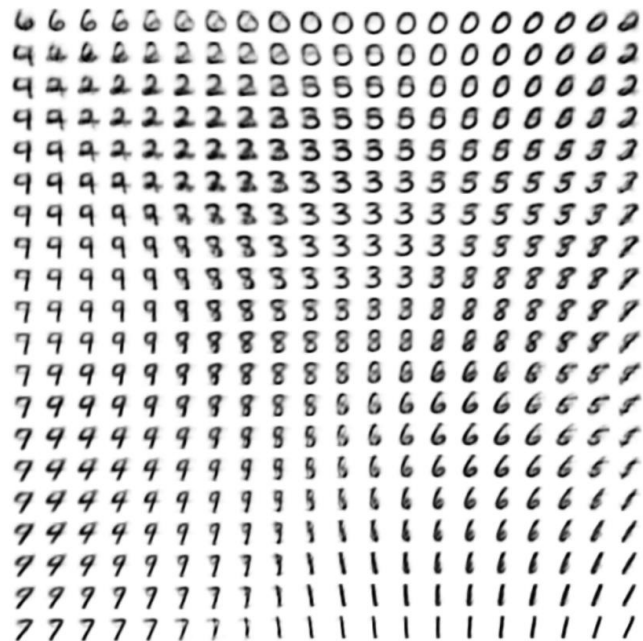
$$E_{z \sim q_\phi(z|x)} [\log p_\Theta(x|z)] - KL(q_\phi(z|x), p(z))$$

1. The input is **encoded** as distribution over the latent space
2. **Encoder output should match prior  $p(z)$**
3. A point from the latent space is sampled from that distribution
4. The sampled point is **decoded**
5. **The reconstruction error is computed**



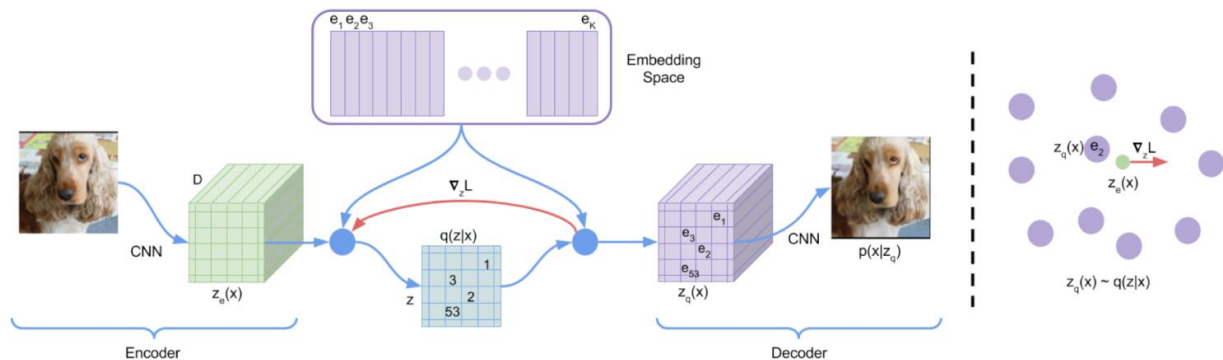
# Variational Autoencoders (VAE)

Learned data manifold for generative models with two-dimensional latent space:



# Discrete Variational Auto-Encoder (dVAE)

- introduced in VQ-VAE 1 [1] and VQ-VAE-2 [2]
- image encoder maps to latent  $32 \times 32$  grid of embeddings
- vector quantization maps to 8k code words
- decoder maps from quantized grid to the image
- copy gradients from decoder input  $z$  to the encoder output



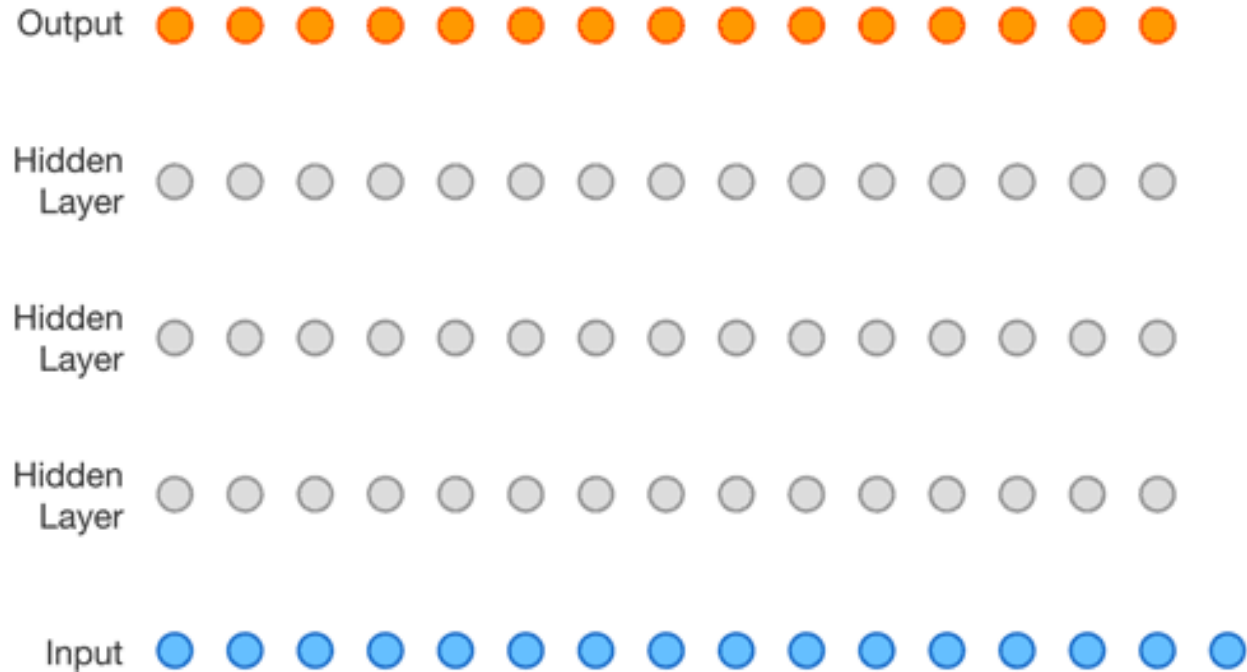
[1] Aaron van den Oord, Oriol Vinyals, Koray Kavukcuoglu. Neural Discrete Representation Learning. NeurIPS 2017 <https://arxiv.org/abs/1711.00937>

[2] Ali Razavi, Aaron van den Oord, Oriol Vinyals. Generating Diverse High-Fidelity Images with VQ-VAE-2. NeurIPS 2019 <https://arxiv.org/abs/1906.00446>

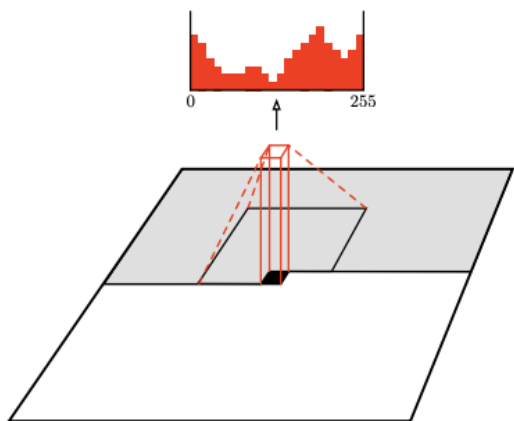
# Outline

- Introduction
- Variational Autoencoders
- Autoregressive Models
  - Image generation
    - PixelCNN
  - Text-to-image generation
    - DALL-E 1
  - Text-to-video generation
    - Phenaki
- Diffusion Models
- Generative Adversarial Networks
- Evaluation

# Autoregressive Models



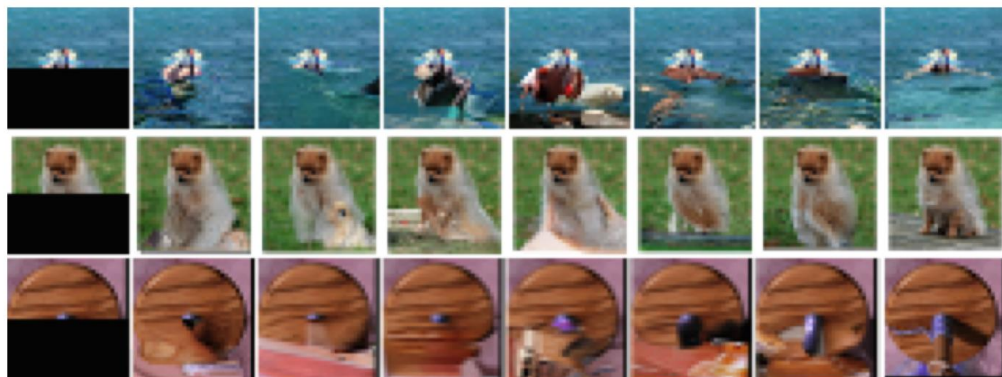
# Pixel Recurrent Neural Networks



occluded

completions

original





# DALL-E 1

- is introduced by OpenAI
- generates 256×256 images from text via dVAE
- autoregressively generates image tokens from textual tokens on a discrete latent space

# DALL-E 1 Training

1. train encoder and decoder image of image into 32x32 grid of 8k possible code word tokens (dVAE)
2. concatenate encoded text tokens with image tokens into single array
3. train to predict next image token from the preceding tokens (autoregressive transformer)
4. discard the image encoder, keep only image decoder and next token predictor

# DALL-E 1 Results



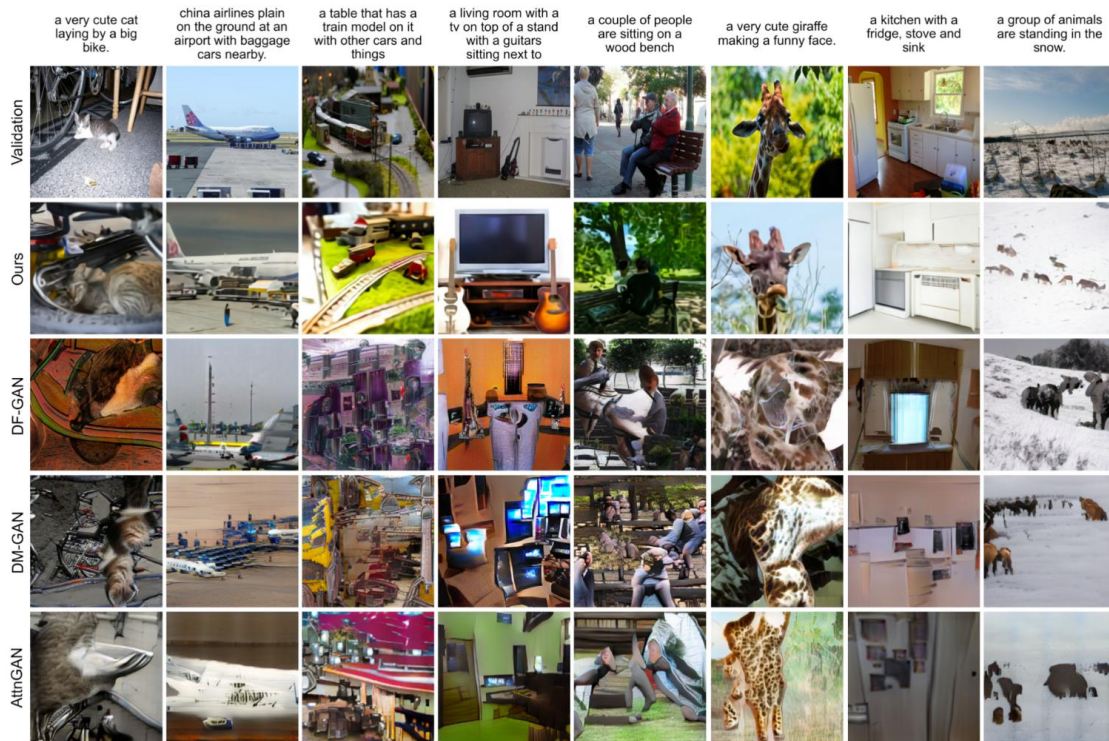
(a) a tapir made of accordion. a tapir with the texture of an accordion.

(b) an illustration of a baby hedgehog in a christmas sweater walking a dog

(c) a neon sign that reads "backprop". a neon sign that reads "backprop". backprop neon sign

(d) the exact same cat on the top as a sketch on the bottom

# DALL-E 1 Results



# Phenaki

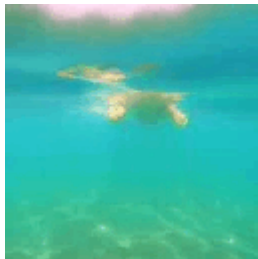
The water is magical



Prompts used:

- A photorealistic teddy bear is swimming in the ocean at San Francisco
- The teddy bear goes under water
- The teddy bear keeps swimming under the water with colorful fishes
- A panda bear is swimming under water

Chilling on the beach



Prompts used:

- A teddy bear diving in the ocean
- A teddy bear emerges from the water
- A teddy bear walks on the beach
- Camera zooms out to the teddy bear in the campfire by the beach

Fireworks on the spacewalk



Prompts used:

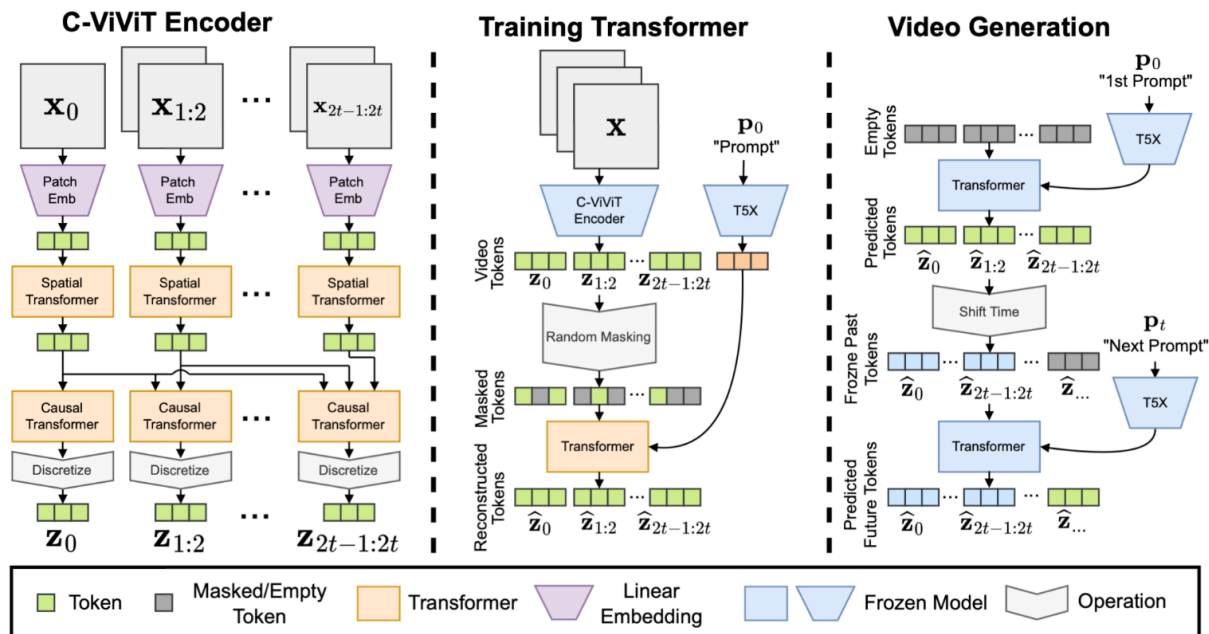
- Side view of an astronaut is walking through a puddle on mars
- The astronaut is dancing on mars
- The astronaut walks his dog on mars
- The astronaut and his dog watch fireworks

# Phenaki



Ruben Villegas, Mohammad Babaeizadeh, Pieter-Jan Kindermans, Hernan Moraldo, Han Zhang, Mohammad Taghi Saffar, Santiago Castro, Julius Kunze, Dumitru Erhan.  
Phenaki: Variable Length Video Generation From Open Domain Textual Description. 2022 <https://arxiv.org/abs/2210.02399>  
<https://phenaki.video>

# Phenaki



Ruben Villegas, Mohammad Babaeizadeh, Pieter-Jan Kindermans, Hernan Moraldo, Han Zhang, Mohammad Taghi Saffar, Santiago Castro, Julius Kunze, Dumitru Erhan.

Phenaki: Variable Length Video Generation From Open Domain Textual Description. 2022 <https://arxiv.org/abs/2210.02399>

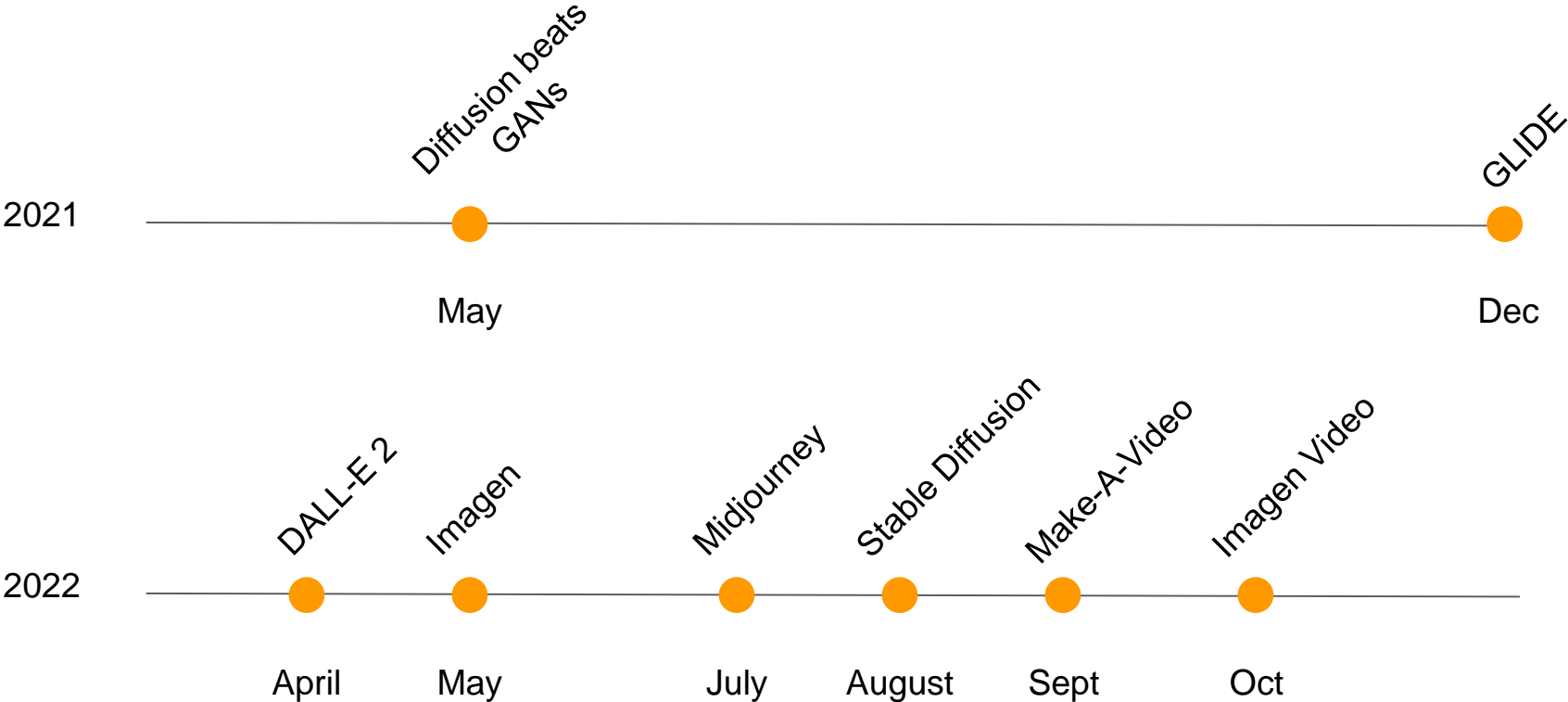
<https://phenaki.video>

# Outline

- Introduction
- Variational Autoencoders
- Autoregressive Models
- Diffusion Models
  - Denoising diffusion models
  - Text-to-image generation
    - DALL-E 2
    - Latent Diffusion
  - Text-to-video generation
    - Make-A-Video
- Generative Adversarial Networks
- Evaluation



# Diffusion Models Timeline



# Diffusion Models

- Diffusion models are inspired by non-equilibrium thermodynamics.
- They define a Markov chain of diffusion steps to slowly add random noise to data and then learn to reverse the diffusion process to construct desired data samples from the noise.
- Unlike VAE or flow models, diffusion models are learned with a fixed procedure and the latent variable has high dimensionality (same as the original data).

# Slide Credits

## **CVPR 2022 Tutorial**

Denoising Diffusion-based Generative Modeling

by Karsten Kreis, Ruiqi Gao and Arash Vahdat

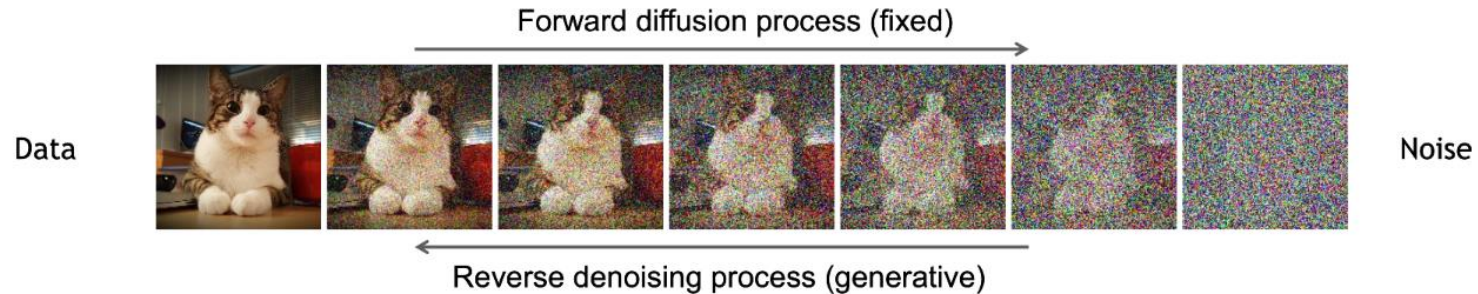
<https://cvpr2022-tutorial-diffusion-models.github.io>

# Denoising Diffusion Models

Learning to generate by denoising

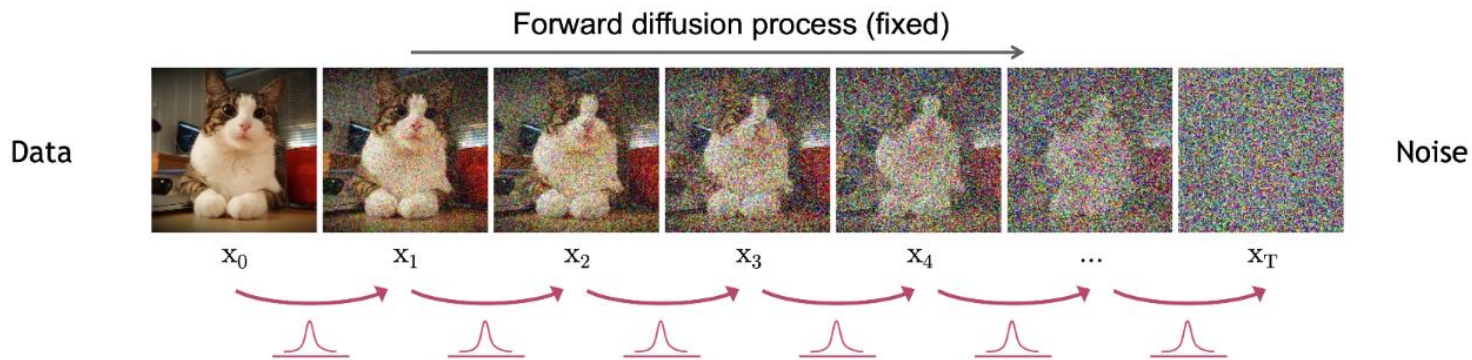
Denoising diffusion models consist of two processes:

- Forward diffusion process that gradually adds noise to input
- Reverse denoising process that learns to generate data by denoising



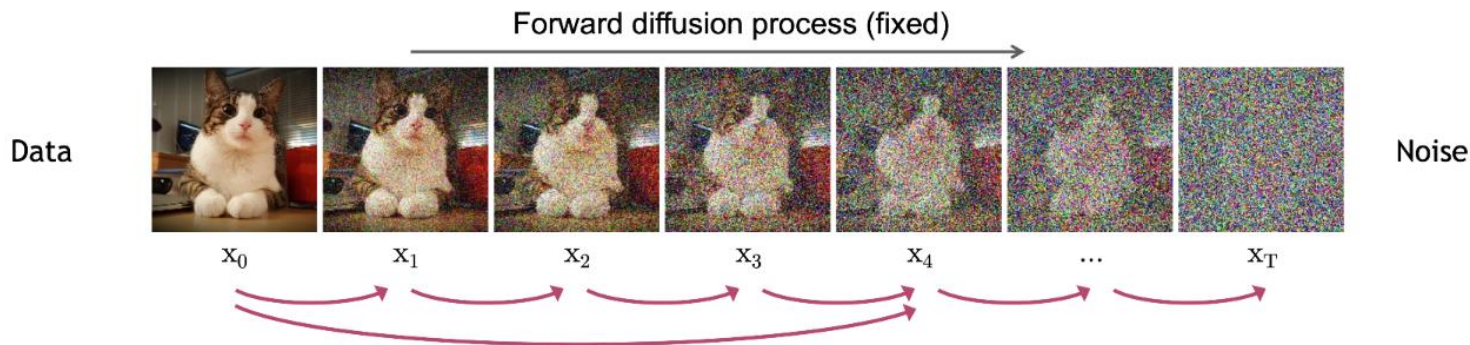
# Forward Diffusion Process

The formal definition of the forward process in T steps:



$$q(\mathbf{x}_t | \mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \sqrt{1 - \beta_t} \mathbf{x}_{t-1}, \beta_t \mathbf{I}) \quad \longrightarrow \quad q(\mathbf{x}_{1:T} | \mathbf{x}_0) = \prod_{t=1}^T q(\mathbf{x}_t | \mathbf{x}_{t-1}) \quad (\text{joint})$$

# Diffusion Kernel



Define  $\bar{\alpha}_t = \prod_{s=1}^t (1 - \beta_s)$      $\rightarrow$      $q(\mathbf{x}_t | \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t; \sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$     (Diffusion Kernel)

For sampling:  $\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$     where  $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

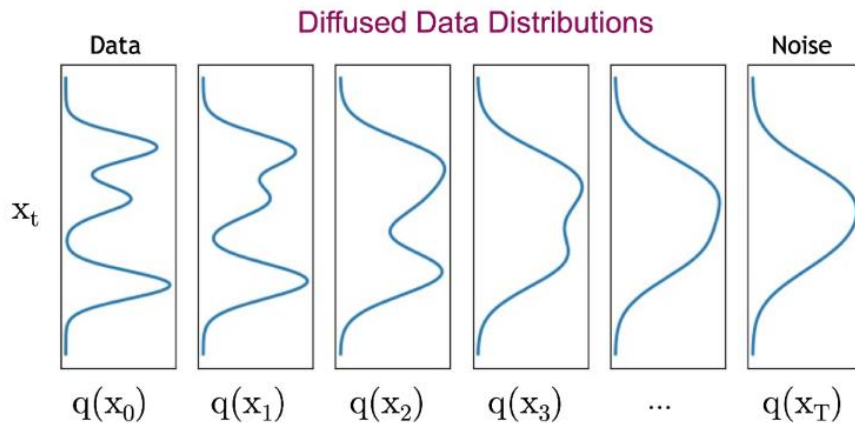
$\beta_t$  values schedule (i.e., the noise schedule) is designed such that  $\bar{\alpha}_T \rightarrow 0$  and  $q(\mathbf{x}_T | \mathbf{x}_0) \approx \mathcal{N}(\mathbf{x}_T; \mathbf{0}, \mathbf{I})$

# What happens to a distribution in the forward diffusion?

So far, we discussed the diffusion kernel  $q(\mathbf{x}_t|\mathbf{x}_0)$  but what about  $q(\mathbf{x}_t)$ ?

$$q(\mathbf{x}_t) = \int \underbrace{q(\mathbf{x}_0, \mathbf{x}_t)}_{\text{Joint dist.}} d\mathbf{x}_0 = \int \underbrace{q(\mathbf{x}_0)}_{\text{Input data dist.}} \underbrace{q(\mathbf{x}_t|\mathbf{x}_0)}_{\text{Diffusion kernel}} d\mathbf{x}_0$$

The diffusion kernel is Gaussian convolution.



We can sample  $\mathbf{x}_t \sim q(\mathbf{x}_t)$  by first sampling  $\mathbf{x}_0 \sim q(\mathbf{x}_0)$  and then sampling  $\mathbf{x}_t \sim q(\mathbf{x}_t|\mathbf{x}_0)$  (i.e., ancestral sampling).

# Generative Learning by Denoising

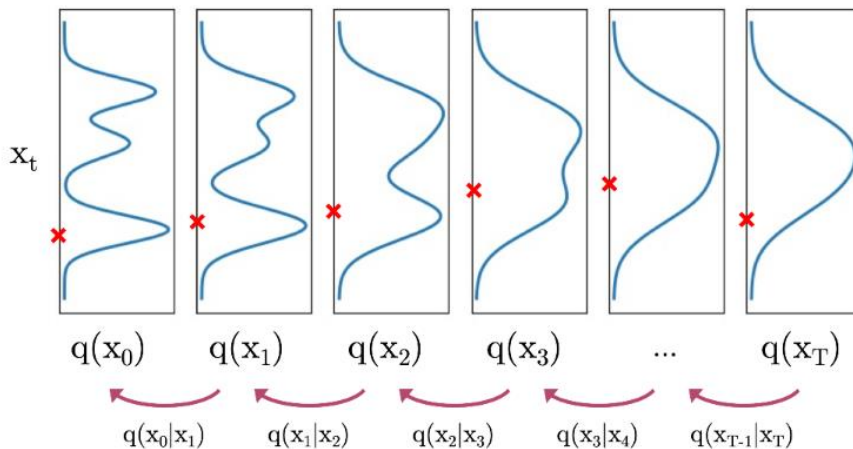
Recall, that the diffusion parameters are designed such that  $q(\mathbf{x}_T) \approx \mathcal{N}(\mathbf{x}_T; \mathbf{0}, \mathbf{I})$

**Generation:**

Sample  $\mathbf{x}_T \sim \mathcal{N}(\mathbf{x}_T; \mathbf{0}, \mathbf{I})$

Iteratively sample  $\mathbf{x}_{t-1} \sim \underbrace{q(\mathbf{x}_{t-1}|\mathbf{x}_t)}_{\text{True Denoising Dist.}}$

Diffused Data Distributions



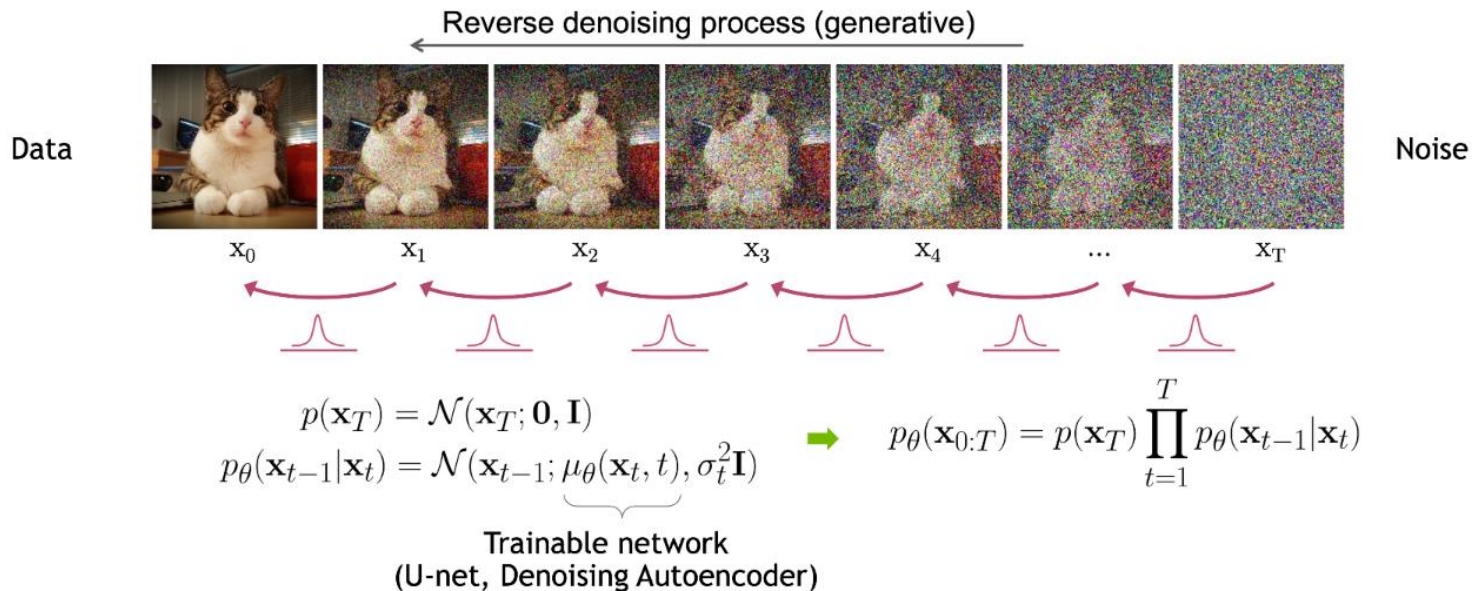
In general,  $q(\mathbf{x}_{t-1}|\mathbf{x}_t) \propto q(\mathbf{x}_{t-1})q(\mathbf{x}_t|\mathbf{x}_{t-1})$  is intractable.

Can we approximate  $q(\mathbf{x}_{t-1}|\mathbf{x}_t)$ ? Yes, we can use a **Normal distribution** if  $\beta_t$  is small in each forward diffusion step.



# Reverse Denoising Process

Formal definition of forward and reverse processes in T steps:



# Learning Denoising Model

## Variational upper bound

For training, we can form variational upper bound that is commonly used for training variational autoencoders:

$$\mathbb{E}_{q(\mathbf{x}_0)} [-\log p_\theta(\mathbf{x}_0)] \leq \mathbb{E}_{q(\mathbf{x}_0)q(\mathbf{x}_{1:T}|\mathbf{x}_0)} \left[ -\log \frac{p_\theta(\mathbf{x}_{0:T})}{q(\mathbf{x}_{1:T}|\mathbf{x}_0)} \right] =: L$$

[Sohl-Dickstein et al. ICML 2015](#) and [Ho et al. NeurIPS 2020](#) show that:

$$L = \mathbb{E}_q \left[ \underbrace{D_{\text{KL}}(q(\mathbf{x}_T|\mathbf{x}_0)||p(\mathbf{x}_T))}_{L_T} + \sum_{t>1} \underbrace{D_{\text{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0)||p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t))}_{L_{t-1}} \underbrace{- \log p_\theta(\mathbf{x}_0|\mathbf{x}_1)}_{L_0} \right]$$

where  $q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0)$  is the tractable posterior distribution:

$$q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_{t-1}; \tilde{\mu}_t(\mathbf{x}_t, \mathbf{x}_0), \tilde{\beta}_t \mathbf{I}),$$

$$\text{where } \tilde{\mu}_t(\mathbf{x}_t, \mathbf{x}_0) := \frac{\sqrt{\bar{\alpha}_{t-1}}\beta_t}{1 - \bar{\alpha}_t} \mathbf{x}_0 + \frac{\sqrt{1 - \beta_t}(1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t \text{ and } \tilde{\beta}_t := \frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t$$

# Parameterizing the Denoising Model

Since both  $q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0)$  and  $p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t)$  are Normal distributions, the KL divergence has a simple form:

$$L_{t-1} = D_{\text{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0)||p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t)) = \mathbb{E}_q \left[ \frac{1}{2\sigma_t^2} \|\tilde{\mu}_t(\mathbf{x}_t, \mathbf{x}_0) - \mu_\theta(\mathbf{x}_t, t)\|^2 \right] + C$$

Recall that  $\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{(1 - \bar{\alpha}_t)} \epsilon$ . [Ho et al. NeurIPS 2020](#) observe that:

$$\tilde{\mu}_t(\mathbf{x}_t, \mathbf{x}_0) = \frac{1}{\sqrt{1 - \beta_t}} \left( \mathbf{x}_t - \frac{\beta_t}{\sqrt{1 - \bar{\alpha}_t}} \epsilon \right)$$

They propose to represent the mean of the denoising model using a *noise-prediction* network:

$$\mu_\theta(\mathbf{x}_t, t) = \frac{1}{\sqrt{1 - \beta_t}} \left( \mathbf{x}_t - \frac{\beta_t}{\sqrt{1 - \bar{\alpha}_t}} \epsilon_\theta(\mathbf{x}_t, t) \right)$$

With this parameterization

$$L_{t-1} = \mathbb{E}_{\mathbf{x}_0 \sim q(\mathbf{x}_0), \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \frac{\beta_t^2}{2\sigma_t^2(1 - \beta_t)(1 - \bar{\alpha}_t)} \|\epsilon - \underbrace{\epsilon_\theta(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t)}_{\mathbf{x}_t}\|^2 \right] + C$$

# Training Objective Weighting

Trading likelihood for perceptual quality

$$L_{t-1} = \mathbb{E}_{\mathbf{x}_0 \sim q(\mathbf{x}_0), \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \underbrace{\frac{\beta_t^2}{2\sigma_t^2(1-\beta_t)(1-\bar{\alpha}_t)}}_{\lambda_t} \|\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t} \epsilon, t)\|^2 \right]$$

The time dependent  $\lambda_t$  ensures that the training objective is weighted properly for the maximum data likelihood training.

However, this weight is often very large for small t's.

[Ho et al. NeurIPS 2020](#) observe that simply setting  $\lambda_t = 1$  improves sample quality. So, they propose to use:

$$L_{\text{simple}} = \mathbb{E}_{\mathbf{x}_0 \sim q(\mathbf{x}_0), \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I}), t \sim \mathcal{U}(1, T)} \left[ \|\epsilon - \epsilon_\theta(\underbrace{\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t} \epsilon}_{\mathbf{x}_t}, t)\|^2 \right]$$

For more advanced weighting see [Choi et al., Perception Prioritized Training of Diffusion Models, CVPR 2022](#).

# Summary

## Training and Sample Generation

---

### Algorithm 1 Training

---

- 1: **repeat**
  - 2:  $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
  - 3:  $t \sim \text{Uniform}(\{1, \dots, T\})$
  - 4:  $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
  - 5: Take gradient descent step on  
$$\nabla_{\theta} \left\| \epsilon - \epsilon_{\theta}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t) \right\|^2$$
  - 6: **until** converged
- 

---

### Algorithm 2 Sampling

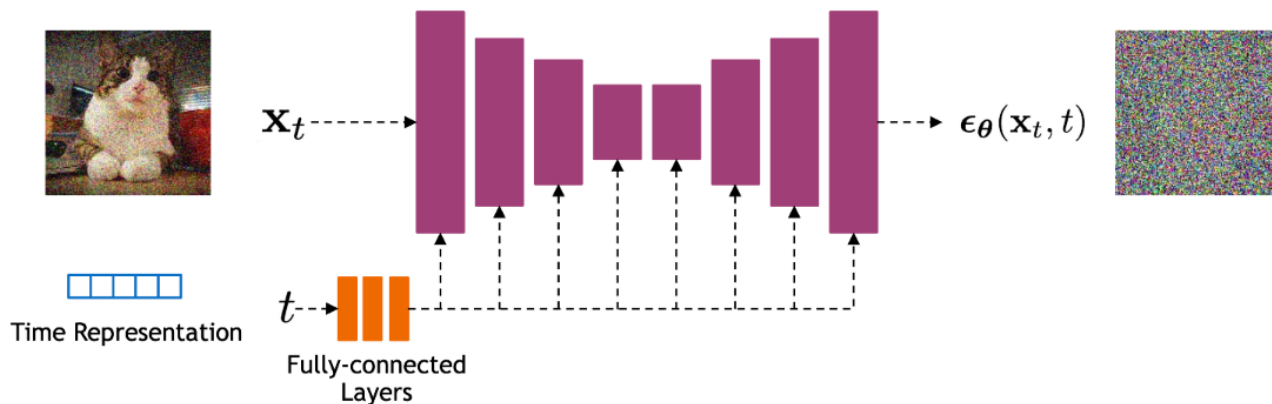
---

- 1:  $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
  - 2: **for**  $t = T, \dots, 1$  **do**
  - 3:  $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
  - 4:  $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left( \mathbf{x}_t - \frac{1 - \alpha_t}{\sqrt{1 - \bar{\alpha}_t}} \epsilon_{\theta}(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$
  - 5: **end for**
  - 6: **return**  $\mathbf{x}_0$
-

# Implementation Considerations

## Network Architectures

Diffusion models often use U-Net architectures with ResNet blocks and self-attention layers to represent  $\epsilon_{\theta}(\mathbf{x}_t, t)$



Time representation: sinusoidal positional embeddings or random Fourier features.

Time features are fed to the residual blocks using either simple spatial addition or using adaptive group normalization layers. (see [Dhariwal and Nichol NeurIPS 2021](#))

# Summary

## Denosing Diffusion Probabilistic Models

In this part, we reviewed denosing diffusion probabilistic models.

The model is trained by sampling from the forward diffusion process and training a denosing model to predict the noise.

We discussed how the forward process perturbs the data distribution or data samples.

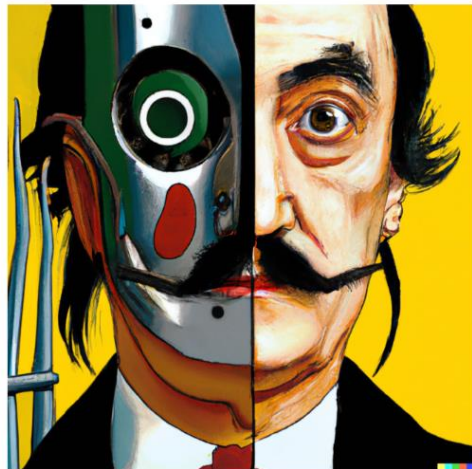
The devil is in the details:

- Network architectures
- Objective weighting
- Diffusion parameters (i.e., noise schedule)

See [“Elucidating the Design Space of Diffusion-Based Generative Models”](#) by Karras et al. for important design decisions.

# DALL-E 2

- is introduced by OpenAI
- generates 1024 x 1024 images from text using diffusion models.



vibrant portrait painting of Salvador Dalí with a robotic half face



a shiba inu wearing a beret and black turtleneck



a close up of a handpalm with leaves growing from it

Aditya Ramesh, Prafulla Dhariwal, Alex Nichol, Casey Chu, Mark Chen.

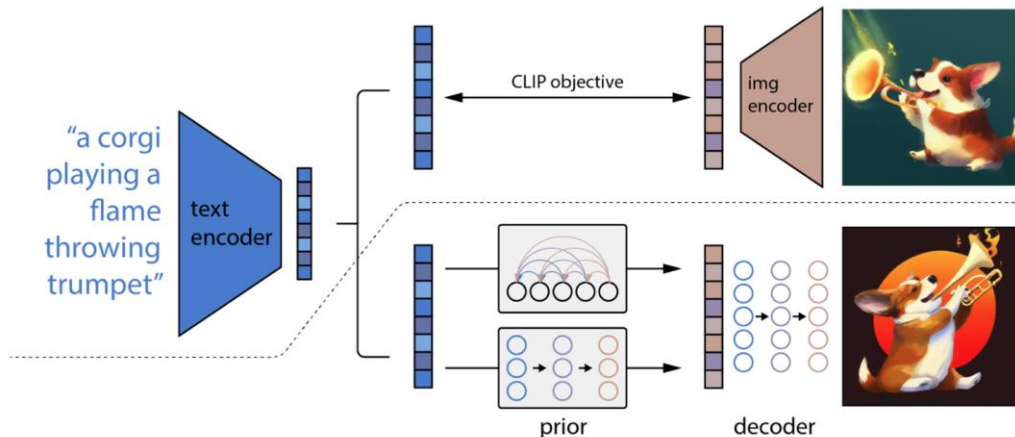
Hierarchical Text-Conditional Image Generation with CLIP Latents. 2022 <https://arxiv.org/abs/2204.06125>

<https://openai.com/product/dall-e-2>



# DALL-E 2

1. generates a CLIP text embedding for text caption
2. “prior” network generates CLIP image embedding from text embedding
3. diffusion decoder generates image from the image embedding



Aditya Ramesh, Prafulla Dhariwal, Alex Nichol, Casey Chu, Mark Chen.

Hierarchical Text-Conditional Image Generation with CLIP Latents. 2022 <https://arxiv.org/abs/2204.06125>

<https://openai.com/product/dall-e-2>

# DALL-E 2 training

Can vary images while preserving style and semantics in the embeddings

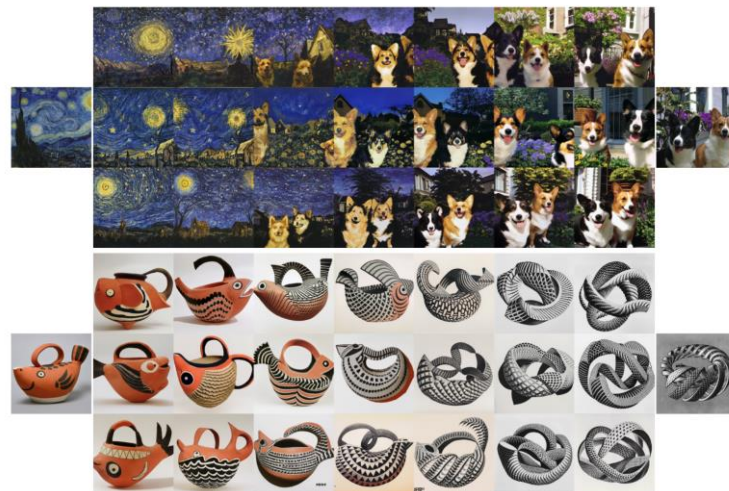
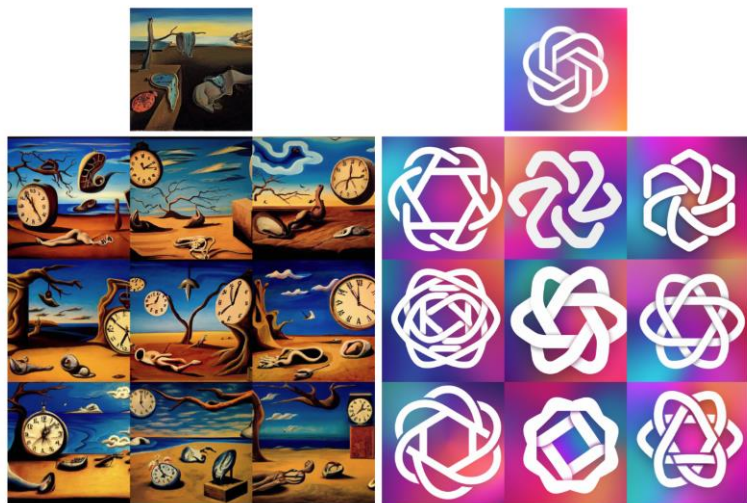


Figure 4: Variations between two images by interpolating their CLIP image embedding and then decoding with a diffusion model. We fix the decoder seed across each row. The intermediate variations naturally blend the content and style from both input images.

Aditya Ramesh, Prafulla Dhariwal, Alex Nichol, Casey Chu, Mark Chen.

Hierarchical Text-Conditional Image Generation with CLIP Latents. 2022 <https://arxiv.org/abs/2204.06125>

<https://openai.com/product/dall-e-2>

# DALL-E 2 Limitations



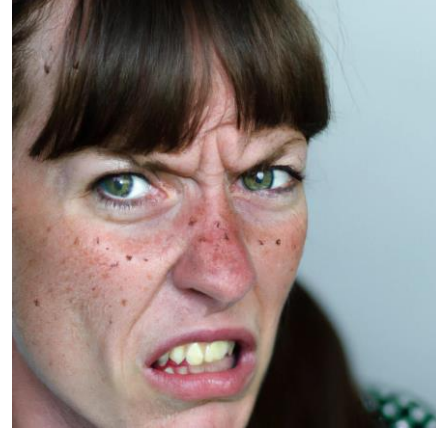
A family dining for Christmas



A hyperrealistic man's face smiling with a pair of sunglasses



A green-eyed woman with freckles showing happiness



A green-eyed woman with freckles showing disgust

# Midjourney



Capture the essence of a young footballer wearing the Paris Saint-Germain cinematic uniform through an extreme close-up portrait, focusing on the intricate details of her face and expressions, very detailed, Octane Render, cinematic, digital art



animated blonde blue eyes love tattoo soft male guy enhanced by ai smile defined --q 2 --s 750



surprised and happy man realistic

# Midjourney

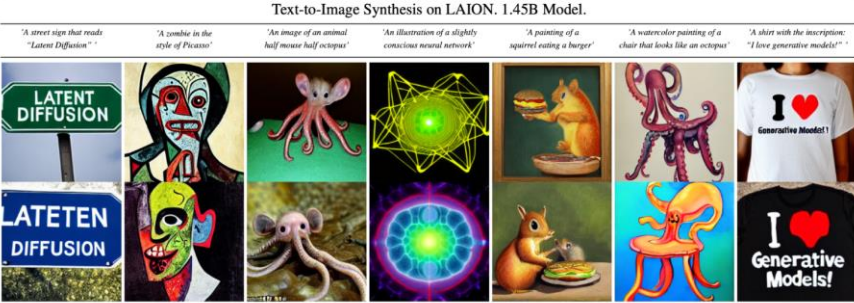


eiffel tower in space with an alien on top



model and robot in fashion show, public,  
cyberpunk,small robot ,red road ,  
cyberpunk,8k rendering

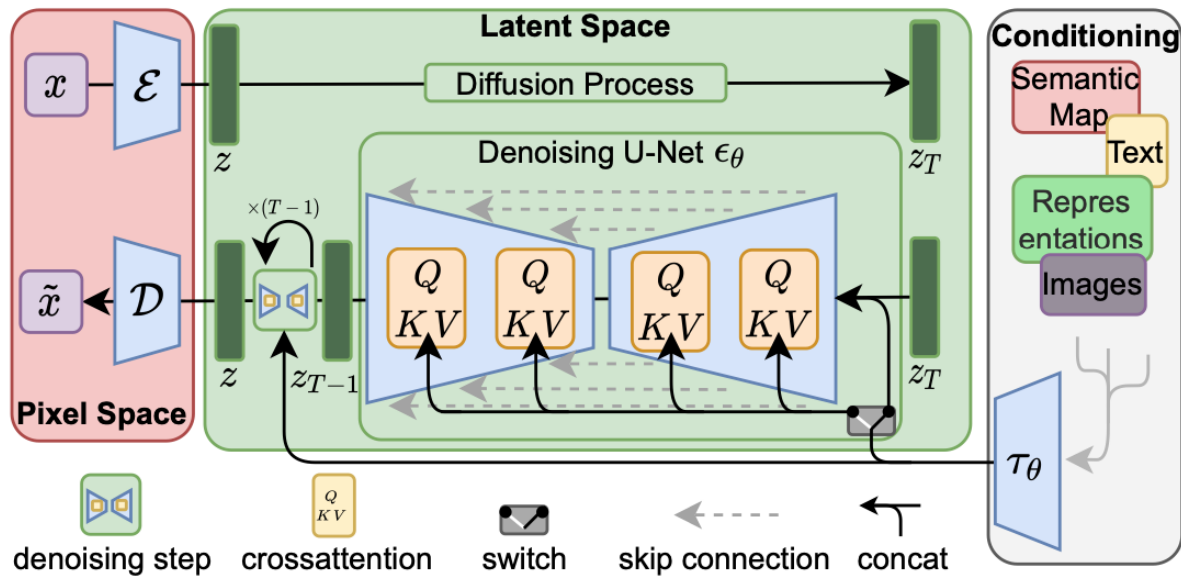
# Latent Diffusion Models



Robin Rombach, Andreas Blattmann, Dominik Lorenz, Patrick Esser, Björn Ommer.

High-Resolution Image Synthesis with Latent Diffusion Models. 2022 <https://arxiv.org/abs/2112.10752>

# Latent Diffusion Models



Robin Rombach, Andreas Blattmann, Dominik Lorenz, Patrick Esser, Björn Ommer.

High-Resolution Image Synthesis with Latent Diffusion Models. 2022 <https://arxiv.org/abs/2112.10752>

# Latent Diffusion Models



Robin Rombach, Andreas Blattmann, Dominik Lorenz, Patrick Esser, Björn Ommer.

High-Resolution Image Synthesis with Latent Diffusion Models. 2022 <https://arxiv.org/abs/2112.10752>



# Stable Diffusion

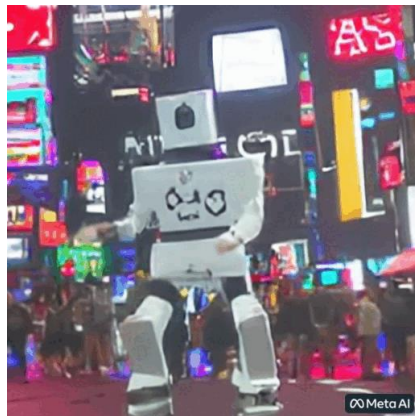
Prompt: realistic smiling woman



# Make-A-Video



A teddy bear painting a portrait



Robot dancing in times square



Cat watching TV with a remote  
in hand



A fluffy baby sloth with an  
orange knitted hat trying to  
figure out a laptop close up  
highly detailed studio lighting  
screen reflecting in its eye

Uriel Singer, Adam Polyak, Thomas Hayes, Xi Yin, Jie An, Songyang Zhang, Qiyuan Hu, Harry Yang, Oron Ashual, Oran Gafni, Devi Parikh, Sonal Gupta, Yaniv Taigman.

Make-A-Video: Text-to-Video Generation without Text-Video Data. 2022 <https://arxiv.org/abs/2209.14792>

<https://makeavideo.studio>

# Make-A-Video architecture

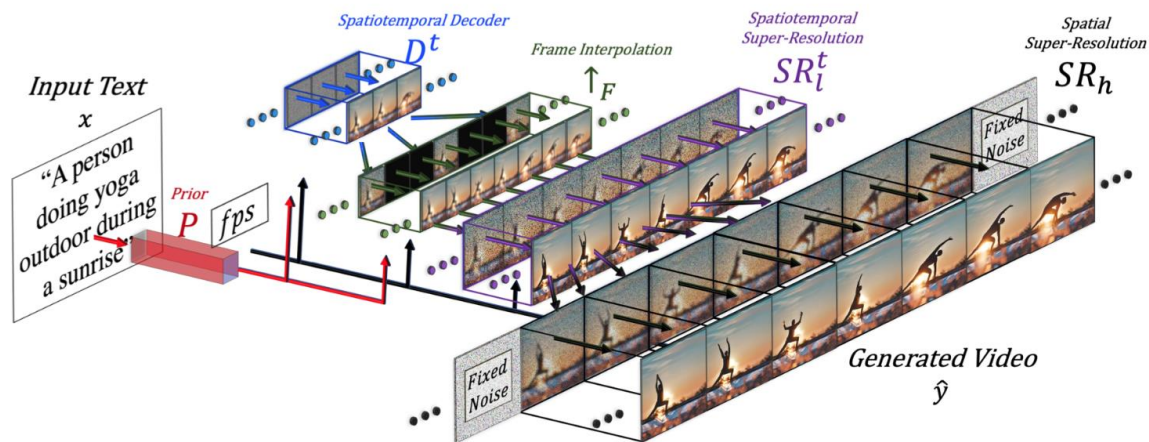


Figure 2: **Make-A-Video high-level architecture.** Given input text  $x$  translated by the prior  $P$  into an image embedding, and a desired frame rate  $fps$ , the decoder  $D^t$  generates 16  $64 \times 64$  frames, which are then interpolated to a higher frame rate by  $\uparrow_F$ , and increased in resolution to  $256 \times 256$  by  $SR_l^t$  and  $768 \times 768$  by  $SR_h$ , resulting in a high-spatiotemporal-resolution generated video  $\hat{y}$ .

Uriel Singer, Adam Polyak, Thomas Hayes, Xi Yin, Jie An, Songyang Zhang, Qiyuan Hu, Harry Yang, Oron Ashual, Oran Gafni, Devi Parikh, Sonal Gupta, Yaniv Taigman.

Make-A-Video: Text-to-Video Generation without Text-Video Data. 2022 <https://arxiv.org/abs/2209.14792>

<https://makeavideo.studio>

# Outline

- Introduction
- Variational Autoencoders
- Autoregressive Models
- Diffusion Models
- Generative Adversarial Networks
  - Architecture and training objective
  - Image generation
  - Video generation
  - Problems of GANs
- Evaluation

# Generative Adversarial Networks

- Setup: Assume we have data  $x_i$  drawn from distribution  $p_{data}(x)$ . Want to sample from  $p_{data}$ .

# Generative Adversarial Networks

- Setup: Assume we have data  $x_i$  drawn from distribution  $p_{data}(x)$ . Want to sample from  $p_{data}$ .
- Idea: Introduce a latent variable  $z$  with simple prior  $p(z)$ .

# Generative Adversarial Networks

- Setup: Assume we have data  $x_i$  drawn from distribution  $p_{data}(x)$ . Want to sample from  $p_{data}$ .
- Idea: Introduce a latent variable  $z$  with simple prior  $p(z)$ .
- Sample  $z \sim p(z)$  and pass to a Generator Network  $x = G(z)$

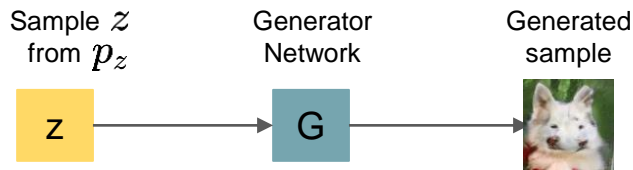
# Generative Adversarial Networks

- Setup: Assume we have data  $x_i$  drawn from distribution  $p_{data}(x)$ . Want to sample from  $p_{data}$ .
- Idea: Introduce a latent variable  $z$  with simple prior  $p(z)$ .
- Sample  $z \sim p(z)$  and pass to a Generator Network  $x = G(z)$
- Then  $x$  is a sample from the Generator distribution  $p_G$ . Want  $p_G = p_{data}$



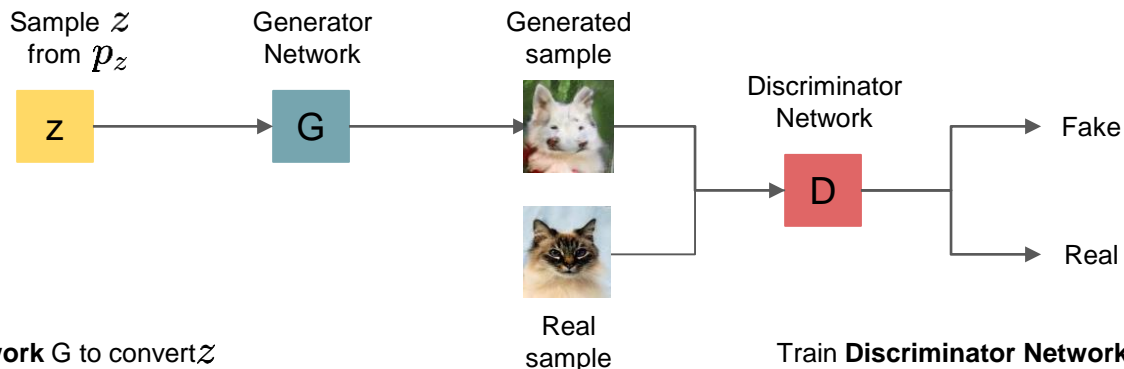
# Generative Adversarial Networks

- Setup: Assume we have data  $x_i$  drawn from distribution  $p_{data}(x)$ . Want to sample from  $p_{data}$ .
- Idea: Introduce a latent variable  $z$  with simple prior  $p(z)$ .
- Sample  $z \sim p(z)$  and pass to a Generator Network  $x = G(z)$
- Then  $x$  is a sample from the Generator distribution  $p_G$ . Want  $p_G = p_{data}$



# Generative Adversarial Networks

- Setup: Assume we have data  $x_i$  drawn from distribution  $p_{data}(x)$ . Want to sample from  $p_{data}$ .
- Idea: Introduce a latent variable  $z$  with simple prior  $p(z)$ .
- Sample  $z \sim p(z)$  and pass to a Generator Network  $x = G(z)$
- Then  $x$  is a sample from the Generator distribution  $p_G$ . Want  $p_G = p_{data}$



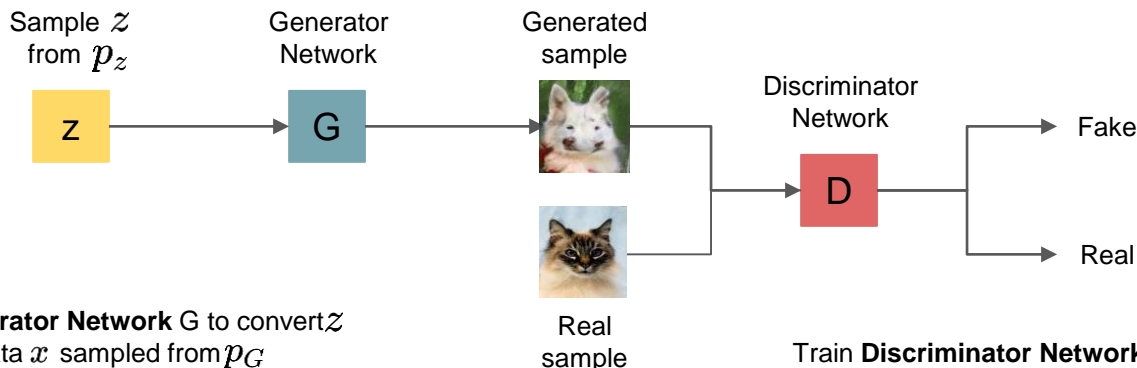
Train **Generator Network G** to convert  $z$  into fake data  $x$  sampled from  $p_G$

Train **Discriminator Network D** to classify data as real or fake (1/0)

# Generative Adversarial Networks: Training Objective

Jointly train generator G and discriminator D with a **minimax game**

$$\min_{\mathbf{G}} \max_{\mathbf{D}} (E_{x \sim p_{data}} [\log \mathbf{D}(x)] + E_{z \sim p(z)} [\log(1 - \mathbf{D}(\mathbf{G}(z)))])$$



Train **Generator Network G** to convert  $z$  into fake data  $x$  sampled from  $p_G$  by **fooling the Discriminator D**

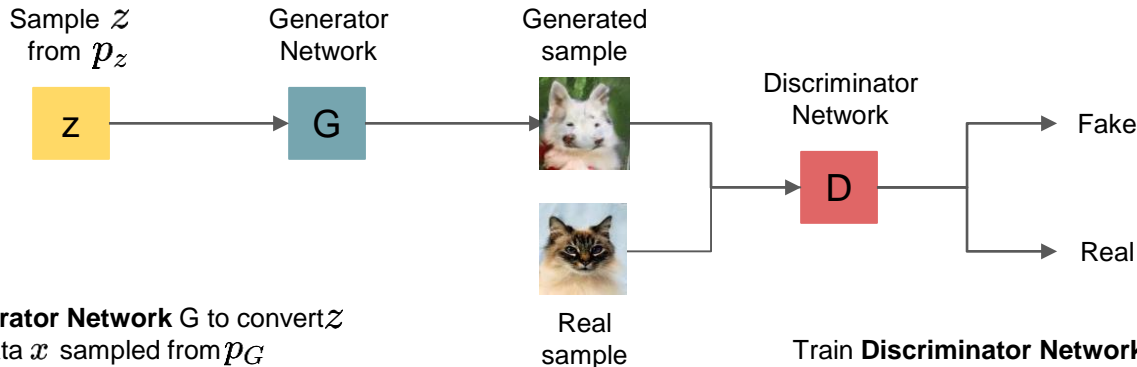
Train **Discriminator Network D** to classify data as real or fake (1/0)

# Generative Adversarial Networks: Training Objective

Jointly train generator  $G$  and discriminator  $D$  with a **minimax game**

Discriminator wants  $D(x)=1$  for  
real data

$$\min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$



Train **Generator Network G** to convert  $z$  into fake data  $x$  sampled from  $p_G$  by **fooling the Discriminator D**

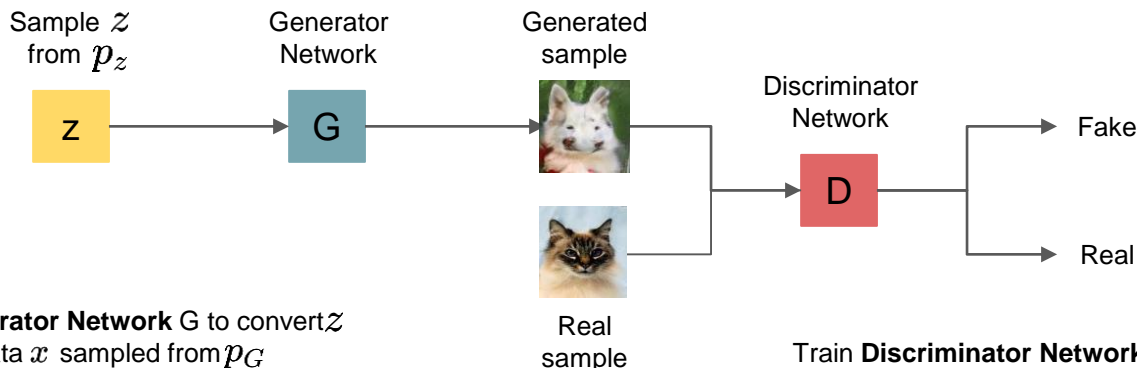
Train **Discriminator Network D** to classify data as real or fake (1/0)

# Generative Adversarial Networks: Training Objective

Jointly train generator  $G$  and discriminator  $D$  with a **minimax game**

Discriminator wants  $D(x)=0$  for  
fake data

$$\min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$



Train **Generator Network G** to convert  $z$  into fake data  $x$  sampled from  $p_G$  by **fooling the Discriminator D**

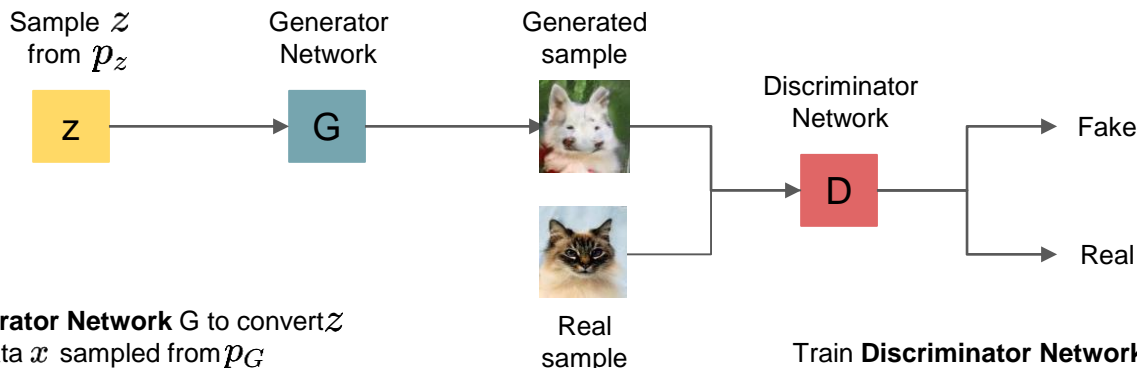
Train **Discriminator Network D** to classify data as real or fake (1/0)

# Generative Adversarial Networks: Training Objective

Jointly train generator  $G$  and discriminator  $D$  with a **minimax game**

$$\min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$

Generator wants  $D(x)=1$  for fake data



Train **Generator Network G** to convert  $z$  into fake data  $x$  sampled from  $p_G$  by **fooling the Discriminator D**

Train **Discriminator Network D** to classify data as real or fake (1/0)

# Generative Adversarial Networks: Training Objective

Jointly train generator  $G$  and discriminator  $D$  with a **minimax game**

$$\min_{\mathbf{G}} \max_{\mathbf{D}} (E_{x \sim p_{data}} [\log \mathbf{D}(x)] + E_{z \sim p(z)} [\log(1 - \mathbf{D}(\mathbf{G}(z)))])$$
$$= \min_{\mathbf{G}} \max_{\mathbf{D}} \mathbf{V}(\mathbf{G}, \mathbf{D})$$

Train  $G$  and  $D$  using alternating gradient updates:

1. Update  $\mathbf{D} = \mathbf{D} + \alpha_{\mathbf{D}} \frac{\delta \mathbf{V}}{\delta \mathbf{D}}$
2. Update  $\mathbf{G} = \mathbf{G} - \alpha_{\mathbf{G}} \frac{\delta \mathbf{V}}{\delta \mathbf{G}}$

# Generative Adversarial Networks: Optimality

$$\min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$



# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \end{aligned}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \max_D \int_X (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \end{aligned}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \max_D \int_X (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \\ &= \min_G \int_X \max_D (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \end{aligned}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))]]) & f(y) = a \log y + b \log(1 - y) \\ & = \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ & = \min_G \max_D \int_X (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \\ & = \min_G \int_X \max_D (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \end{aligned}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \max_D \int_X (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \\ &= \min_G \int_X \max_D (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \end{aligned}$$

$$f(y) = a \log y + b \log(1 - y)$$

$$f'(y) = \frac{a}{y} - \frac{b}{1-y}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \max_D \int_X (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \\ &= \min_G \int_X \max_D (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \end{aligned}$$

$$f(y) = a \log y + b \log(1 - y)$$

$$f'(y) = \frac{a}{y} - \frac{b}{1-y}$$

$$f'(y) = 0 \Leftrightarrow y = \frac{a}{a+b}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \max_D \int_X (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \\ &= \min_G \int_X \max_D (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \end{aligned}$$

$$f(y) = a \log y + b \log(1 - y)$$

$$f'(y) = \frac{a}{y} - \frac{b}{1-y}$$

$$f'(y) = 0 \Leftrightarrow y = \frac{a}{a+b}$$

**Optimal Discriminator:**

$$D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \int_X (p_{data}(x) \log D_G^*(x) + p_G(x) \log(1 - D_G^*(x))) dx \end{aligned}$$

$$\text{Optimal Discriminator: } D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$$



# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \int_X (p_{data}(x) \log D_G^*(x) + p_G(x) \log(1 - D_G^*(x))) dx \\ &= \min_G \int_X (p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} + p_G(x) \log \frac{p_G(x)}{p_{data}(x) + p_G(x)}) dx \end{aligned}$$

**Optimal Discriminator:**  $D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \int_X (p_{data}(x) \log D_G^*(x) + p_G(x) \log(1 - D_G^*(x))) dx \\ &= \min_G \int_X (p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} + p_G(x) \log \frac{p_G(x)}{p_{data}(x) + p_G(x)}) dx \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}]) \end{aligned}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \int_X (p_{data}(x) \log D_G^*(x) + p_G(x) \log(1 - D_G^*(x))) dx \\ &= \min_G \int_X (p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} + p_G(x) \log \frac{p_G(x)}{p_{data}(x) + p_G(x)}) dx \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}]) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{2 \times p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{2 \times p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \end{aligned}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))] ) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{2 \times p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{2 \times p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \end{aligned}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))] ) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{2 \times p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{2 \times p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \end{aligned}$$

$$\text{Kullback-Leibler Divergence: } KL(p, q) = E_{x \sim p} [\log \frac{p(x)}{q(x)}]$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))] ) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{2 \times p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{2 \times p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \\ &= \min_G (KL(p_{data}, \frac{p_{data} + p_G}{2}) + KL(p_G, \frac{p_{data} + p_G}{2}) - \log 4) \end{aligned}$$

**Kullback-Leibler Divergence:**  $KL(p, q) = E_{x \sim p} [\log \frac{p(x)}{q(x)}]$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{2 \times p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{2 \times p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \\ &= \min_G (KL(p_{data}, \frac{p_{data} + p_G}{2}) + KL(p_G, \frac{p_{data} + p_G}{2}) - \log 4) \end{aligned}$$

**Jensen-Shannon Divergence:**  $JSD(p, q) = \frac{1}{2}KL(p, \frac{p+q}{2}) + \frac{1}{2}KL(q, \frac{p+q}{2})$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{2 \times p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{2 \times p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \\ &= \min_G (KL(p_{data}, \frac{p_{data} + p_G}{2}) + KL(p_G, \frac{p_{data} + p_G}{2}) - \log 4) \\ &= \min_G (2 \times JSD(p_{data}, p_G) - \log 4) \end{aligned}$$

**Jensen-Shannon Divergence:**  $JSD(p, q) = \frac{1}{2}KL(p, \frac{p+q}{2}) + \frac{1}{2}KL(q, \frac{p+q}{2})$



# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{2 \times p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{2 \times p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \\ &= \min_G (KL(p_{data}, \frac{p_{data} + p_G}{2}) + KL(p_G, \frac{p_{data} + p_G}{2}) - \log 4) \\ &= \min_G (2 \times JSD(p_{data}, p_G) - \log 4) \end{aligned}$$

JSD is always nonnegative and zero when the two distributions are equal

=> the global minimum is  $p_{data} = p_G$

$$\text{Jensen-Shannon Divergence: } JSD(p, q) = \frac{1}{2} KL(p, \frac{p+q}{2}) + \frac{1}{2} KL(q, \frac{p+q}{2})$$

# Generative Adversarial Networks: Optimality

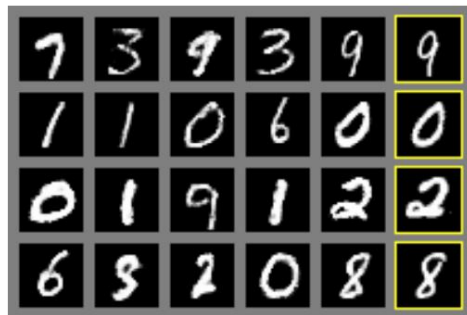
$$\min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$

$$= \min_G (2 * JSD(p_{data}, p_G) - \log 4)$$

**Summary:** The global minimum of the minimax game happens when:

1.  $D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$  (Optimal discriminator for any G)
2.  $p_G(x) = p_{data}(x)$  (Optimal generator for optimal D)

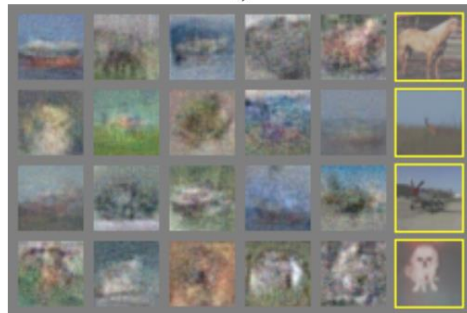
# Generative Adversarial Networks: results



a)



b)

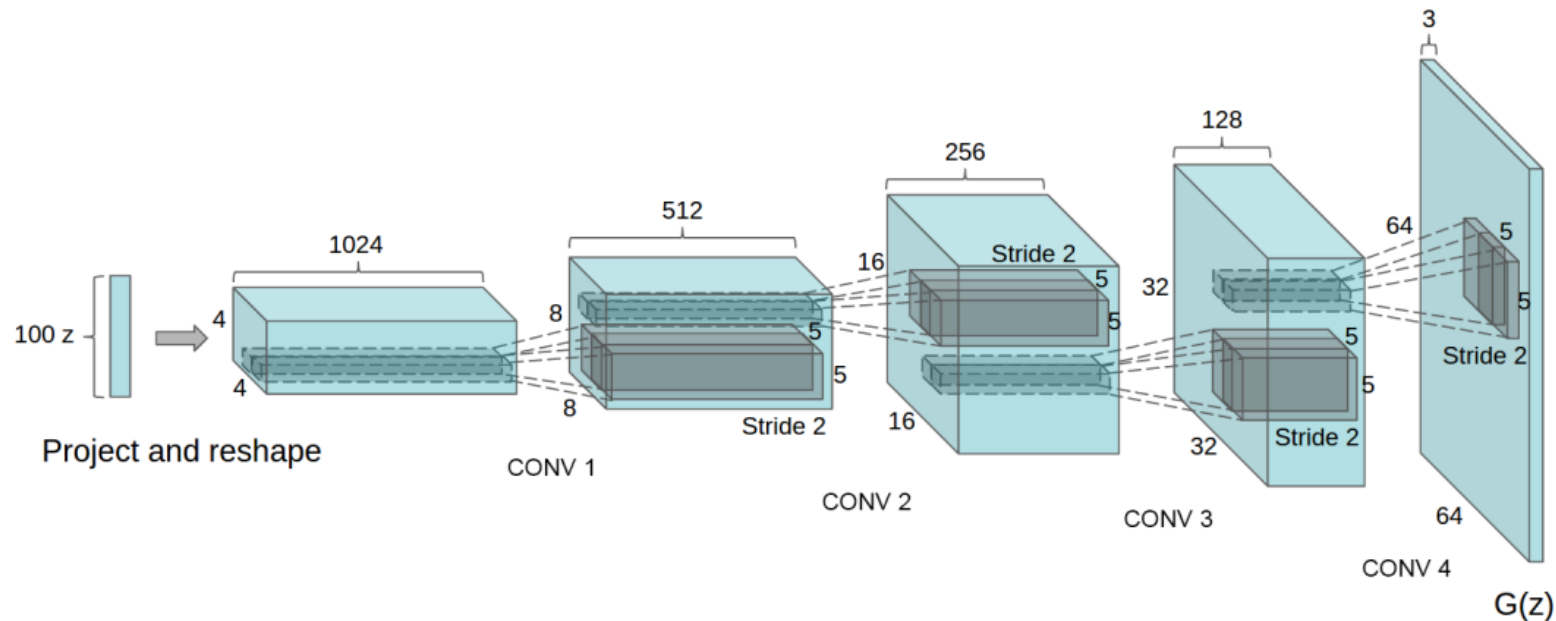


c)

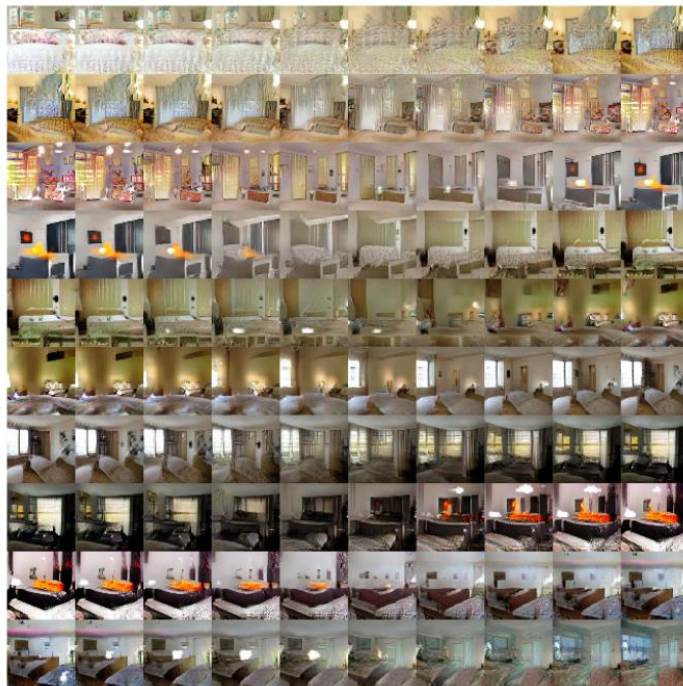


d)

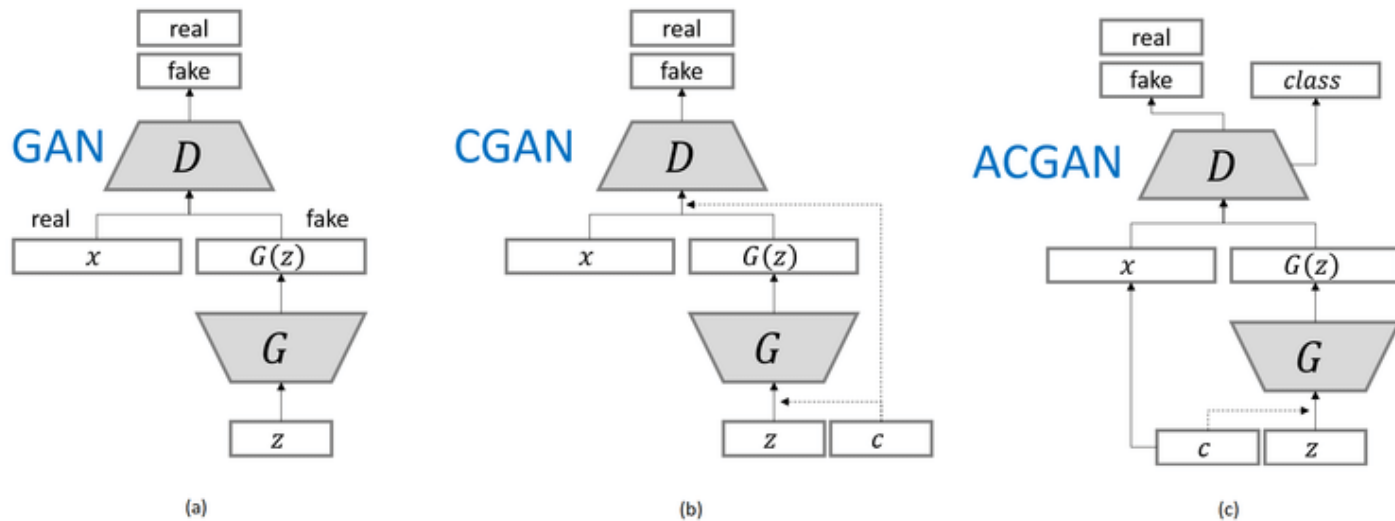
# Generative Adversarial Networks: DC-GAN



# Generative Adversarial Networks: Interpolation



# Conditional GANs



[b] Mehdi Mirza, Simon Osindero. Conditional Generative Adversarial Nets. 2014

[c] Augustus Odena, Christopher Olah, Jonathon Shlens. Conditional Image Synthesis With Auxiliary Classifier GANs. ICML 2016

# Conditional GANs



monarch butterfly



goldfinch



daisy

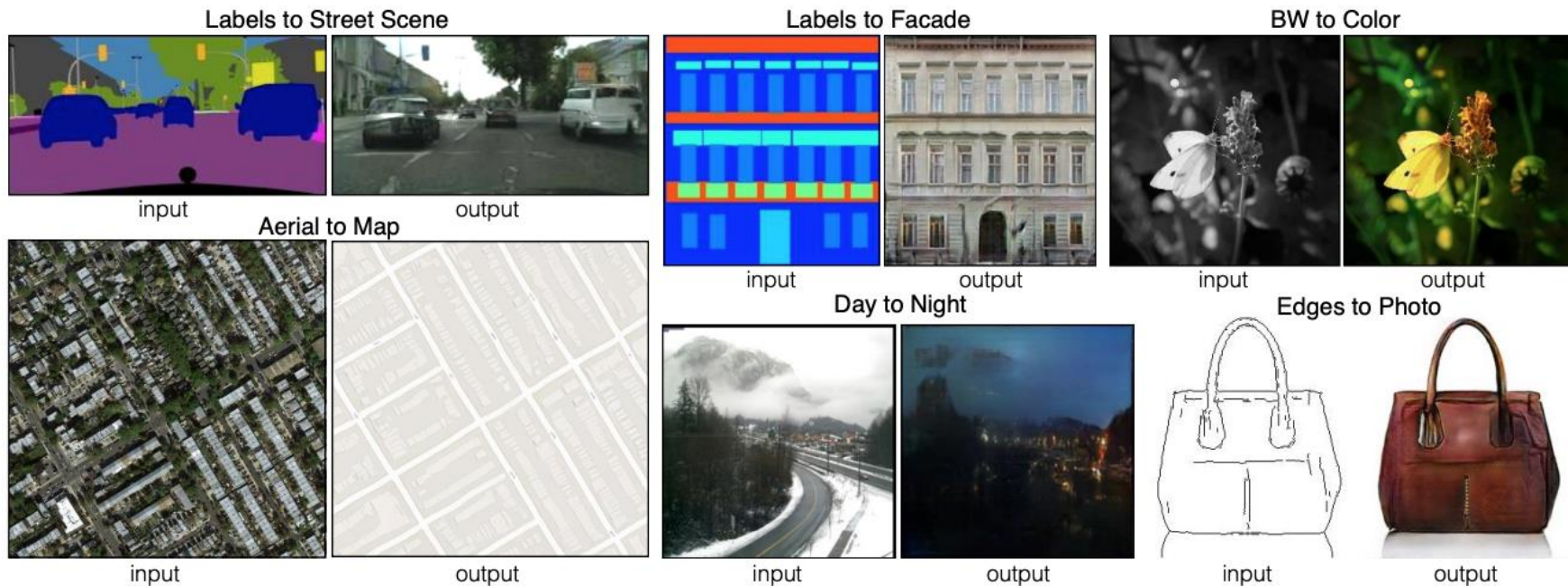


redshank



grey whale

# Image-to-Image Translation: Pix2Pix





# Image-to-Image Translation: Pix2Pix

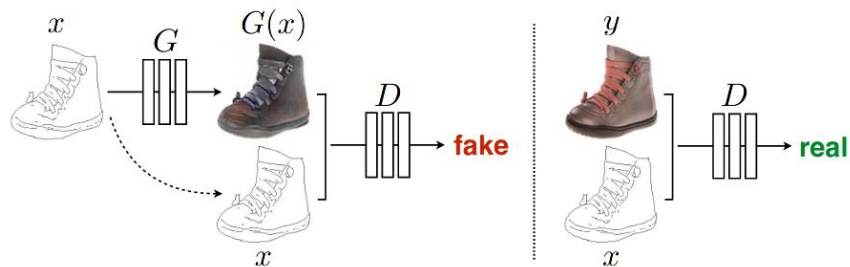
Objective:

$$G^* = \arg \min_G \max_D \mathcal{L}_{cGAN}(G, D) + \lambda \mathcal{L}_{L1}(G)$$

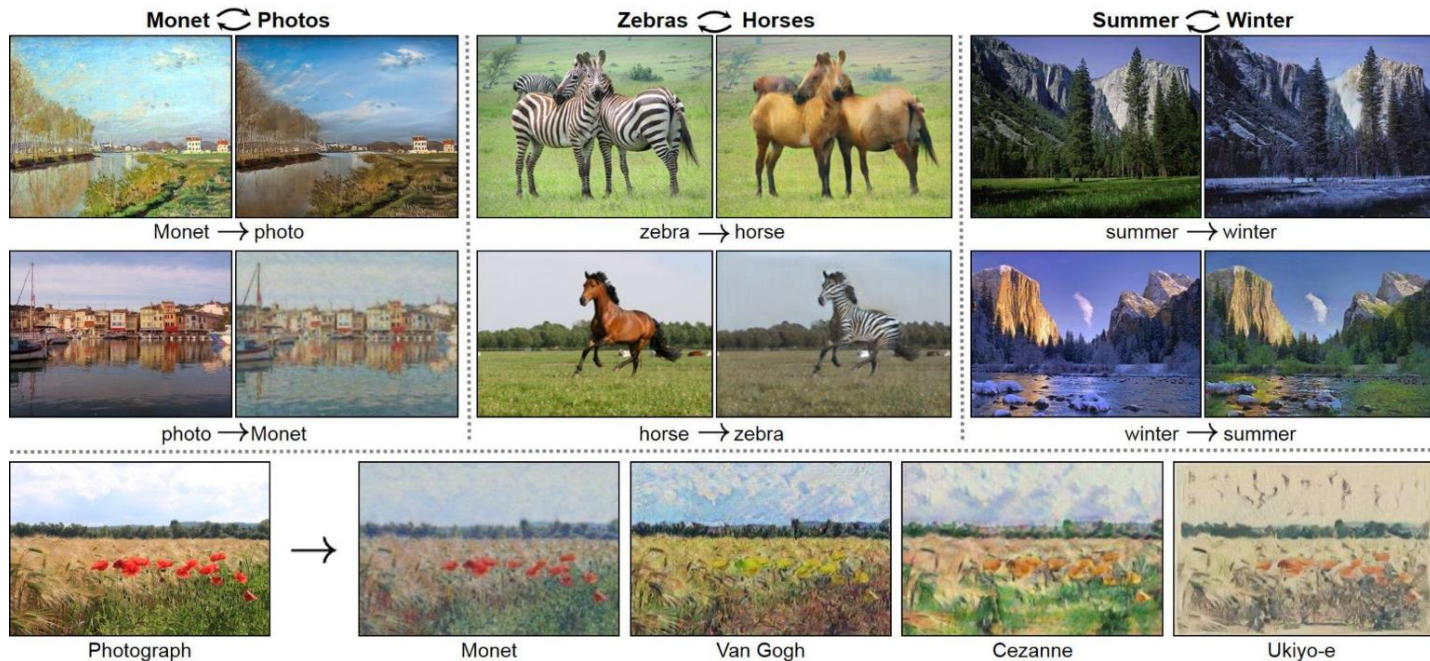
where

$$\mathcal{L}_{cGAN}(G, D) = \mathbb{E}_{x,y}[\log D(x, y)] + \mathbb{E}_{x,z}[\log(1 - D(x, G(x, z)))]$$

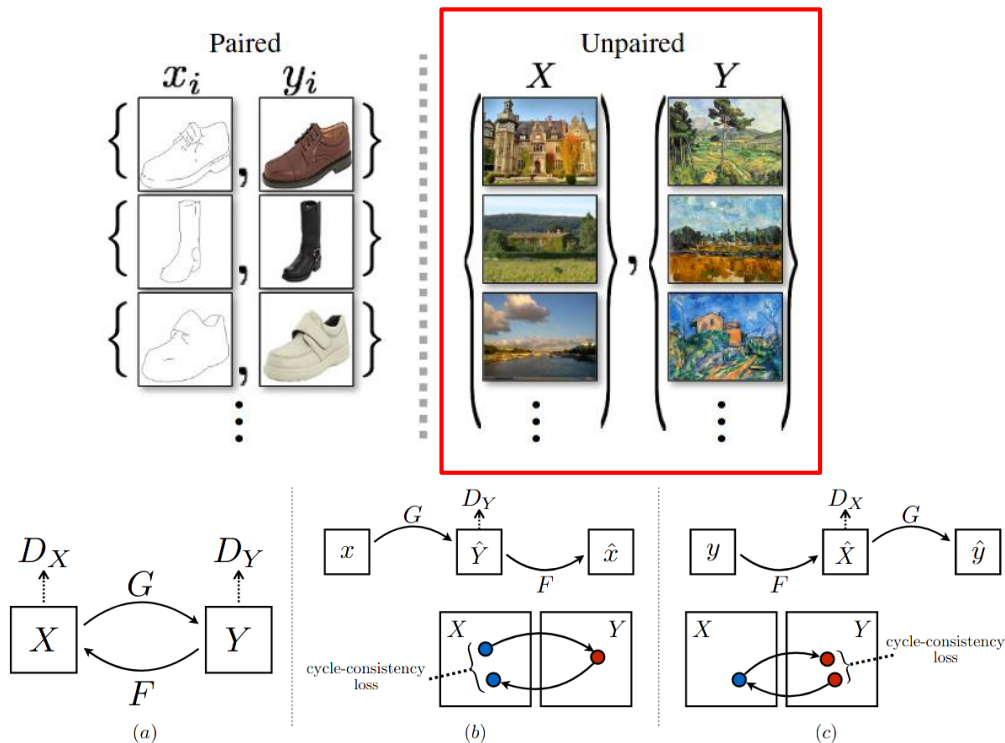
$$\mathcal{L}_{L1}(G) = \mathbb{E}_{x,y,z}[\|y - G(x, z)\|_1]$$



# Unpaired Image-to-Image Translation: CycleGAN



# Unpaired Image-to-Image Translation: CycleGAN



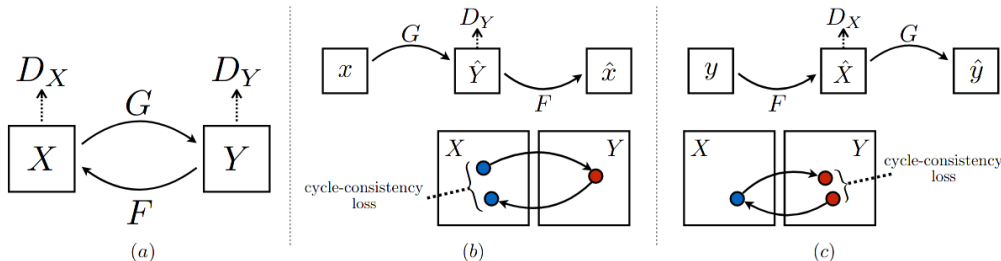
# Unpaired Image-to-Image Translation: CycleGAN

Objective:

$$\begin{aligned}\mathcal{L}(G, F, D_X, D_Y) = & \mathcal{L}_{\text{GAN}}(G, D_Y, X, Y) \\ & + \mathcal{L}_{\text{GAN}}(F, D_X, Y, X) \\ & + \lambda \mathcal{L}_{\text{cyc}}(G, F),\end{aligned}$$

where

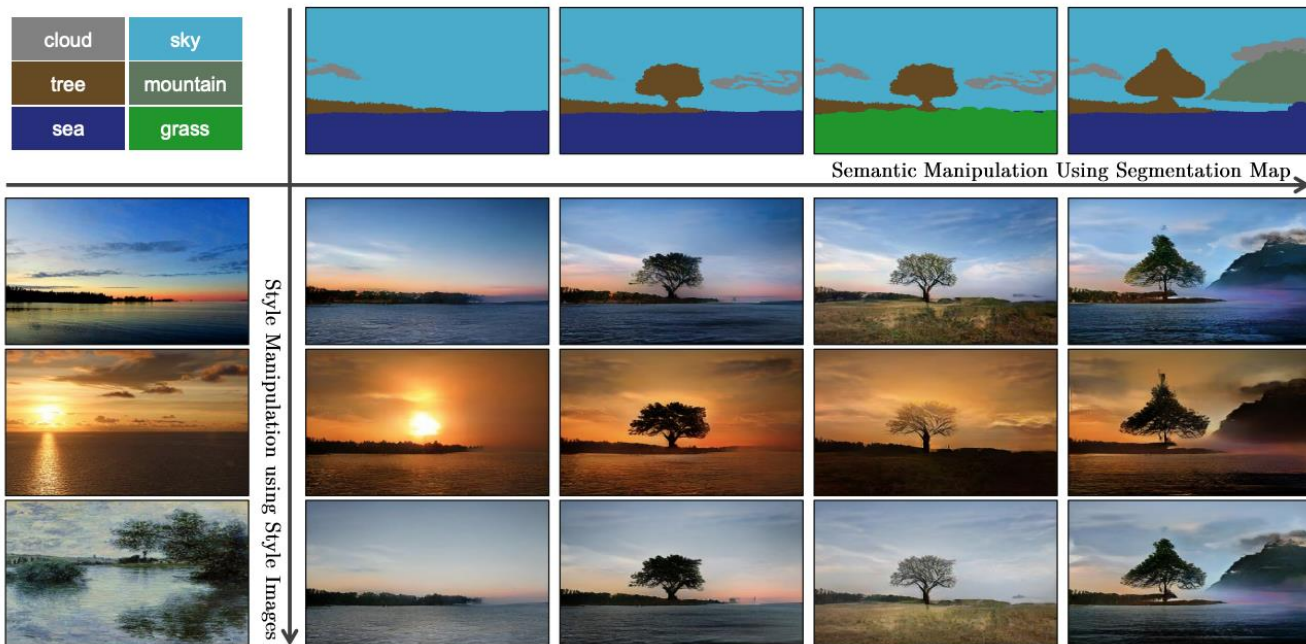
$$\begin{aligned}\mathcal{L}_{\text{GAN}}(G, D_Y, X, Y) = & \mathbb{E}_{y \sim p_{\text{data}}(y)} [\log D_Y(y)] \\ & + \mathbb{E}_{x \sim p_{\text{data}}(x)} [\log(1 - D_Y(G(x)))] , \\ \mathcal{L}_{\text{cyc}}(G, F) = & \mathbb{E}_{x \sim p_{\text{data}}(x)} [\|F(G(x)) - x\|_1] \\ & + \mathbb{E}_{y \sim p_{\text{data}}(y)} [\|G(F(y)) - y\|_1].\end{aligned}$$



# Unpaired Image-to-Image Translation: CycleGAN



# Label Map to Image



# Label Map to Image

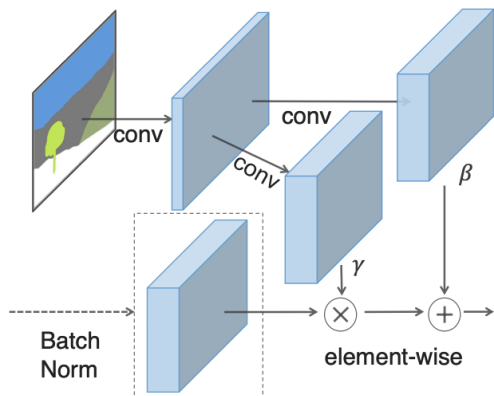


Figure 2: In the SPADE, the mask is first projected onto an embedding space and then convolved to produce the modulation parameters  $\gamma$  and  $\beta$ . Unlike prior conditional normalization methods,  $\gamma$  and  $\beta$  are not vectors, but tensors with spatial dimensions. The produced  $\gamma$  and  $\beta$  are multiplied and added to the normalized activation element-wise.

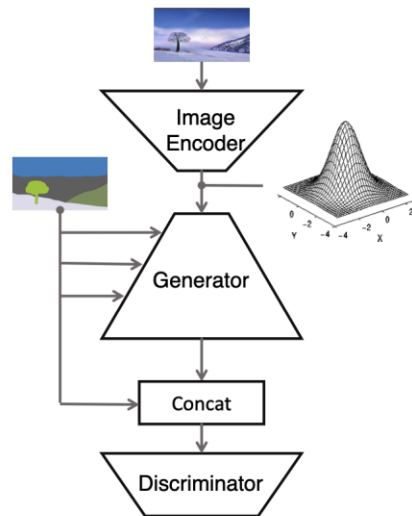
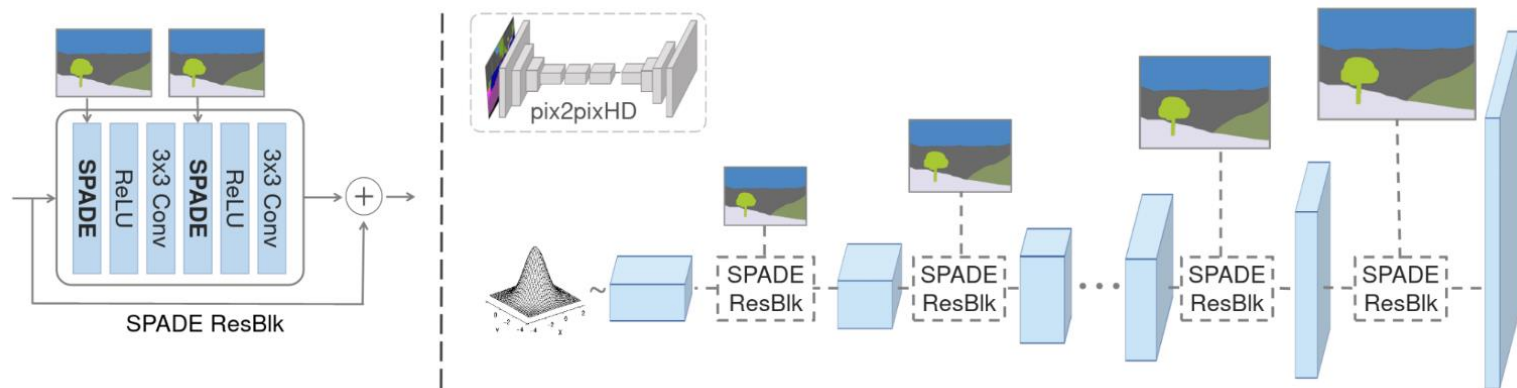


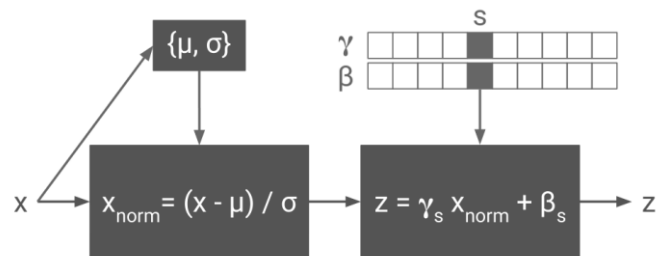
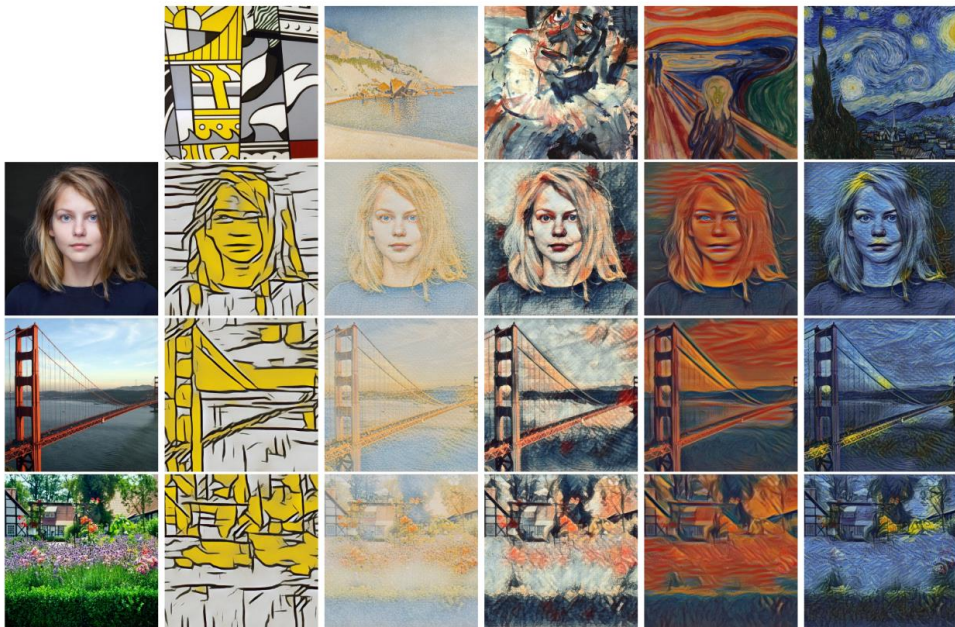
Figure 15: The image encoder encodes a real image to a latent representation for generating a mean vector and a variance vector. They are used to compute the noise input to the generator via the reparameterization trick [28]. The generator also takes the segmentation mask of the input image as input via the proposed SPADE ResBlks. The discriminator takes concatenation of the segmentation mask and the output image from the generator as input and aims to classify that as fake.

# Label Map to Image



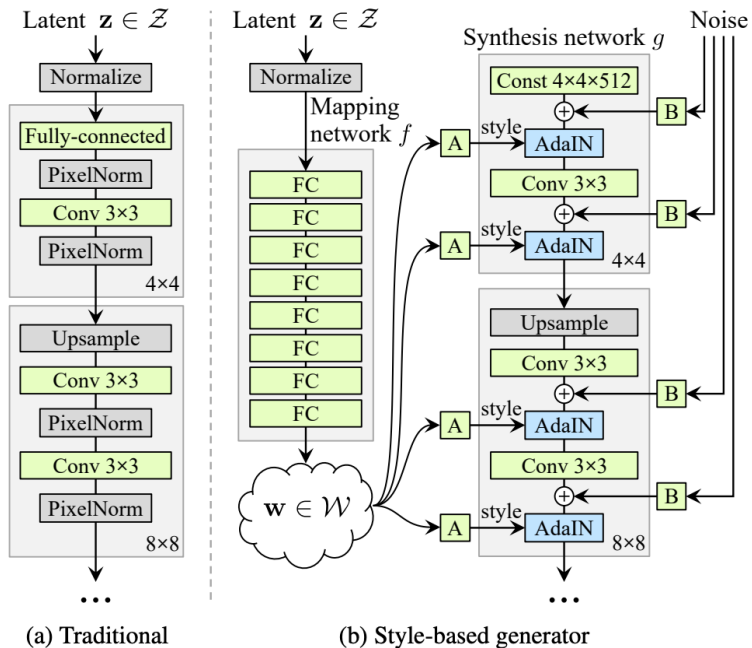


# Conditional GANs: Conditional Instance Normalization



The input activation  $x$  is normalized across spatial dimensions and scaled and shifted using style-dependent parameter vectors  $\gamma_s, \beta_s$  where  $s$  indexes the style label.

# StyleGAN



# StyleGAN

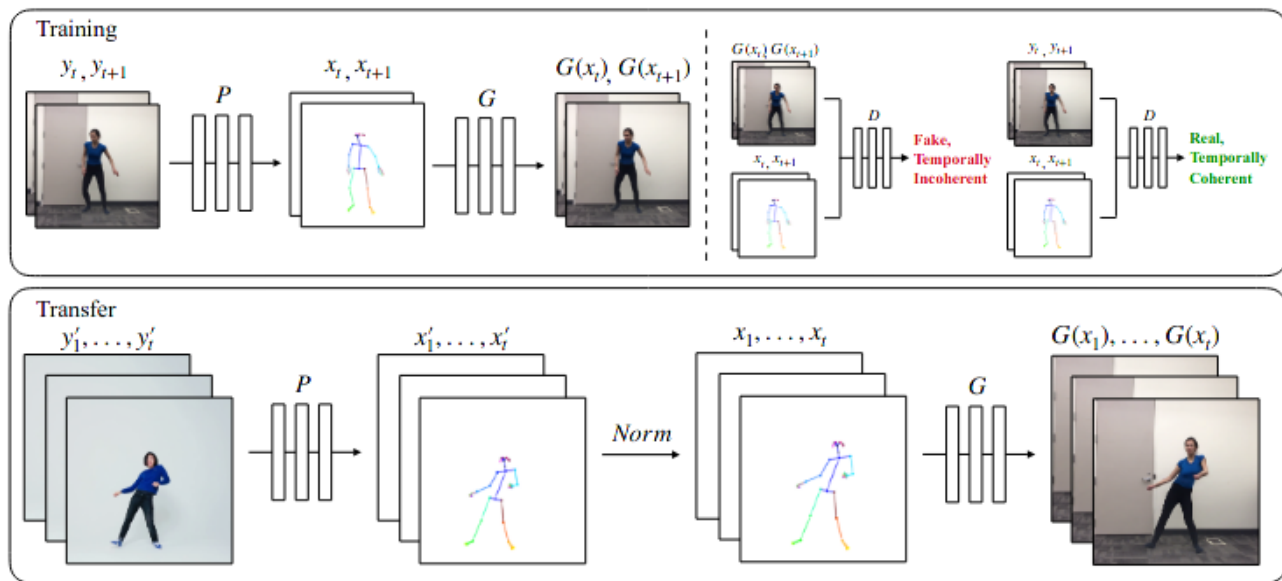


Tero Karras, Samuli Laine, Timo Aila. A Style-Based Generator Architecture for Generative Adversarial Networks. CVPR 2019

# Video Generation. Everybody Dance Now



# Video Generation. Everybody Dance Now



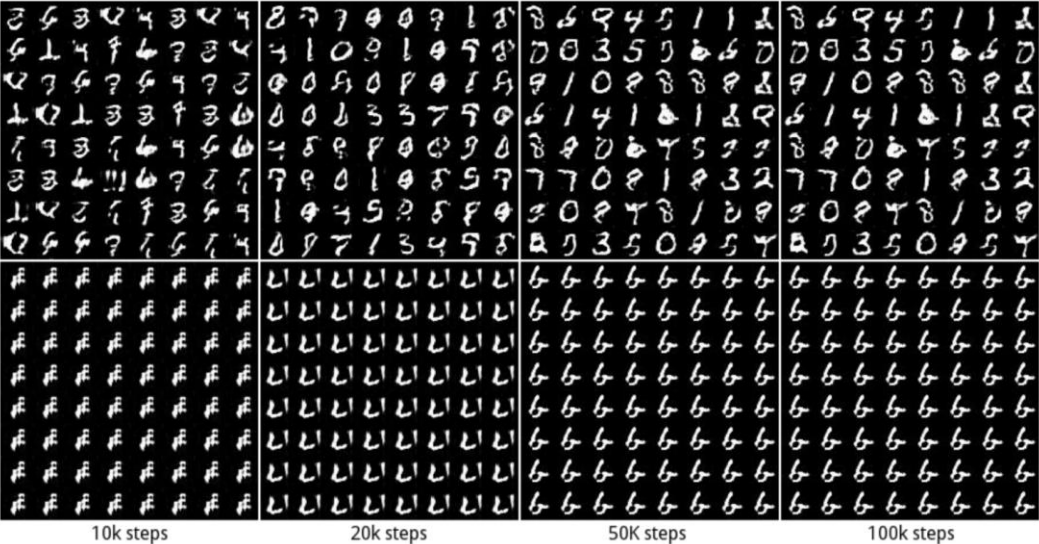
# Problems of GANs

- **Mode collapse:**
  - G collapses providing limited sample variety
- **Non-convergence:**
  - model parameters oscillate, destabilize and never converge
- **Diminished gradient:**
  - D is so successful that the G gradient vanishes and learns nothing

# Mode collapse

Real-life data is multimodal (10 in MNIST)

Mode collapse: when few modes generated



# Partial mode collapse

The generator produces realistic and diverse samples, but much less diverse than the real-world data distribution.

(A) 100 Epoch



(B) 199 Epoch



(C) 300 Epoch





# Solution to mode collapse

## Wasserstein loss

- Trains the discriminator to optimality without worrying about vanishing gradients.
- If the discriminator doesn't get stuck in local minima, it learns to reject the outputs that the generator stabilizes on.

# Generative Models Evaluation

Which of these images looks better?



# Generative Models Evaluation

Which of these images looks more realistic?



# Generative Models Evaluation

Which of these images appears to be more similar to the text prompt?



Prompt: The saying "BE EXCELLENT TO EACH OTHER" written on a red brick wall with a graffiti image of a green alien wearing a tuxedo. A yellow fire hydrant is on a sidewalk in the foreground.

# Generative Models Evaluation

- Human-based ratings and preference judgments
- Inception Score (IS) [1]
- Frechet Inception Distance (FID) [2]
- Structured Similarity Index Metric (SSIM) [3]
- CLIP score [4]

[1] Tim Salimans, Ian Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, Xi Chen. Improved Techniques for Training GANs. NeurIPS 2016

<https://arxiv.org/abs/1606.03498>

[2] Martin Heusel, Hubert Ramsauer, Thomas Unterthiner, Bernhard Nessler, Sepp Hochreiter. GANs Trained by a Two Time-Scale Update Rule Converge to a

Local Nash Equilibrium. NeurIPS 2017 <https://arxiv.org/abs/1706.08500>

[3] Zhou Wang; A.C. Bovik; H.R. Sheikh; E.P. Simoncelli. Image quality assessment: from error visibility to structural similarity. IEEE Transactions on Image Processing 2004 <https://www.cns.nyu.edu/pub/eero/wang03-reprint.pdf>

[4] Jack Hessel, Ari Holtzman, Maxwell Forbes, Ronan Le Bras, Yejin Choi. CLIPScore: A Reference-free Evaluation Metric for Image Captioning. EMNLP 2021 <https://arxiv.org/abs/2104.08718>

# Inception Score (IS)

IS measures:

- the **quality** of the generated images
- their **diversity**

# Inception Score (IS)

CIFAR10 dataset:

airplane



automobile



bird



cat



deer



dog



frog



horse



ship

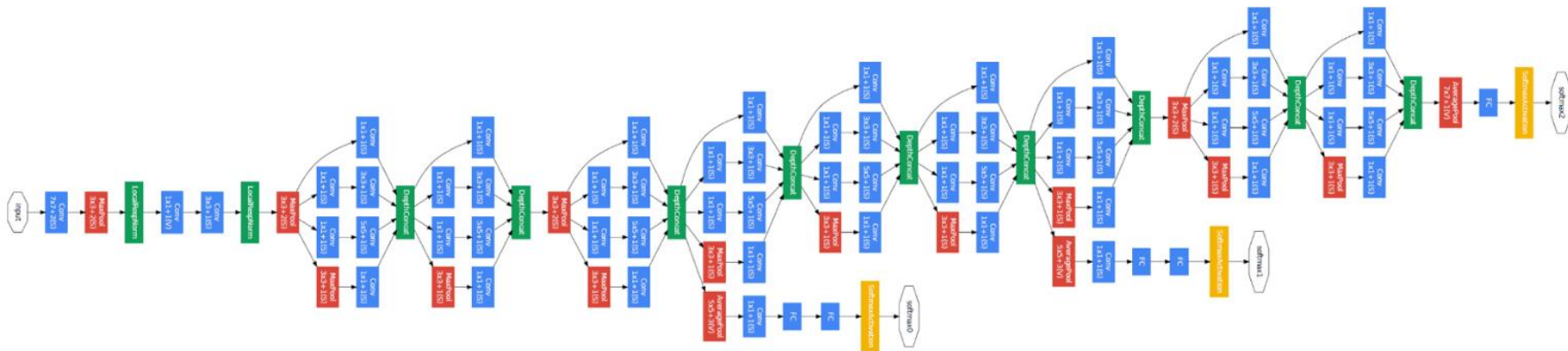


truck



# Inception Score (IS)

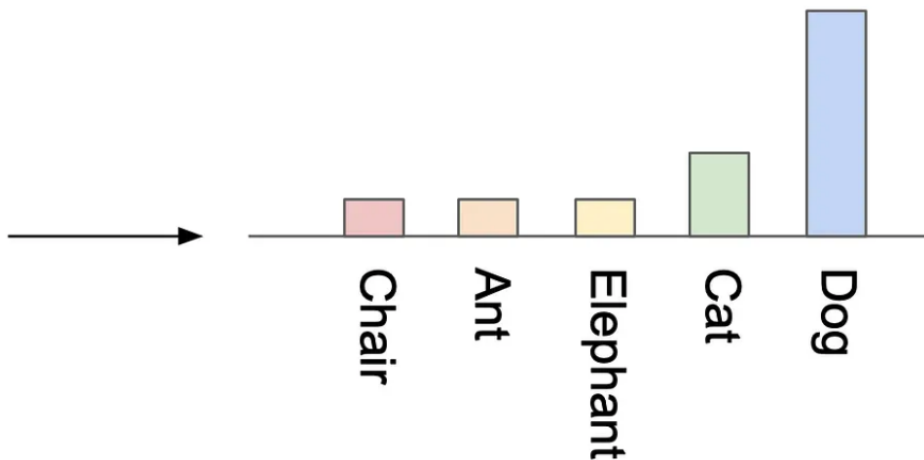
Generate 50000 images by the **Inception** image classifier pre-trained on CIFAR10





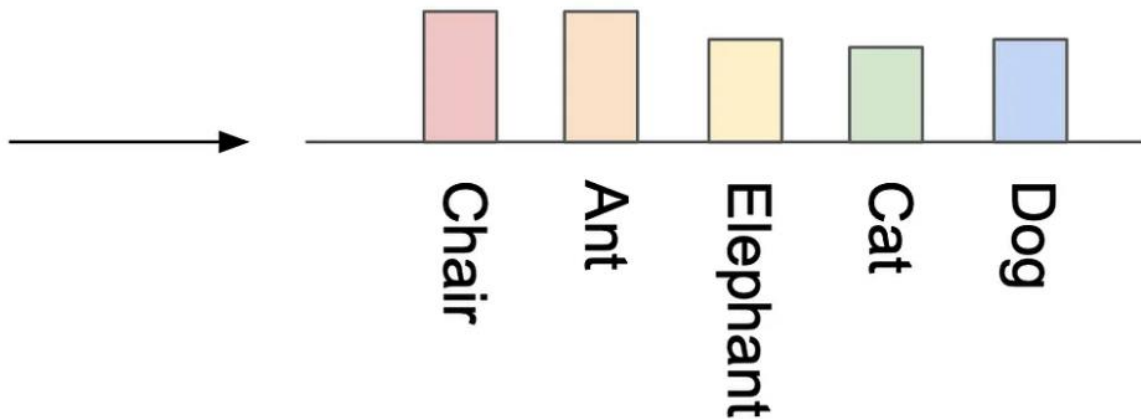
# Inception Score (IS)

Generated images are fed into the Inception image classifier network pre-trained on the CIFAR10 dataset predict conditional probability  $p(y|x)$  — where  $y$  is the label and  $x$  is the generated data



# Inception Score (IS)

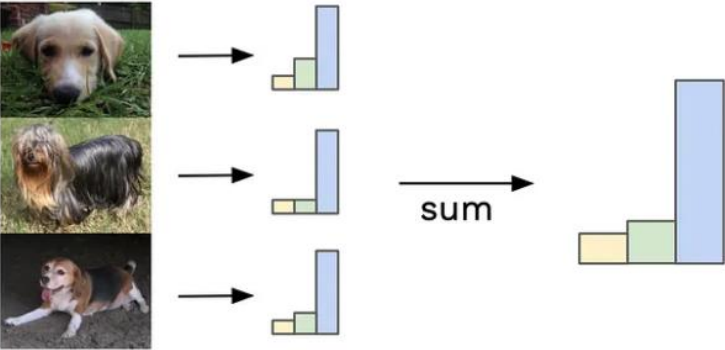
If the probability scores are widely distributed then the generated image is of low quality:



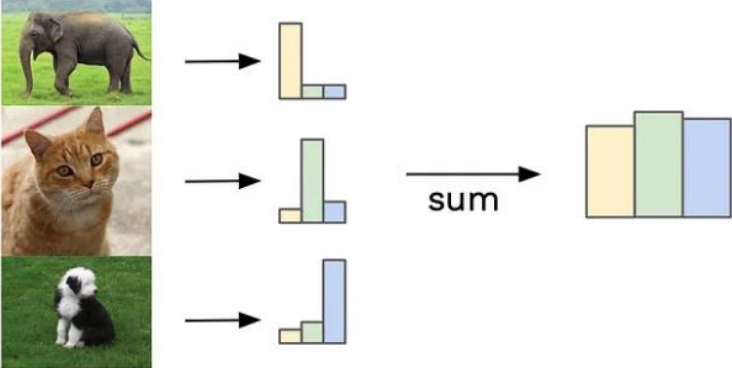
# Inception Score (IS)

Calculate marginal probability  $p(y) = \int_z p(y|x = G(z))dz$

Similar labels sum to give focussed distribution



Different labels sum to give uniform distribution



# Inception Score (IS)

- **Quality**: conditional probability  $p(\mathbf{y}|\mathbf{x})$
- **Diversity**: marginal probability  $p(\mathbf{y})$

We want

- the conditional probability  $p(\mathbf{y}|\mathbf{x})$  to be highly predictable (**low entropy**) i.e. given an image, we should know the object type easily
- the marginal probability  $p(\mathbf{y})$  to be uniform (**high entropy**)

## Inception Score (IS)

Compute their KL-divergence to combine these two criteria:

$$IS(G) = \exp(\mathbb{E}_{x \sim p_g} KL(p(y|x) || p(y)))$$

# Inception Score (IS) limitations

IS is limited by what the Inception classifier can detect, which is linked to the training data

# Frechet Inception Distance (FID)

FID compares the distribution of generated images with the distribution of real images

# Frechet Inception Distance (FID)

- Uses the **Inception network** to extract features from an intermediate layer



# Frechet Inception Distance (FID)

- Uses the Inception network to extract features from an intermediate layer
- Models data distribution for these features using a multivariate Gaussian distribution with mean  $\mu$  and covariance  $\Sigma$

# Frechet Inception Distance (FID)

- Uses the Inception network to extract features from an intermediate layer
- Models data distribution for these features using a multivariate Gaussian distribution with mean  $\mu$  and covariance  $\Sigma$
- The FID between the real images  $x$  and generated images  $g$  is:

$$FID(x, g) = \|\mu_x - \mu_g\|_2^2 + Tr(\Sigma_x + \Sigma_g - 2(\Sigma_x \Sigma_g)^{\frac{1}{2}})$$

where  $Tr$  sums up all the diagonal elements

# Frechet Inception Distance (FID)

- **Lower** FID values mean **better** image quality and diversity
- FID is sensitive to mode collapse
- FID is more robust to noise than IS. If the model only generates one image per class, the distance will be high

# Structured Similarity Index Metric (SSIM)

The Structural Similarity Index (SSIM) metric mimicks the human visual perception system which is highly capable of identifying **structural information** from a scene.

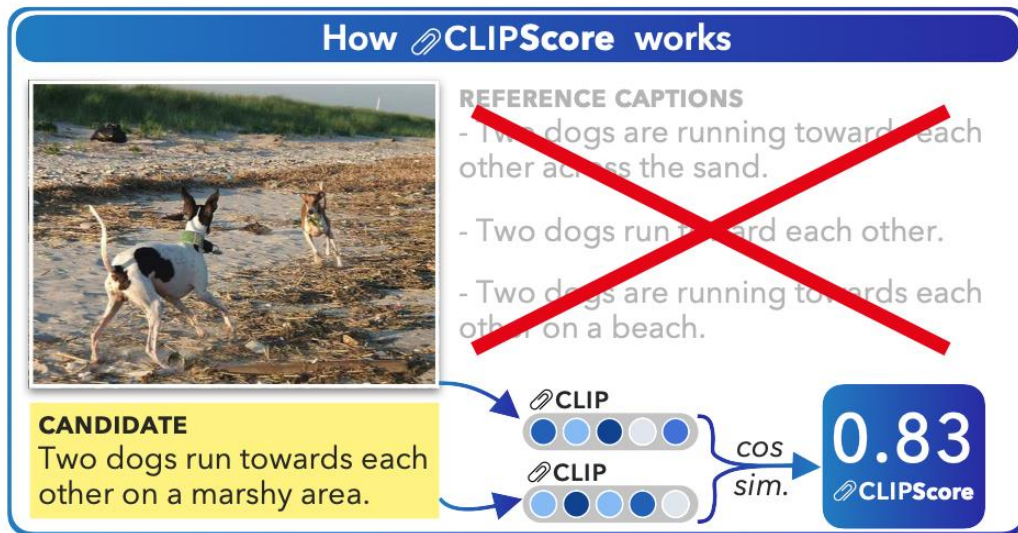
SSIM extracts 3 features from an image on a **pixel-wise** basis:

- Luminance
- Contrast
- Structure

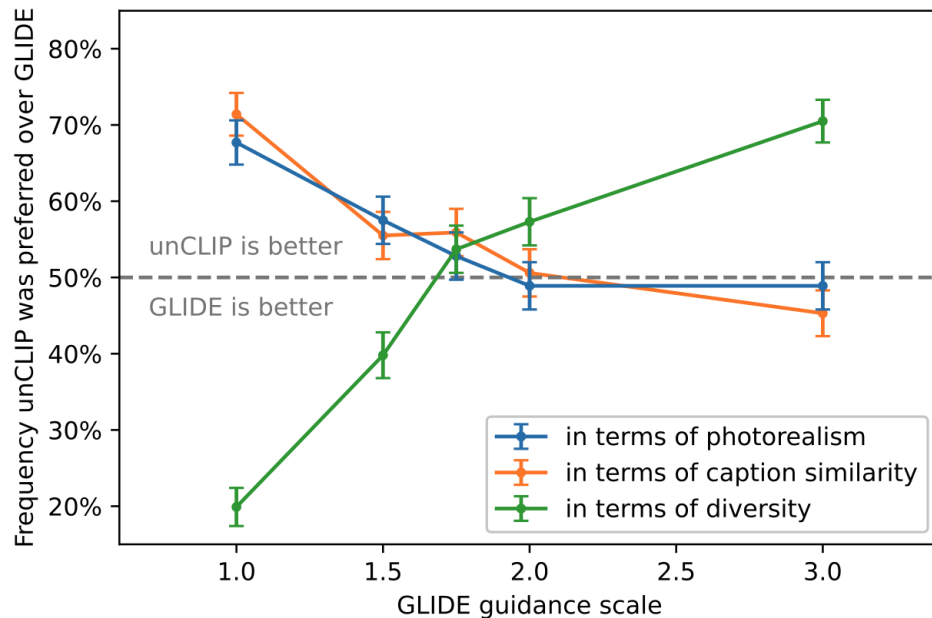
The comparison between the two images is performed on the basis of these 3 features.

# CLIP score

- CLIP score measures the compatibility of image-caption pairs.
- Higher CLIP scores imply higher compatibility



# DALL-E 2 evaluation results



Increasing guidance scale improves sample quality at the cost of diversity

Aditya Ramesh, Prafulla Dhariwal, Alex Nichol, Casey Chu, Mark Chen.

Hierarchical Text-Conditional Image Generation with CLIP Latents. 2022 <https://arxiv.org/abs/2204.06125>

<https://openai.com/product/dall-e-2>

# TP9

<https://colab.research.google.com/drive/1PYx6qESrgTmm38eCYHpkN-1UovsK4UD7?usp=sharing>

<https://colab.research.google.com/drive/1NWoFAZjLeaCMb3JsMUPJX5bm0PenCRYy?usp=sharing>

<https://colab.research.google.com/drive/1xOfQOR-XCP1uw0pOqR4nXITZU6wsXGp2?usp=sharing>

Q&A